SIX: Proposing an Alternative Skewness Index

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Abstract

This paper studies the dynamics of risk-neutral skewness in the S&P 500 index (SPX) market. We use a state-preference pricing approach to develop a forward-looking market symmetric index (SIX). SIX, as an \textit{ex ante} risk-neutral measure, estimates how negatively skewed the SPX market return distribution is in the next 30 days. Using SPX options data from 1996 to 2013, we conduct a horse race between SIX and the Chicago Board Options Exchange (CBOE) SKEW index. We study the contemporaneous relationship between the daily change of risk-neutral volatility and skewness of the index return and find a negative relationship between the risk-neutral volatility and skewness, where the former is modelled by VIX and the latter by SIX. The opposite is found between VIX and SKEW. We investigate the predictability of these \textit{ex ante} skewness measures on the one-day ahead SPX market returns. We show that only the daily change of SIX adds significant explanatory power on the one-day ahead return. The same is not found with SKEW. Furthermore, we find that SIX is a biased but efficient forecast of future physical skewness, while there is no statistically significant relationship for SKEW. Our results suggest that SIX is an important indicator of institutional anxiety regarding stock market uncertainty, and is a useful complement to the existing VIX index.

\textbf{Keywords:} Lower Partial Moment; Upper Partial Moment; Skewness Forecasting; Skewness Index; Volatility Index; Risk-Neutral Moment; SKEW; VIX; State-Preference Pricing.

\textbf{JEL:} D81, G10, G17.

1 Introduction

The first two moments of a return distribution have been extensively studied and applied in the field of asset pricing. The past six decades have also seen the mean

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and variance occupying prominent roles in both financial theory and empirical applications. The latter, as a summary statistic that takes account of deviations from both sides of the mean, however, is a poor measure of risk if the return distribution is asymmetric. Indeed, there is mounting evidence in both theoretical and empirical literature, including Kraus and Litzenberger (1976), Harvey and Siddique (2000), Ang, Chen, and Xing (2006), Xing, Zhang, and Zhao (2010), Neuberger (2012) and including the more recent Chang, Christoffersen, and Jacobs (2013), which suggests that skewness is an important factor for consideration in asset pricing and investment management.

In this paper, we study the dynamics of risk-neutral skewness in the S&P 500 index (SPX). Using a state-preference pricing approach, we develop a forward-looking market symmetric index (SIX) as a forecast of the 30-day skewness return of SPX market returns. This ex ante measure estimates how negatively skewed the market return distribution is in the coming month, and is directly comparable to the recently announced Skew index (ticker symbol: SKEW), published by the Chicago Board Options Exchange (CBOE) since February 2011. SKEW is derived from the price of SPX return skewness that measures the perceived left tail of the return distribution.

Using SPX index options data from 1996 to 2013, we conduct a horse race between SIX and SKEW among a number of hypotheses. Including periods of both bull and bear regimes as well as the recent Global Financial Crisis (GFC), this rich period enables us to uncover the dynamics of risk-neutral skewness that is more manifest given the presence of relatively rare events. First, we examine the contemporaneous relationship between the daily changes of risk-neutral volatility and skewness of the index return. There is mixed evidence in the literature. Han (2008) examines the role investor sentiment plays in the stock market and finds that the risk-neutral skewness of the index is less negative in the presence of higher market volatility.

By examining individual stock options, Dennis and Mayhew (2002) find that the implied skewness of stock options tends to be more negative in periods of high market volatility and when the risk-neutral density for index options is more negatively skewed. Conrad, Dittmar, and Ghysels (2012) find minimal relation between volatility and skewness estimates in their sample of individual securities.

We argue that as the expected market risk-neutral volatility increases, the return distribution should become more negatively skewed. Our rational is that advances in market volatility usually imply a rally in fear among investors and is often associated
with a drop in market return, reflecting a more negatively skewed market return distribution\textsuperscript{3}. Using the CBOE volatility index (VIX) as a candidate for risk-neutral volatility, our results cannot reject this hypothesis for SIX. We obtain contrary results using SKEW.

Second, we study the contemporaneous relationship between daily SPX returns and changes in our skewness measures. In particular, we investigate whether these skewness indices contain any \textit{fear} information from the options market when there is a market drop.

It has been well-documented in the literature that there is a negative correlation between the rate of change in VIX and daily market returns (see, for example, Carr and Wu (2009)). As expected market volatility increases, investors may demand a higher rate of return on stocks. Hence, stock prices fall and can ultimately lead to a drop in market returns. Similarly, as the expected market skewness decreases, investors may expect a greater chance of a significant decline in returns if the market goes down. This may lead to demands of higher rate of returns on stocks and, hence, a fall in market returns. We show that the rate of change in daily market returns is negatively proportional to the changes in SIX, while the contrary is found for SKEW. In particular, only SIX has a statistically significant asymmetric response to market drops as opposed to market advances.

Third, we investigate the predictability of these \textit{ex ante} skewness measures on future SPX market returns. Xing et al. (2010) examine individual stock options in the US market and argue that the shape of the volatility skew has predictive power for future equity returns. Further, they find that an increase in the volatility skew (i.e. a decrease in skewness) is associated with a decrease in equity returns, with the result unchanged after including other market control variables. Similar findings are documented in Rehman and Vilkov (2012). By examining individual stock options from 1996 to 2007, Rehman and Vilkov proxy the \textit{ex ante} skewness with a model-free implied skewness and show that stocks with high implied skewness outperform counterparts with low implied skewness.

The contrary empirical findings are presented in Conrad et al. (2012). The authors estimate the \textit{ex ante} higher moments of the underlying stocks risk-neutral return distributions using the method suggested in Bakshi, Kapadia, and Madan (2003). They find that more negatively (positively) \textit{ex ante} skewed returns are associated with subsequent higher (lower) returns. Note that they average the daily estimates of risk-neutral moments for each stock over time, and thus each firm in their sample has a single observation for volatility, skewness and kurtosis for each stock.

\textsuperscript{3}VIX is known as the \textit{investor fear gauge} due to its spikes during periods of market turmoil. Whaley (2009) documents that changes in VIX increase at a higher absolute rate when the stock market declines compared to when it rises.
maturity (of one, three, six and twelve months) from 1996 to 2005\textsuperscript{4}.

This finding from Conrad et al. (2012) is further supported by Bali and Murray (2012). They investigate whether risk-neutral skewness is able to predict the cross-section of equity option portfolio returns, by creating skewness assets comprised of two positions in the option and one position in the underlying stock. These delta neutral and vega neutral skewness assets are designed to increase in value if risk-neutral skewness increases. A strong negative relation between the skewness asset returns and risk-neutral skewness is found. As they create the delta neutral and vega neutral skewness assets on the second trading day following each monthly option expiration and hold them to expiration, the risk-neutral moments of their skewness assets are inevitably assumed to be constant over the holding period. However, this assumption contradicts the initial premise that risk-neutral moments are evolving. It is possible that these skewness assets are in fact not immune from delta and vega shocks, and that the results cannot be interpreted fully without assessing the impact from such possibilities.

In this paper, we argue that a decrease in risk-neutral skewness (an increase in SIX and SKEW) may imply that more negative returns are expected. There could be a negative relationship between the daily changes of SIX and SKEW with that of one-day ahead SPX returns (or equivalently, a positive relationship between the \textit{ex ante} skewness and the expected return). Our findings with SIX support this hypothesis and show that the more negatively skewed the risk-neutral distribution is, the lower the future returns are in the SPX market. We find the model with SKEW does not have any explanatory power on the future return.

Fourth, we investigate whether these \textit{ex ante} risk-neutral skewness measures are reasonable proxies for the physical return skewness of SPX returns in the following month. We find that SIX is a biased but efficient forecast of future physical skewness, while there is no statistically significant relationship for SKEW. As discussed in Neuberger (2012), the physical realized skewness is hard to measure precisely compared to the first two moments of the return distribution. Merton (1980) and Harvey and Siddique (2000) show that the estimates of higher moments require a long time series and are likely to be measured with errors. This is further discussed in the study by Bakshi et al. (2003) that, after with using sample sizes of 350, 400 and 450 days, the quality of the estimation is improved with a larger sample size. Our physical monthly realized skewness measure is computed using high-frequency intra-day return data, which partially resolves this problem by using more data points in each sample period.

However, it is important to understand that there is no clear theory or empirical

\textsuperscript{4}In this context, this averaging may have removed the time variation in these risk neutral moments and hence the results may not be directly comparable to ours.
findings suggesting any relationship between measures from these two frameworks. As articulated in Carr and Wu (2009), the implied density generally differs substantially from the density under the physical measure. In this respect, this investigation is explorative rather than explanatory. As a publicly accessible market index, SKEW is available to investors with little cost. From an investor’s point of view, it is critical to learn and determine whether such a skewness index provides any information about the physical skewness in the next 30-day period.

This paper emphasises the importance of higher-moment risk in asset pricing and investment management, and contributes to the literature in a number of ways. First, to the author’s best knowledge, this is the first study to examine the behaviour of the recently published CBOE SKEW index. By comparing SKEW to an alternative measure of risk-neutral skewness, we show that SIX provides more meaningful information than SKEW in terms of its relationship with VIX, and its contemporaneous relationship with daily market returns and predictability of future returns. Thus, this possibly explains why SKEW has not received the popularity and attention enjoyed by the CBOE VIX index.

Second, the findings from this paper present a fundamental concern about the underlying methodology of SKEW. The theoretical basis of SKEW originates from the model-free risk-neutral moments in Bakshi et al. (2003). However, as one translates the theoretical model to equations for empirical use with real world data, approximation errors are inevitably introduced. One such problem includes approximating an integration of an infinite number of strike prices by the available amount for one security traded in the market. This approximation error is more pronounced as the amount of strike prices available decreases. In other words, if SKEW fails to explore and deliver the risk-neutral skewness information from the SPX options market, the findings of the risk-neutral skewness in individual stock options become questionable.

For example, in a recent study by Friesen, Zhang, and Zorn (2012), the average number of out-of-the-money (OTM) puts/calls used for estimating the Bakshi et al. (2003) measure of skewness is 3.4. It is not difficult to see that an entire return distribution could hardly be approximated by 4 data points (option strike prices) without any bias. There are numerous recent studies that share this concern from Friesen et al. (2012). In this paper, we do not attempt to improve the Bakshi et al. (2003) method; rather, we provide an alternative way to estimate risk-neutral moments by using a state-preference pricing approach with Black-Scholes analytical derivatives as state prices.

The remainder is organised as follows. In Section 2, we review the methodology to develop the CBOE SKEW index and discuss potential drawbacks with its methodology. Section 3 presents the data and methodology used to construct SIX.
Section 4 compares the performance of SIX and CBOE SKEW as a risk-neutral skewness forecast against various hypotheses of this paper. In addition, another candidate for the skewness index, the third moment index (TIX), is discussed to ensure robustness of our results. Section 5 concludes.

2 CBOE SKEW Index

2.1 Index Construction

CBOE started publishing SKEW on a real-time basis throughout each trading day from February 23, 2011. It has become the benchmark measure for perceived future tail risk of the SPX return distribution. As outlined in the CBOE SKEW Index White Paper (CBOE, 2010), SKEW is a model-free measure of future risk-neutral skewness of market returns. It is an option-based indicator that is constructed using prices of SPX options.

Its methodology is based on the idea proposed by Bakshi and Madan (2000) and Bakshi et al. (2003) that any security payoff can be spanned and priced using an explicit positioning across option strikes. Similar to other indices provided by CBOE such as VIX and VXN, SKEW is not directly investable or tradable. Currently, there are no derivatives trading for SKEW nor any instrument planned for the near future.

The CBOE SKEW is defined as

\[
\text{SKEW} = 100 - 10 \times S
\]

(1)

where \( S \) is the market price of the risk-neutral skewness payoff. \( S \) is transformed for two main reasons: first, unlike most other market indices, such as SPX and VIX, \( S \) tends to be constantly negative. Indeed, from the sample provided by CBOE, there is no single positive observation in \( S \) from 1990 to 2013. Thus, the negative figures of SKEW could be confusing and misleading when compared with other indices. Second, \( S \) tends to vary within a narrow range. For example, between 1990 and 2013, \( S \) ranges from -0.10 to -4.69. Therefore, scaling is required to make the readings more presentable and meaningful when quoted in the media.

Given equation (1), SKEW represents how negatively skewed the market risk-neutral distribution is. That is, a decrease in the expected risk-neutral skewness (equivalently a decrease in \( S \)) means an increase in SKEW. This is further discussed when comparing SKEW with SIX in Section 3.

\( S \) is estimated as an expectation of the power payoffs of the 30-day logarithmic returns of SPX, \( R \), \( R^2 \) and \( R^3 \); where \( R \) is the 30-day log-return of SPX. Theoretically, \( S \) is the price of a portfolio of SPX options that intends to mimic a pure
exposure to a skewness payoff. This formula is:

\[ S = \frac{E_Q(R^3) - 3E_Q(R)E_Q(R^2) + 2E_Q^3(R)}{(E_Q(R^2) - E_Q^2(R))^{3/2}} =: \frac{P_3 - 3P_1P_2 + 2P_1^3}{(P_2 - P_1^2)^{3/2}} \]  

(2)

where \( E_Q \) represents the expectation performed in the risk-neutral world. The \( P_i \) variables are approximated as:

\[ P_1 = \sum_i \frac{-\Delta K_i}{K_i^2} e^{rT} Q(K_i) \left( 1 + \ln \left( \frac{F_0}{K_0} \right) - \frac{F_0}{K_0} \right) \]  

(3)

\[ P_2 = \sum_i \frac{2\Delta K_i}{K_i^2} e^{rT} Q(K_i) \left( 1 - \ln \left( \frac{K_i}{F_0} \right) \right) + 2 \ln \left( \frac{K_0}{F_0} \right) \left( \frac{F_0}{F_0} - 1 \right) + \frac{1}{2} \ln^2 \left( \frac{K_0}{F_0} \right) \]  

(4)

\[ P_3 = \sum_i \frac{3\Delta K_i}{K_i^2} e^{rT} Q(K_i) \left( 2 \ln \left( \frac{K_i}{F_0} \right) - \ln^2 \left( \frac{K_i}{F_0} \right) \right) + 3 \ln^2 \left( \frac{K_0}{F_0} \right) \left( \frac{1}{3} \ln \left( \frac{K_0}{F_0} \right) - 1 + \frac{F_0}{K_0} \right) \]  

(5)

where \( T \) is the time to expiration\(^5\), \( F_0 \) is the forward index level for maturity \( T \) derived from SPX option prices, \( K_i \) is the strike price of the \( i \)th OTM option, \( \Delta K_i \) is the strike price interval\(^6\), \( r \) is the risk-free interest rate to expiration, \( Q(K_i) \) is the average of the highest bid price and lowest ask price for each option at \( K_i \) and \( K_0 \) is the strike price immediately below the forward index level \( F_0 \). \( \varepsilon_j \) \((j \in \{1, 2, 3, \})\) is an adjustment term to compensate for the difference between \( K_0 \) and \( F_0 \) (CBOE, 2010). They are also used in calculating CBOE VIX.

Rarely are there options with maturities of exactly 30 days. Therefore, to determine the 30-day measure of expected skewness, \( S_j \)'s from two maturities around 30 days are inter- or extra-polated, as such:

\[ S = S_1 \left[ \frac{N_{T_2} - N_{30}}{N_{T_2} - N_{T_1}} \right] + S_2 \left[ \frac{N_{30} - N_{T_1}}{N_{T_2} - N_{T_1}} \right] \]  

(6)

where \( N_x \) denotes the number of minutes to some future date \( x \) and \( T_1 < T_2 \).

In this paper, we compare two different methods of constructing risk-neutral

\(^5\)Since August 24, 1992, SPX options expire at the open on the third Friday of the contract month. \( T \) is the summation of minutes remaining until midnight of the current day, total minutes in the calendar days between current day and settlement day and minutes from midnight until 8:30 a.m. (Chicago Time) on SPX settlement day.

\(^6\)\( \Delta K_i = (K_{i+1} - K_{i-1})/2 \) or \( \Delta K_i = K_{2\text{nd lowest}} - K_{\text{lowest}} \) or \( \Delta K_i = K_{\text{highest}} - K_{2\text{nd highest}} \).
skewness measures. Since SIX and the realized skewness measure do not have any artificial scaling, we scale SKEW down for a more consistent comparison in terms of magnitude. SKEW is only available from CBOE and we define SKEW* in this study as such:

\[
SKEW^* = \frac{SKEW - 100}{10} = -S
\]  

This essentially eliminates the scaling but retains the sign effect of SKEW. Hence, the statistical result of SKEW can be directly interpreted as that of SKEW*. In other words, SKEW* describes how negatively skewed the market risk-neutral distribution is without scaling. For simplicity, we rename SKEW* to SKEW for the rest of the paper, unless otherwise stated.

The idea of extracting market risk-neutral skewness from traded options has been well discussed in the literature. Prior to SKEW, many recent studies, including Dennis and Mayhew (2002), Han (2008), Atilgan, Bali, and Demirtas (2010), Bali and Murray (2012), Neumann and Skiadopoulos (2012), Conrad et al. (2012), adopt the model-free skewness measure from the seminal paper by Bakshi et al. (2003). Note that although the same principle and methodology are applied in both CBOE SKEW and a recent study by Neumann and Skiadopoulos (2012), the result from the latter cannot be interpreted the same way as in SKEW. As outlined in their paper, rather than using the raw SPX options data, new options are created by interpolating across the implied volatilities with a cubic smoothing spline.

### 2.2 Drawbacks of SKEW

Despite the appeal of the model-free SKEW, there are a number of potential drawbacks worth noting. Many of these concerns are shared with the current VIX methodology.

First, there are several types of approximation errors from transforming the BKM’s model-free skewness formula to the S formula in equation (2), including discretization and truncation errors. Discretization errors arise from approximating the numerical integrations with discretized summations, while truncation errors are induced by truncating the infinity range of strike prices to a finite range restricted by the limited trading data. Since the lowest and highest strike price vary over time, the truncation error may change substantially.

In a study of VIX by Jiang and Tian (2005), these errors are shown to be negligible if the downside and upside limits span at least three standard deviations from the at-the-money (ATM) level. Similar critiques are also seen in Dennis and Mayhew (2002). Thus, it is reasonable to assume the truncation error to be less prominent when the SPX is more volatile, where deeper-OTM options are more
heavily traded. Dennis and Mayhew (2002) suggest the use of the largest range of strike prices at each time such that the domain of integration is symmetric. In data filtration, they suggest that there should be at least two calls and two puts for each maturity. This last criterion has become the standard requirement in the current literature.

To understand how these errors may have a role in distorting SKEW, we present two groups of statistics here. Table 1 reports the summary statistics of numbers of OTM SPX option strikes in the SKEW calculation\(^7\). Our daily SPX option quotes are obtained from the Ivy DB database of OptionMetrics\(^8\). This sample covers the period from January 4, 1996 to August 30, 2013. This dataset is further discussed in Section 3.3.

According to CBOE (2010), on any given day, there are two series of SPX options with different maturities chosen for SKEW calculation, namely a near-term series and a next-term series. For both series, it is apparent that on average the number of OTM put option strikes is more than double of that of OTM call options. This is in line with findings from Bollen and Whaley (2004) that there are more demands for the OTM index put options for insurance purpose. Another interesting observation is that while there is a general upward trend in the number of option strikes, these numbers peak around 2011 and slightly drop down in the last two years. It is critical to understand that the drop in the number of option strikes do not necessarily translate to a decrease in trading volume of those options. All it says is that there has been a decrease in demands for the more extreme OTM options.

A more worrisome observation from the above table is the minimum number of option strikes. For example, in the near-term series, the minimum number of OTM put option strikes is 2 out of all business days in 2007, which implies that on that particular day, there are only two OTM put options available for calculating SKEW in equation (5). This suggests that both truncation and discretization errors could have serious impacts on approximating the true SKEW.

Table 2 reports the summary statistics of strike price ranges of OTM SPX options in SKEW calculation. Bottom is defined as a ratio of the lowest strike price of OTM

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\(^7\)Since OptionMetrics only provides option quotes at the end of each trading day, there could be potentially some inconsistencies comparing to ones that are actually used by CBOE for calculating SKEW.

\(^8\)It is important to note that OptionMetrics do not distinguish the monthly SPX options series from the weekly and quarterly counterparts. It is understood that CBOE only uses SPX monthly series in VIX and SKEW calculations. For consistency, we remove all weekly and quarterly SPX options.
put options over the SPX index level on one day. Top is defined as a ratio of the highest strike price of OTM call options over the SPX index level on that day. It is important to note that the lowest strike price of OTM put options is one that with positive bid price and it is not beyond two consecutive OTM put options with zero bid price. The same is also held for the highest strike price of OTM call options. More discussion on the data filtration can be found in CBOE (2010).

By comparing the mean statistics, it shows that OTM put options span bigger ranges than OTM call options, as seen from the mean of the bottom is much further away from 1 than that of the top. This is consistent with what we find in Table 1 that OTM put options are more demanded. The minimum of the bottom is 0.21 from one of 252 trading days in 2008. The maximum of the top is 1.59 from that same year.

What could potentially cause problems is when either the bottom or the top is close to 1. This would pose a serious concern on truncation errors. For example, the maximum of the bottom is 0.94 from one of trading days in 2007. This suggests that on that particular day, the market option prices could only provide 6% of information to span the entire distribution on the left-hand side of the forward index level.

The second potential drawback with the SKEW methodology is that, the linear interpolation of near and next term maturities to form a fixed 30-day measure may induce a bias, if the model-free skewness does not follow a linear function of maturity. In fact, the current literature shows that the term structure is neither linear nor monotonic.

Third, the CBOE uses the average of the lowest ask price and the highest bid price as a proxy for the option price. As documented in the literature (e.g., Figlewski (2010)), bid-ask spreads are quite wide for options that are less liquidly traded. The mid-point of a wide spread may bias the true information about the implied volatility at that particular strike price level. This impact is more substantial for OTM SPX calls than OTM puts, as the latter is more heavily traded as portfolio insurance.

Fourth, a close examination of the $P^i$ formulas from equations (3) to (5) reveals that an OTM put option receives more weight than an OTM call option, even at the same moneyness level$^9$. In the original set up of BKM, where an infinite number of strike prices are assumed, this problem is less of a concern because an OTM put option is cheaper than an equal OTM call option in the standard Black model (Carr & Lee, 2009). However, as there are only a finite number of strike prices available even for the most liquidly traded SPX options, this bias towards OTM put options may lead to a bias in SKEW and an overestimation of left-tail risk.

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$^9$By definition, the strike price of an OTM put option is lower than an OTM call option. Combined with the fact that the weight $1/K_i^2$ in equation (5) is a decreasing function of $K$, more weights have been placed on OTM put than OTM call options.
Fifth, the criterion for being included in the pool of OTM options for the calculation of SKEW does not consider trade volume. As described in CBOE (2010), an OTM option is selected as long as it has a non-zero bid price and not beyond two consecutive strike prices with zero bid prices. Thus, a possible SKEW manipulation strategy may involve placing higher bids (to increase the average of the bid-ask spread) across a range of OTM options, or even placing bids on deep OTM options (that originally have zero bids and would otherwise be excluded from the SKEW calculation) for them to be updated in the SKEW calculation. This strategy may be more effective by trading OTM put options, because mechanically higher weights are allocated to OTM puts. Although SKEW is not directly investable nor is there any derivative instrument written on it currently, this would be a problem and concern if futures or options on SKEW become tradable in the future.

In this paper, we do not attempt to address all the aforementioned problems to improve SKEW. Instead, we propose an alternative measure, based on a different methodology, and examine whether it outperforms SKEW as a forward-looking risk-neutral skewness measure.

3 Proposing a Simple Solution: SIX

Rather than improving on the methodology used to construct SKEW, we propose a simple alternative: the market symmetric index (SIX). We first review the state-preference pricing framework and discuss the logic used to create SIX, followed by the data description and methodology employed in this paper. Using the same methodology as Liu (2014a), we reproduce relevant measures to construct SIX. Some of the discussion below are also presented in Liu (2014a, 2014b).

3.1 State-Preference Pricing

There are three basic constraints in asset pricing models, namely the absence of arbitrage, single-agent optimality and market equilibrium. The unifying implication from these three constraints is the existence of a state price, also known as a positive discount factor, for each future state, such that the price of any asset is the state price weighted sum of its future payoffs (Duﬃe, 2001). This concept originates from the general equilibrium model of security markets in Arrow and Debreu (1954).

Arrow (1964) and Debreu (1959) propose that the fundamental value of any financial asset should be the sum of future payoffs multiplied by state prices. This infers that, given the market price of an asset and its future payoffs, one can estimate the fair discount rate to match the fundamental value with the observed price.

In the state-preference framework, uncertainty is modelled by assuming that
there are a number of different states \( s = \{1, \ldots, S\} \) in each future period of the world. Each investor has their own assessment of the probability that a particular state will occur in a given time period. The main result in this framework is that the price of a security can be expressed as a simple function of the payoff in each state and the corresponding state price. This follows the results presented in Harrison and Kreps (1979) and Harrison and Pliska (1981).

In a *complete* market setting, the stochastic discount factor can capture investors’ marginal rate of substitution between current consumption and consumption in some future state \( s \) at time \( t \). In this regard, one may argue that the entire risk-neutral distribution of the underlying asset can be extracted, given the market price of a call or put option. As discussed in Ross (1976), even simple options can span the whole state space and thus the price of any option contains information on state prices.

A substantial amount of literature (e.g., Merton (1973), Breeden (1979) and Cox, Ingersoll, and Ross (1985)) incorporates state prices in modern asset pricing models and, with the stream of research pioneered by Breeden and Litzenberger (1978). This study shows that the price of an elementary security, or the state price, may be modelled as the second derivative of a call or a put option price. Building on this work, Rubinstein (1994, 1998) proposes an alternative method to infer state-contingent prices from option prices. Similar extensions are found in Derman and Kani (1994, 1998) and Derman, Kani, and Chriss (1996).

Britten-Jones and Neuberger (2000) was a breakthrough study in this field, which shows that, under diffusion assumptions, option prices fully specify the risk-neutral integrated returns variance between the current date and some future date. We refer to Jiang and Tian (2005) and Carr and Wu (2009) for a recent discussion on the application of this approach.

The basic form of the state-preference pricing equation is:

\[
P_t = \sum_{s=1}^{S} (\Phi_{s,t+1} d_{s,t+1})
\]

where summing over all possible states \( \{1, \ldots, S\} \), \( P_t \) is the price of some asset at time \( t \), \( \Phi_{s,t+1} \) is the state price and \( d_{s,t+1} \) is the payoff of this asset at state \( s \) and time \( t + 1 \). Defining the state payoff \( d_{s,t+1} \) is discussed in the next section.

Breeden and Litzenberger (1978) show that the state price can be modelled as the second derivative of a call or a put option price, such that:

\[
\Phi(T, \ldots) = \frac{\partial^2 C(K, T)}{\partial K^2} = \frac{\partial^2 P(K, T)}{\partial K^2}
\]
In the risk-neutral framework, the put option price can be defined as:

\[ P = \int_0^K e^{-rT}(K - S_T)f(S_T) dS_T \]  

(10)

where \( f(S_T) \) is the risk-neutral probability density function. Note that, when the asset price dynamic is discrete rather than continuous, the above equation can be approximated as

\[ P = \sum_s e^{-rT}(K - s)p(s) \]  

(11)

where \( p(\cdot) \) is the risk-neutral probability of asset price being \( s \in (0, K) \).

We take the partial derivative in the first equation with respect to the strike price \( K \):

\[
\frac{\partial P}{\partial K} = \frac{\partial}{\partial K} \left\{ \int_0^K e^{-rT}(K - S_T)f(S_T) dS_T \right\} \\
= e^{-rT} \left\{ (K - K)f(K) + \int_0^K f(S_T)dS_T \right\} \\
= e^{-rT} F(K)
\]

where \( F(\cdot) \) is the risk-neutral distribution function. Next, we take the second derivative with respect to strike price \( K \):

\[
\frac{\partial^2 P}{\partial K^2} = \frac{\partial}{\partial K} \left\{ e^{-rT} F(K) \right\} = e^{-rT} f(K)
\]

Rearranging the above function, it gives:

\[ f(K) = e^{rT} \frac{\partial^2 P}{\partial K^2} \]

By examining the put-call parity, it can be shown that for call options, the same relationship is held:

\[ f(K) = e^{rT} \frac{\partial^2 C}{\partial K^2} \]

The problem is reduced to estimating the second derivative of option prices with respect to strike price. In this paper, we consider both model-based and model-free approaches to estimate state prices. While both provide similar estimates in a controlled environment, the model-based approach is chosen for its ability to avoid the negative state price issue in the data.

In the model-free approach, the state price is computed as the mathematical

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10 By examining at the portfolio construction of butterfly spreads, an alternative derivation is presented in Appendix A.
approximation of the second derivative:

\[ \phi_i = e^{rT} \frac{\partial^2 Q_i}{\partial K^2} \approx e^{rT} \frac{(Q_{i-1} - Q_i) - (Q_i - Q_{i+1})}{(\Delta K)^2} = e^{rT} \frac{Q_{i-1} - 2Q_i + Q_{i+1}}{(\Delta K)^2} \]  

where \( Q_i \) represents a call or a put option price. More precisely, the above equations can be extended using the Taylor series (Eberly, 2008):

\[ \phi_i = e^{rT} \frac{\partial^2 Q_i}{\partial K^2} \approx e^{rT} \frac{-Q_{i-2} + 16Q_{i-1} - 30Q_i + 16Q_{i+1} - Q_{i+2}}{12(\Delta K_i)^2} \]  

These can be directly estimated using observed SPX option prices. To analyse the approximation errors, Liu (2014b) performs simulations to estimate the forward-looking volatility using the model-free approach. Although the model-free approach gives reasonable estimates of true volatility with simulated data; it fails to accord exactly with real world SPX options data, as shown in three phenomena.

First, option prices for deep OTM options are occasionally equal across a consecutive series of strikes and thus a zero state price is obtained. This is more common for less volatile trading days, where less trades/quotes are observed in each end of the strike series.

Second, irrational bids are seen in deep OTM options. For example, an OTM put with lower strike price has higher average of bid-ask spread than another OTM put with higher strike price. Theoretically the former is strictly less than the latter. This would return a negative state price.

Third, even when the price of an OTM put option is an increasing function of its strike price, it is still possible to obtain a negative state price whenever \( P_{i-1} - 2P_i + P_{i+1} < 0 \). It is worth noting that this does not necessarily lead to an arbitrage opportunity as the price of an option is defined as the average of the highest bid and lowest ask.

Given the above failures of the model-free approach, we adopt a model-based approach. In the Black-Scholes framework, Breeden and Litzenberger (1978) show that the state price may be approximated by the ‘delta security’, as shown:

\[ \Phi(K_i, K_{i+1}) = e^{-rT} (N(d_2(K_i)) - N(d_2(K_{i+1}))) \]  

where \( K_i < K_{i+1} \) and

\[ d_2(K) = \frac{\ln(S_0/K) + (r - d - \sigma^2/2)T}{\sigma \sqrt{T}} \]  

where \( d \) is the dividend yield. The key input in equation (15) is the volatility parameter \( \sigma \), which is estimated as the average of the Black-Scholes implied volatilities.
from two ATM call and put options from two maturities that are closest to a 30-day period (i.e. an average of four implied volatilities). A substantial amount of empirical studies document that ATM Black-Scholes implied volatility is an efficient forecast of subsequent realized volatility.

Latane and Rendleman (1976) study the volatilities implied by call option prices on individual stocks. They show that the implied volatilities predict future volatility better than other predictors based on historical stock price data.

By using the constant proportional dividend yield model in Merton (1973) as an extension on Latane and Rendleman (1976), Chiras and Manaster (1978) find that implied variances are better predictors of future stock return variances than those based on historic stock price information. They develop a trading strategy from informational content of the implied variance and show that it produces abnormally high returns.

Beckers (1981) reaches a similar conclusion by extending the analysis to include dividends and optimal weighting schemes of different options on the same stock. Day and Lewis (1988) study the volatility of the stock market around the quarterly expirations of stock index futures contracts and non-quarterly expirations of stock index options. All call options on the Major Market Index, the New York Stock Exchange Composite Index and S&P 100 Index from 1983 and 1986 are examined. They show that the behaviour of implied volatilities reflects increases in the volatility of stock indices around expiration dates.

By investigating implied volatilities of both call and put options on the S&P 100 index, Fleming (1998) finds that the implied volatility dominates historical volatility in terms of ex ante forecasting power. He studies the forecast error from the implied volatility and shows that it is orthogonal to parameters linked to conditional volatility, such as those found in ARCH specifications. Christensen and Prabhala (1998) reach the same conclusion in its study of the S&P 100 index market.

Similarly, positive evidence of the predictability of ATM implied volatilities is found in the currency market. Jorion (1995) examines implied volatilities of short-term ATM options on the German deutsche mark, the Japanese yen and the Swiss franc, and finds that implied volatility outperforms time-series forecasts, such as a moving average and GARCH estimate, in terms of both informational content and predictive power.

The other concern is that the true stochastic price movements may be better characterised by stochastic volatility and jumps. Bates (2000) argues that the original Black-Scholes implied volatilities are very similar to those computed when considering jump risk. Collectively these studies support the empirical use of implied volatility as a suitable input in our state prices.

In a related study by Liu (2014b), a state-preference pricing approach is applied
with Black-Scholes analytic second derivatives to develop a forward-looking state-price volatility index (SVX), as a forecast of the next 30-day market risk-neutral volatility. Using SPX option prices from 1996 to 2013, Liu finds that SVX is 99% correlated with VIX, and is a better estimator for the short-term realized volatility of SPX returns.

### 3.2 Methodology

SIX is a ratio of the lower partial moment (LPM) volatility to the upper partial moment (UPM) volatility. Our intuition is the following: a unimodal symmetrical distribution is defined as a distribution that has the mean (if defined), median and mode coinciding at the same point. A common example is the popular normal distribution used in financial modelling. One feature of a symmetrical distribution is that two sides of the distribution should mirror each other when a vertical line is drawn through the median. In other words, if the risk-neutral distribution is symmetric (i.e., the left-hand side of median point in one distribution is the same as the right-hand side), the ratio of its LPM volatility to the UPM volatility should be equal to 1. We define SIX as

\[
SIX = \frac{BEX}{BUX} \tag{16}
\]

where the Bear Index (BEX) and the Bull Index (BUX) is the lower and upper partial moment volatility index of market returns, respectively (Liu, 2014a). If the risk-neutral distribution is negatively (positively) skewed, SIX should be greater (less) than 1\(^{11}\).

Using the same methodology from Liu (2014a), we reproduce BEX and BUX to compute SIX measure. Much of the discussion below are also presented in Liu (2014a). The definition of the \(n^{th}\) order LPM, or LPM degree \(n\), for some continuous distribution \(F\) is given as the following (Bawa & Lindenberg, 1977):

\[
LPM_n(h; F) \equiv \int_{-\infty}^{h} (h - R)^n dF(R) = \mathbb{E}(\min(0, R - h)^2)|_{n=2} \tag{17}
\]

Clearly, the LPM volatility is the LPM degree 2. With similar analogy, the \(n^{th}\) order upper partial moment (UPM), or UPM degree \(n\), of some continuous distribution \(F\)

\(^{11}\)Alternatively, this ratio could be defined as BUX/BEX. Since this ratio measures the right-hand side of the distribution over the left-hand side, it increases as the return distribution becomes more positively skewed. Since we are comparing SIX directly with SKEW and SIX measures how negatively skewed the distribution is, it makes more sense to adopt BEX/BUX as the definition for SIX.
is defined as the following:

$$\text{UPM}_n(h; F) \equiv \int_h^\infty (R - h)^n dF(R) = \mathbb{E}((\max(0, R - h))^2)|_{n=2}$$  \hspace{1cm} (18)

The target rate of return $h$ can be set to any arbitrary threshold for different purposes in each study. Popular candidates for $h$ include, 1) 0 for the avoidance of losses; 2) $R_f$ for the avoidance of returns below the risk-free rate; and 3) mean return rate $E(R)$ for comparison with the original mean-variance analysis. Psychological studies show that there is no clear determination of the LPM threshold (Mao, 1970; Unser, 2000; Veld & Veld-Merkoulova, 2008).

In a related study, Liu (2014a) examines BEX and BUX as the upper- and lower-partial moment volatility index of market returns, respectively. Liu follows the industry standard for the zero mean assumption in 30-day realized variance of market returns (see for example, Carr and Wu (2006)) and sets the above threshold $h$ to be 0. Using the state price function as described in the previous section, Liu (2014a) defines the state payoff function $d_{S_T}$ of BEX and BUX as the followings.

$$\text{BEX}^2 : d_{S_T} = \left( \ln \left( \frac{S_T + 0.05}{S_0} \right) \right)^2 I_{S_T < S_0}$$  \hspace{1cm} (19)

$$\text{BUX}^2 : d_{S_T} = \left( \ln \left( \frac{S_T + 0.05}{S_0} \right) \right)^2 I_{S_T > S_0}$$  \hspace{1cm} (20)

Here $S_0$ is the current SPX level, $S_T$ is an arbitrary future SPX level at maturity, and $T$ is the time to maturity for the selected option\textsuperscript{12}. Here $T$ is set to 30 calendar days exactly (or 22 trading days). $I$ is an indicator function that is 1 if the condition is met or 0 otherwise.

An additional of 0.05 term is added in the above numerators. The state price shown in equation (14) is an interval from $K_i$ to $K_{i+1}$ and we need to modify the state payoffs to get to the center of the state price interval. Given the minimum tick size in the SPX index is 0.1, we add in the 0.05 term in order to get to the center of the 10 cents interval for a more precise approximation. Summing all products of the state payoff and the state price, we obtain the $\text{BEX}^2$ and $\text{BUX}^2$. BEX and BUX can then be calculated as the squared-roots of these two measures, respectively.

3.3 Data

We obtain daily SPX option quotes from the Ivy DB database of OptionMetrics, which provides historical prices of options based on closing quotes at the CBOE.

\textsuperscript{12}Since August 24, 1992, SPX options expire at the open on the third Friday of the contract month. Hence the time to expiration is the number of calendar days remaining less one.
from January 3, 1996 to August 30, 2013. Including both bull and bear regimes as well as the recent GFC, this rich period enables us to uncover the dynamics of risk-neutral skewness that is more evident given the existence of relatively rare events. This data set includes the highest closing bid and ask quotes for each SPX option and the Black-Scholes implied volatilities based on the average of best bid and ask prices. The option price is approximated by the average of best bid and ask, which is a standard approach adopted by CBOE and other researchers.

We use US one-month and three-month Treasury-bill yields, adjusting for dividend yields, as the risk-free interest rates. Treasury-bills and dividend yields are obtained from the Federal Reserve Bulletin and OptionMetrics, respectively. For SPX options with shorter (longer) maturities than either of the two Treasury-bill maturities, the risk-free rate is approximated by the dividend-adjusted one-month (three-month) yield. Daily quotes of SKEW are computed according to the guideline provided in CBOE (2010). We match our SKEW estimates with the one obtained from CBOE SKEW Historical Price Data\(^{13}\).

As discussed in the previous section, the state price at each interval from \(K_i\) to \(K_{i+1}\) is calculated as in equation (14). In this framework, the model no longer requires inputs from each traded SPX option. In order to obtain a smoother state price density, we create states with 0.10 increments, ranging from 0.1 to 9,999. To understand this, a state of 200 indicates the SPX will be at 200 after one month.

In Section 2.2, we present the summary statistics of strike price ranges of OTM SPX options in SKEW calculation in Table 2. The minimum of the bottom is 0.21 from one of trading days in 2008. The maximum of the top is 1.59 from that same year. Given that SPX fluctuates from 598.48 to 1709.67 from 1996 to 2013, with our choice of states from 1 to 3600, this covers a much bigger range than that of SKEW.

Summary statistics of VIX, SIX and SKEW indexes are presented in Table 3\(^{14}\). The sample mean and median of SIX during this period are 1.01 and 1.01, respectively. Given the benchmark for a symmetrical distribution is 1, and that the larger the number the more negatively skewed it is, these figures suggest that the risk-neutral distribution of the market return is, on average, negatively skewed. It is important to notice that the minimum of SIX is 0.85, which suggests that risk-neutral distribution were positively skewed in some days. The sample mean and median of SKEW, which are 1.75 and 1.72, respectively. The standard deviation of

\(^{13}\)This dataset is available at http://www.cboe.com/publish/scheduledtask/mktdata/datahouse/Skewdailyprices.csv.

\(^{14}\)The Third Moment Index (TIX) is discussed in Section 4.6.
SIX is 0.07, which is significantly less than that of SKEW at 0.56 using student’s t-test. The sample mean and median of the realized physical skewness are -0.02 and -0.03, respectively. This suggests the realized return distribution is slightly negatively skewed, which can also be seen from the skewness of ΔSPX at -0.22.

Figure 1 presents histograms of daily changes of SPX, SIX and SKEW. One apparent observation can be made from the comparison is that ΔSKEW is a lot more volatile than ΔSIX, which can also be seen in Table 3 that ΔSKEW has a much higher standard deviation comparing to that of ΔSIX.

Table 4 reports the cross-correlations among SIX, SKEW, SPX, VIX and their corresponding daily changes. The correlation between SIX and SKEW is 0.078 and that of daily counterparts is -0.274 This suggests that, overall, there is only a small correlation between SIX and SKEW but negative correlation in the daily changes. The correlation between SIX and SPX is -0.244 and that of daily counterparts is -0.637. These negative correlations imply that the more negatively skewed the risk-neutral distribution is, the lower the index return. The opposite relationship is observed for SKEW and SPX. The correlation at levels and daily changes is 0.303 and 0.285, respectively. These figures suggest that as the expected left-tail risk becomes larger, the index return is larger. Further discussion on these correlations is presented in the next section.

Figure 2 illustrates the levels of the CBOE SKEW, SIX and SPX at the close of trading in our sample. Reading from top to bottom, the first solid line in black is SPX. The second trajectory in red is SKEW. The third dotted line in blue is SIX. The fourth green line is TIX. Figure 2 is interesting in the following aspects.

There is an overall trend that as SPX goes up (down), SIX tends to go down (up); which is also reflected in the negative correlation between the changes of SIX and that of SPX. Three turning points in SIX are noteworthy. The first turning point occurs around September 2000, which sees SPX reaching its local peak and SIX hitting its local trough. From that point in time, there is a clear rising trend of SIX from late 2000 to late 2002. This corresponds to the dot com bubble burst and can be seen as a sign of collective nervousness in the market.

The second turning point takes place around late 2003. This sees SIX reaching a local peak before its downward trend in the period of 2003 to 2007. Climbing down
from the previous peak level, it shows that investors may have regained confidence, with fewer premiums priced into the index options. This may be seen as an indication that another bubble was formed in this period, which later burst in 2007 as the subprime crisis.

The third turning point arises around July 2007. This sees SIX heading on an upward trend after it reaches its the trough in 2007. It is particularly interesting to observe that SIX appears to persist at relatively higher levels since 2007. This can be interpreted as a sign that investors expect a higher chance for those left-tail events and are willing to pay more premiums on index options. The persistence implies that the general confidence has not been restored to pre-GFC level. In addition, there are several other observations worth noting. There are two sharp spikes in SIX around October 1997 and October 1998 - the former occurs following a stock market sell-off in which the Dow Jones market fell 555 points, while the latter corresponds to the general nervousness in the stock market during that period.

In contrast, there is no obvious relationship between SPX and SKEW in this time frame. For example, as the market hits the bottom during the subprime crisis in late 2008, SKEW does not provide a clear indication because of fluctuations in that period. This makes it difficult to believe that market’s expectation about Black Swan events would fluctuate so dramatically during a crisis, if SKEW did indeed measure tail risks perceived from the options market. There are many more peaks and troughs in the historical movements of SKEW than SIX and SPX, with extreme movements are more frequent in SKEW as well. For example, there are many spikes in the period of 2003 to 2007, while a general upward trend is presented in the SPX market. There is no clear reasoning to support the abnormal behaviour in SKEW during this period.

3.4 Nonparametric Measure

We calculate alternative measures of risk-neutral skewness by taking differences in the implied volatility of SPX options at different strikes. BKM show that the differences of implied volatilities can be good proxies for implied skewness. The nonparametric risk-neutral negative skewness is defined as the difference in implied volatilities between an OTM put option with -0.25 delta and an OTM call option with 0.25 delta:\footnote{We choose an OTM put (call) option with a delta that is closest to the -0.25 (0.25) threshold. We take an average of these measures from two series of SPX options with different maturities (which are used to interpolate the 30-day CBOE SKEW).}

\[
\text{Skew}^{\text{NonPar}} = \text{PIV}_{-0.25} - \text{CIV}_{0.25}
\]  

(21)

We choose an OTM put (call) option with a delta that is closest to the -0.25 (0.25) threshold. We take an average of these measures from two series of SPX options with different maturities (which are used to interpolate the 30-day CBOE SKEW).
This definition is similar to Xing et al. (2010). It is important to note that some researchers (e.g., Bali, Hu and Murray (2014)) define this nonparametric skewness measure as \( CIV_{25} - PIV_{25} \). Our definition is adopted for a consistent comparison with CBOE SKEW and SIX, which the negative skewness are measured.

4 Hypotheses and Results

Several hypotheses are tested. First, we examine the relationship between the risk-neutral volatility and skewness. To do that, we investigate the relationship between the daily changes of VIX and that of SIX and SKEW, respectively. There is mixed evidence in the literature. Since an increase in VIX is usually associated with a drop in the market decreasing the risk-neutral skewness, we argue that there is a positive relationship between the changes in VIX and the changes in SIX and SKEW.

Second, we study how the daily changes of SIX and SKEW respond to changes in SPX returns. It is well-documented that VIX responds differently to a decrease in SPX returns compared to an increase in SPX returns (Whaley, 2009). Thus, we examine whether a similar behaviour exists in SIX and SKEW.

Third, we investigate the predictability of SIX and SKEW on future market returns. Since a decrease in the risk-neutral skewness (i.e. an increase in SIX and SKEW) implies more negative returns are expected, we test whether there is a negative relationship between the daily change of SIX and SKEW with that of one-day ahead SPX return.

Fourth, can SIX or SKEW predict future realized skewness? The implied density generally differs substantially from the density under the physical measure (Carr & Wu, 2009). There is no clear theoretical support on whether there is any predictability of the realized skewness. While our test is explorative rather than explanatory, we believe it is still useful in evaluating SKEW as a publicly accessible market index.

Finally, we compare SIX with the third moment index (TIX) as a robustness check of our state price modelling, the latter of which is an extension of the discussion in Liu (2014b).

4.1 Risk-Neutral Skewness and Volatility

We investigate the contemporaneous relationship between the market risk-neutral volatility and skewness, by examining the rate of change in SIX/SKEW and that of
VIX. This is assessed by estimating regressions in the following forms:

\[
\begin{align*}
\Delta \text{SIX}_t &= \alpha_0 + \alpha_1 \Delta \text{VIX}_t + \varepsilon_t \\
\Delta \text{SKEW}_t &= \beta_0 + \beta_1 \Delta \text{VIX}_t + \varepsilon_t \\
\Delta \text{Skew}_{\text{NonPar}}^t &= \gamma_0 + \gamma_1 \Delta \text{VIX}_t + \varepsilon_t 
\end{align*}
\] (22)

where the daily change is defined as, for example, \(\Delta \text{SIX}_t = \ln(\text{SIX}_{t+1}/\text{SIX}_t)\).

Unlike the well-documented existence of strong negative correlation between the rate of change in VIX and equity market returns, there is mixed evidence in the literature about the interaction between the second and the third moment. By examining individual stock options, Dennis and Mayhew (2002) find that implied skewness tends to be more negative in periods of high market volatility by examining stock options.

In a recent study by Neuberger (2012), the author examines the term structure of skewness in the SPX options market from 1997 to 2009, and shows that both the realized and implied second and third moments are highly negatively correlated with each other, with correlations in excess of -0.9. Similar empirical findings are documented in Kozhan, Neuberger, and Schneider (2013). By examining SPX options from 1996 to 2012, the authors find that the implied variance is negatively correlated with skewness. When the implied variance increases, the absolute size of the skewness also increases (i.e. the magnitude of negative skewness increases).

It is worth noting that Han (2008) obtains the opposite result. The author finds that index risk-neutral skewness is less negative when market volatility is higher. This result is likely due to the differences in the data sample periods. Han employs daily SPX options data from January 1988 to June 1997 - this sample period is distinct from ours and aforementioned studies’ sample periods.

We argue that if the expected market risk-neutral volatility increases, the return distribution should become more negatively skewed i.e. SIX/SKEW increases. Our logic is the following. An advance in market volatility usually implies a rally in investor fear and is often associated with a drop in market return. This can be reflected as the market return distribution expected to be more negatively skewed. In other words, we expect a positive contemporaneous relationship between the changes in VIX and the changes in SIX and SKEW.

Based on the reasoning above, two hypotheses can be tested using equation (22). First, if the rate of change in VIX is related to that of SIX and SKEW, the slope coefficient \(\alpha_1\) and \(\beta_1\) should be nonzero. In particular, we should expect the slope coefficients to be positive. Second, the intercepts should be zero if this relationship is unbiased.

We report the ordinary least-squares results in Table 5. The heteroskedasticity-
consistent standard errors and covariance matrix is computed according to Newey and West (1994).

[Insert Table 5 here.]

Table 5 presents the result. The estimate of $\alpha_1$ is 0.0786 and is statistically significant against a null of $\alpha_1 = 0$ at all levels. The intercept term $\alpha_0$ is not statistically significant against a null of zero. The adjusted $R^2$ is 47.62%. That is, the relationship tends to be unbiased. This is consistent with our aforementioned reasoning. This reflects the positive relation between movements in the expected risk-neutral variance and the left-tail risk.

However, the opposite is found for SKEW, as presented in Panel B. The estimate of $\beta_1$ is -0.4652 and is statistically significant against a null of $\beta_1 = 0$. This relationship also tends to be unbiased as the intercept is not statistically significant against a null of zero. It is worth noting that the adjusted $R^2$ from the first model is almost ten times of that of the second model.

With the nonparametric skewness measure Skew$^{\text{NonPar}}$, we see a similar result with that of SIX. The slope coefficient is 1.4260 and it is significant at all levels. The intercept term is not statistically significant against a null of zero. The adjusted $R^2$ for the model is 5.32%.

### 4.2 Fear Indicator

In this section, we study the contemporaneous relationship between rates of change in SIX/SKEW and daily SPX returns. In particular, we investigate whether these skewness indices contain any fear information from the options market. Generally, a fall in the stock market usually means a rally in investor fear. If this proposition holds true, SIX and SKEW can be regarded as the ‘fear barometer’.

VIX is well-known as investors’ fear gauge, with empirical studies finding that the index responds more to negative changes in SPX returns than positive changes (Whaley, 2009). The interest of our paper is to determine whether this feature is shared by the skewness index, i.e. that there is an asymmetric response from SIX and SKEW to a market drop than an advance. This proposition can be assessed by estimating regressions in the following form:

\[
\begin{align*}
\Delta \text{SIX}_t &= \alpha_0 + \alpha_1 \Delta \text{SPX}_t + \alpha_2 \Delta \text{SPX}^-_t + \varepsilon_t \\
\Delta \text{SKEW}_t &= \beta_0 + \beta_1 \Delta \text{SPX}_t + \beta_2 \Delta \text{SPX}^-_t + \varepsilon_t \\
\Delta \text{Skew}^{\text{NonPar}}_t &= \gamma_0 + \gamma_1 \Delta \text{SPX}_t + \gamma_2 \Delta \text{SPX}^-_t + \varepsilon_t 
\end{align*}
\]  

(23)

where the daily change is defined as, for example, $\Delta \text{SIX}_t = \ln(\text{SIX}_{t+1}/\text{SIX}_t)$. $\Delta \text{SPX}^-_t$ is defined as $\min(\Delta \text{SPX}, 0)$. 

---

23
It has been well-documented in the literature that there is a negative correlation between the rate of changes in VIX and daily market returns (see for example, Carr and Wu (2009)). As the expected market volatility increases, investors should demand a higher rate of return on stocks and hence prices fall, which may ultimately lead a drop in market return. A similar relationship should hold for skewness. As the expected market skewness decreases, investors may expect that there will be a higher probability of a more negative return if the market goes down. This translates to a demand for higher rates of return on stocks, and hence a fall in market returns. Therefore, the relation between the rates of change in SIX/SKEW should be proportional to market returns.

Furthermore, as argued in Bollen and Whaley (2004) and Whaley (2009), during a market fall, there is an increasing demand to buy ATM and OTM index puts. Hence, we should also expect that the changes in SIX/SKEW rise at a higher absolute rate when the equity market falls, compared to when it rises.

Given the above reasons, three hypotheses can be tested using equation (23). First, if the rate of change in SIX and SKEW is related to the daily market return, the slope coefficients $\alpha_1$ and $\beta_1$ should be nonzero. In particular, we should expect the slope coefficients to be negative. Second, the coefficients $\alpha_2$ and $\beta_2$ should be positive to reflect an asymmetric response to a market drop. Third, if this relationship is unbiased, we should find the intercepts to be zero.

We report the ordinary least-squares results in Table 6. The heteroskedasticity-consistent standard errors and covariance matrix is computed according to Newey and West (1994).

Table 6 presents the result. For SIX, the estimate of $\alpha_1$ is -0.3100 and is statistically significant against a null of $\alpha_1 = 0$ at all levels. The estimate of $\alpha_2$ is -0.0493 and it is statistically significant against a null of $\alpha_2 = 0$ at 5% level. The intercept is not statistically significant against a null of $\alpha_0 = 0$. Similar to the findings in VIX literature, this reflects not only the inverse relationship between movements in SIX and movements in the SPX market, but also the asymmetry of movements from an increasing demand in index put options. These coefficients can be interpreted as follows. If the market goes up by 100 basis points, the SIX will likely fall by an amount of:

$$\Delta\text{SIX}_t = -0.31 \times 0.01 = -0.31\%$$

That is, a rise in the market is associated with a decrease in market negative skewness, or equivalently, an increase in the market skewness. On the other hand, if the
market drops by 100 basis points, the SIX will likely rise by an amount of:

$$\Delta\text{SIX}_t = -0.3100 \times (-0.01) + (-0.0493 \times (-0.01)) = 0.0036\%$$

That is, a drop in the market is associated with an increase in market negative skewness, or equivalently, a decrease in market skewness. Therefore, SIX can be considered as a barometer of investors’ fear.

For SKEW, the estimate of $\beta_1$ is 3.1132 and is statistically significant against a null of $\beta_1 = 0$. The estimate of $\beta_2$ is -0.5276 and it is not statistically significant against a null of $\beta_2 = 0$ at all levels. The intercept is not statistically significant against a null of $\beta_0 = 0$, that is, the relationship tends to be unbiased. This is contrary to our results using SIX and contradicts our earlier hypotheses. The results of SKEW implies that a rise (fall) in the market is associated with an increase (decrease) in the left-tail risk. However, this is inconsistent with the intuition behind SKEW as a measure of perceived likelihood of Black Swan events in the market. In addition, there is no strong asymmetrical relationship between the rate of changes in SKEW and the SPX market returns. Comparing the adjust $R^2$, we can see that the model with SIX at 36.90% is almost five times more than that of SKEW at 7.73%.

For the nonparametric skewness measure $\text{Skew}^{\text{NonPar}}$, we see a similar result with that of SIX. The estimate of $\gamma_1$ is -3.1993 and it is significant at all levels. The estimate of $\gamma_2$ is -1.3844 but not significant against a null of zero. These two variables have the same sign as those with SIX. The intercept term is not statistically significant against a null of zero. The adjusted $R^2$ for the model is 1.67%.

### 4.3 Predictability of Market Returns

We investigate the predictability of SIX and SKEW on the 1-day ahead future market returns. These are assessed by estimating regressions in the following forms:

$$\Delta\text{SPX}_{t+1} = \alpha_0 + \alpha_1 \Delta\text{SIX}_t + \alpha_2 \Delta\text{SPX}_t + \epsilon_1$$
$$\Delta\text{SPX}_{t+1} = \beta_0 + \beta_1 \Delta\text{SKEW}_t + \beta_2 \Delta\text{SPX}_t + \epsilon_2$$
$$\Delta\text{SPX}_{t+1} = \gamma_0 + \gamma_1 \Delta\text{Skew}^{\text{NonPar}}_t + \gamma_2 \Delta\text{SPX}_t + \epsilon_3$$
$$\Delta\text{SPX}_{t+1} = \theta_0 + \theta_1 \Delta\text{SIX}_t + \theta_2 \Delta\text{SKEW}_t + \theta_3 \Delta\text{Skew}^{\text{NonPar}}_t + \theta_4 \Delta\text{SPX}_t + \epsilon_4$$

The daily change is defined as the following, using the daily close-to-close values:

$$\Delta\text{SPX}_{t+1} = \ln \left( \frac{\text{SPX}_{t+1}}{\text{SIX}_t} \right)$$
$$\Delta\text{SIX}_t = \ln \left( \frac{\text{SIX}_t}{\text{SIX}_{t-1}} \right)$$

If an informed trader chooses to trade any information in the options market prior to the equity market, and the latter is slow to absorb this information from
the options market, we expect that risk-neutral moments embedded in the options market to have predictive power on future stock market returns. More specifically, if the expected market risk-neutral skewness decreases (or the market becomes more left skewed), investors will demand a higher rate of return on stocks, and hence stock prices will fall.

The idea is that, given the return in the market today, does SIX or SKEW any prediction power on the return tomorrow? At least three hypotheses can be tested using equation (24). First, if SIX and SKEW have any informational content about future market returns, the slope coefficients $\alpha_1$ and $\beta_1$ should be nonzero. In particular, we should expect the slope coefficients to be negative. Second, if the forecast is unbiased, we should find the intercepts to be zero. Third, if SIX is a more superior forecast than SKEW, we should expect to find $\gamma_1$ is significant but not $\gamma_2$.

In a recent study by Xing et al. (2010), the authors examine individual stock options in the US market and argue that the shape of the volatility skew has predictive power for future equity returns. In their study, volatility skew is defined as the difference between the implied volatilities of OTM puts and ATM calls. Given this definition, the increase in the volatility skew is equivalent to a decrease in the skewness of return distribution\(^{16}\). Further, they find that an increase in the volatility skew (i.e. a decrease in the skewness) is associated with a decrease in equity returns. This result is unchanged after including other market control variables into the model.

Similar findings are documented in Rehman and Vilkov (2012). By examining individual stock options, these authors proxy the \textit{ex ante} skewness by using model-free implied skewness to show that stocks with high implied skewness outperform counterparts with low implied skewness.

We report the ordinary least-squares results in Table 7. The heteroskedasticity-consistent standard errors and covariance matrix is computed according to Newey and West (1994).

[Insert Table 7 here.]

For all four regressions, intercepts are both small in magnitude and not statistically significant. In addition, an increase in the market return today generally associates with a market decline tomorrow, as indicated by the statistically significant negative coefficients of $\Delta\text{SPX}$. In column (1), the estimate of $\alpha_1$ is -0.0695 and is statistically significant against a null of $\alpha_1 = 0$. The intercept of SIX is not\(^{16}\)

\(^{16}\)For coterminal options on the same underlying assets, an OTM put has a smaller strike price than an ATM call. If the difference between the implied volatility of an OTM put and an ATM call becomes wider, the slope of the volatility smile is steeper and hence the distribution is more negatively skewed. For further explanation, we refer readers to Bakshi et al. (2003)
statistically significant against a null of \( \alpha_0 = 0 \). Our result implies that if SIX increases, this is likely to associate with a drop in the market return on the following day. This is consistent with our hypothesis that as expected returns distribution becomes more negatively skewed, returns are expected to fall on average.

In column (2), the coefficient of \( \Delta \text{SKEW} \) is -0.0012 with a standard error of 0.0018. It has the same sign as that of \( \Delta \text{SIX} \), however, it is not significant with a p-value of 0.4853. This finding is further supported by the result presented in column (5). When both contemporaneous \( \Delta \text{SIX} \) and \( \Delta \text{SKEW} \) are included to explain the one-day ahead return, only \( \Delta \text{SIX} \) and \( \Delta \text{Skew}^{\text{NonPar}} \) are statistically significant. The coefficient of \( \Delta \text{SIX} \) is -0.0644, which is quite similar to the one in column (1).

We do want to point out that for all three models, the adjusted \( R^2 \) is around 0.50%. Nonetheless, only the daily changes of SIX add small but significant explanatory power on the one-day ahead market return.

### 4.4 Predictability of Future Realized Skewness

In this section, we examine the informational content of SIX and SKEW to determine whether they have any explanatory power on forecasting future realized skewness. This is assessed by estimating a regression in the following forms:

\[
\begin{align*}
\text{RSkew}_{t,t+30} &= \alpha_0 + \alpha_1 \text{SIX}_t + \epsilon_t \\
\text{RSkew}_{t,t+30} &= \beta_0 + \beta_1 \text{SKEW}_t + \epsilon_t \\
\text{RSkew}_{t,t+30} &= \gamma_0 + \gamma_1 \text{Skew}^{\text{NonPar}}_t + \epsilon_t \\
\text{RSkew}_{t,t+30} &= \theta_0 + \theta_1 \text{SIX}_t + \theta_2 \text{SKEW} + \theta_3 \text{Skew}^{\text{NonPar}} + \epsilon_t
\end{align*}
\]

(25)

where \( \text{RSkew}_{t,t+30} \) denotes the subsequent monthly realized skewness in calendar days convention at time \( t \).

The literature documents that unlike the first two moments of the return distribution, the realized skewness is difficult to measure precisely (Neuberger, 2012). Merton (1980) and Harvey and Siddique (2000) show that the estimates of higher moments require a long time series and that it is quite possible to be measured with errors. This is further confirmed in the study by Bakshi et al. (2003) that with using sizes of 350, 400 and 450 days, the quality of the estimation is improved from a longer sample size.

One problem shared by these previous studies, including Bakshi et al. (2003), is that they calculate the skewness measure based on daily returns from the end-of-day market quote. If a size of 350 days from Bakshi et al. (2003) is suggested based on a requirement of more data to calculate the skewness measure, then the monthly skewness measure is meaningful given the presence of intra-day data. With the aid of 5-minute interval intra-day SPX quotes, we obtain 77 returns on a daily basis and
1694 returns for a month\(^{17}\). The formula is presented below, where \(N = 1694\):

\[
\text{RSkew}_{t,t+30} = \frac{N^{1/2} \sum_{i=1}^{N} \ln \left( \frac{S_{t+i}}{S_{t+i-1}} \right)^3}{\sum_{i=1}^{N} \left( \ln \left( \frac{S_{t+i}}{S_{t+i-1}} \right)^2 \right)^{3/2}}
\]  

(26)

In calculating the realized skewness, we apply filtering to remove extreme overnight returns that are greater than 50%\(^{18}\).

Two hypotheses can be tested using the above regressions. First, if SIX and SKEW contain some information about future realized skewness, the slope coefficients should be non-zero. Second, if SIX and SKEW are unbiased forecasts of future realized skewness, one should expect that the intercept coefficient is zero and the slope coefficient is -1. That is, since SIX and SKEW measure how negatively skewed the market return distribution is, there should be a perfect negative correlation between SIX and SKEW with realized skewness.

Ordinary least-squares estimates of equation (25) are presented in Table 8. The heteroskedasticity-consistent standard errors and covariance matrix is computed according to Newey and West (1994).

[Insert Table 8 here.]

In column (1), the estimate of \(\alpha_1\) is -0.6461 and is statistically significant against a null of \(\alpha_1 = 0\). Hence, SIX contains some information about future realized skewness. In particular, since we are interested in whether SIX is an unbiased forecast, we run the Wald test to determine whether we can reject the null of \(\alpha_1 = -1\). The F-statistic from the Wald test is 1.7414 with a p-value of 0.1870, which shows that we cannot reject this null hypothesis. However, SIX seems to be a biased forecast given the intercept \(\alpha_0\) is statistically significant against a null of \(\alpha_0 = 0\). However, this should not be problematic if the magnitude of the bias is constant and known.

In comparison, column (2) presents that the estimate of \(\beta_1\) is -0.0408 and is not statistically significant against a null of \(\beta_1 = 0\). Hence, this suggests that SKEW does not contain any information about future realized skewness. In addition, to determine whether SKEW is an unbiased forecast, we run the Wald test to see

\(^{17}\)There are 77 5-minute intervals in a trading day from 9:35am to 4:00pm EST. Assuming there are 22 trading days in one month on average, we obtain 1694 intervals.

\(^{18}\)In particular, there is one large negative overnight return on Thursday April 11\(^{th}\), 1996. The market was closed on 6\(^{th}\) and 7\(^{th}\) due to the Good Friday on 5\(^{th}\). For some unknown reason, the index was completely stale from 8\(^{th}\) to 10\(^{th}\). When the market was reopened, a huge spike was observed and caused a huge negative overnight return. If this return was included, it would translate to a huge spike in the realized skewness for the next 22 trading days.
whether we can reject the null of $\beta_1 = -1$. The F-statistic from the Wald test is 0, which shows that the null hypothesis is strongly rejected. The intercept $\beta_0$ is not statistically significant against a null of $\beta_0 = 0$. Further, the adjusted $R^2$ is almost 0, which indicates that this model has little explanatory power on future realized skewness.

In column (4), however, the slope coefficient of Skew$_t^{\text{NonPar}}$ is 2.5192 and significant at all levels. This challenges our previous findings that SIX and Skew$_t^{\text{NonPar}}$ provide consistent results. In the full model, column (5) gives a similar picture. The coefficient of SIX is -1.3138 and significant at all levels. The coefficient of SKEW is -0.0496 is not significant. Skew$_t^{\text{NonPar}}$ has a positive coefficient 3.9329 and is significant at all levels.

Overall, we find that SIX is a biased estimate with some explanatory power of future realized skewness, while SKEW does not seems to have any informational content of future realized skewness in our sample.

4.5 SIX and the Third Moment Index

As a robustness check, the \textit{ex ante} risk-neutral skewness can be alternatively defined as sharing the same state price with BEX and BUX but having a different state payoff function. The third moment index (TIX) is defined as follows.

\[
\text{TIX} = \frac{365}{30} \left( \frac{-1}{V^3} \sum_{s=1}^{S} \Phi_s \left( \ln \left( \frac{K_s + 0.05}{S_0} \right) \right)^3 \right)
\]

(27)

where $\Phi_s$ is the state price function and

\[
V = \sqrt{\sum_{s=1}^{S} \Phi_s \left( \ln \left( \frac{K_s + 0.05}{S_0} \right) \right)^2}
\]

(28)

Note that, the state payoff function in TIX assumes that the 30-day market return mean is 0. This is consistent with the zero mean assumption in the calculation of SVX. Alternatively, we use the short-term T-bills and the first moment mean return estimated using same state prices as the proxy for mean return. We obtain similar results with these alternative set ups.

SIX measures skewness by assessing the symmetry of the distribution, while TIX measures skewness by defining a state payoff function that measures the actual negative skewness statistic. It is clear that SIX and TIX rely on different principles to measure skewness. Our results reflect the qualitative similarities between the two indices. Figure 2 shows the levels of the CBOE SKEW, SIX, TIX and the SPX at the close of trading days through January 1996 to August 2013. This figure illustrates
that SIX and TIX mimic each other in its peaks and troughs. Furthermore, an examination of the cross-correlations from Table 4 reveals that the correlation of these two measures is almost 100% in terms of levels and 100% in terms of daily changes. Regression results are presented in the corresponding tables. In summary, similar findings are found with TIX.

5 Conclusion

In this paper, we study the dynamics of the risk-neutral skewness in the SPX market. Using a state-preference pricing approach, we develop a forward-looking market symmetric index (SIX) as a forecast of the 30-day skewness returns of SPX. This \textit{ex ante} measure estimates how \textit{negatively skewed} the market return distribution is in the coming month. It is directly comparable to the recently announced CBOE SKEW index derived from the price of SPX return skewness (that measures the perceived left tail of the return distribution).

Using SPX options data from 1996 to 2013, we conduct a horse race between SIX and SKEW among a number of hypotheses. First, we examine the contemporaneous relationship between daily changes of risk-neutral volatility and skewness of index returns. We argue that if the expected market risk-neutral volatility increases, the return distribution should become more negatively skewed. Using the CBOE VIX as a candidate for risk-neutral volatility, our result statistics cannot reject this hypothesis for SIX. However, we find a contrary relationship between VIX and SKEW.

Second, we study the contemporaneous relationship between daily SPX returns and changes in the skewness measures. In particular, we investigate whether these skewness indices contain any \textit{fear} information from the options market when there is a market fall. We show that the rate of change in daily market returns is negatively proportional to the changes in SIX, while the opposite result is found for SKEW. In particular, only SIX has a statistically significant asymmetric response to the market drop compared to a market advance.

Third, we investigate the predictability of these \textit{ex ante} skewness measures on future SPX market returns. We argue that as a decrease in the risk-neutral skewness (i.e. an increase in SIX and SKEW) implies more negative returns are expected, there should be a negative relationship between the daily changes of SIX and SKEW with that of one-day ahead SPX returns (or equivalently, a positive relationship between the \textit{ex ante} skewness and the future return). Our findings with SIX support this hypothesis: the more negatively skewed the risk-neutral distribution, the lower the future returns in the SPX market. We show that the model with SKEW does not have any explanatory power.
Last, we investigate whether these \textit{ex ante} risk-neutral skewness measures are suitable proxies for the physical return skewness of SPX returns in the following month. We find that SIX is a biased but efficient forecast of future physical skewness, while there is no statistically significant relationship for SKEW.

These findings raise a fundamental concern about the underlying methodology of SKEW. The theoretical basis of SKEW originates from the model-free risk-neutral moments in Bakshi et al. (2003). However, as one translates the theoretical model to equations for empirical use with real world data, approximation errors are inevitably introduced. One such problem includes approximating an integration of an infinite number of strike prices by the available amount for one security traded in the market. This approximation error is more pronounced as the amount of strike prices available decreases. In other words, if SKEW fails to deliver the risk-neutral skewness information from the SPX options market, the findings of the risk-neutral skewness in individual stock options become unreliable. We hope this paper can pioneer subsequent research on this matter.

A Appendix

A.1 State Prices in Breeden and Litzenberger (1978)

Breeden and Litzenberger (1978) show that the state price may be modelled as the second derivative of a call or a put option price.

\[
\frac{\partial^2 C(K, T)}{\partial K^2} = \frac{\partial^2 P(K, T)}{\partial K^2}
\]

In section 3.3 we present a mathematical derivation of this state price. Here, we present a more intuitive approach by constructing a butterfly spread to long one call with strike \(M - \Delta M\), long one call with strike \(M + \Delta M\) and short two calls with strike \(M\) (Barraclough, 2007). At maturity \(T\), the payoff of this portfolio is illustrated in the following table.

<table>
<thead>
<tr>
<th>(S_T &lt; M - \Delta M)</th>
<th>(M - \Delta M &lt; S_T &lt; M)</th>
<th>(M &lt; S_T &lt; M + \Delta M)</th>
<th>(M + \Delta M &lt; S_T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long 1 call with (M - \Delta M)</td>
<td>0</td>
<td>(S_T - (M - \Delta M))</td>
<td>(S_T - (M - \Delta M))</td>
</tr>
<tr>
<td>Short 2 calls with (M)</td>
<td>0</td>
<td>0</td>
<td>(-2(S_T - M))</td>
</tr>
<tr>
<td>Long 1 call with (M + \Delta M)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total at (t = T)</td>
<td>0</td>
<td>(\Delta M + (S_T - M))</td>
<td>(\Delta M - (S_T - M))</td>
</tr>
</tbody>
</table>

That is, the payoff is exactly \(\Delta M\) if \(S_T = M\) at maturity. Thus the cost of butterfly spread that produces a payment of $1 if the future state is \(S_T = M\) is:

\[
P(M; \Delta M) = \frac{C(M - \Delta M, T) - 2C(M, T) + C(M + \Delta M, T)}{\Delta M}
\]
If we divide the above by the step size $\Delta M$ and in the limit as $\Delta M \to 0$, we obtain:

$$\lim_{\Delta M \to 0} \frac{P(M; \Delta M)}{\Delta M} = \lim_{\Delta M \to 0} \frac{C(M - \Delta M, T) - 2C(M, T) + C(M + \Delta M, T)}{\Delta M^2}$$

$$= \frac{\partial^2 C(K, T)}{\partial K^2} \bigg|_{K=M}$$

Thus the price of a security $f$ with payoff $d_M^f$ at some future state $M$ is

$$P^f = \int_M d_M^f \frac{P(M; \Delta M)}{\text{payoff}} = \int_M d_M^f \left( \frac{\partial^2 C(K, T)}{\partial K^2} \bigg|_{K=M} \right) dM$$

References


Figure 1: Histograms of Daily Returns
Figure 2: Indices Daily Trajectories
Table 1: Summary of Number of OTM Strike Prices
Summary statistics of average numbers of Out-of-the-Money (OTM) SPX option strikes in VIX calculation. Daily SPX option quotes are obtained from the Ivy DB database of OptionMetrics. This sample covers the period from January 4, 1996 to August 30, 2013. Near term represents the first series of SPX options with maturity closer to today but longer than 7 days. Next term represents the second series of SPX options with maturity longer than that of the near term. We exclude any option with zero bid price, as well as ones beyond two consecutive options with zero bid price. More details on data filtration can be found in CBOE (2010).

<table>
<thead>
<tr>
<th>Year</th>
<th>Near Term Call Mean</th>
<th>Near Term Put Mean</th>
<th>Next Term Call Mean</th>
<th>Next Term Put Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>1996</td>
<td>9.9</td>
<td>17</td>
<td>4</td>
<td>19.9</td>
</tr>
<tr>
<td>1997</td>
<td>13.4</td>
<td>25</td>
<td>3</td>
<td>27.7</td>
</tr>
<tr>
<td>1998</td>
<td>14.3</td>
<td>33</td>
<td>5</td>
<td>29.2</td>
</tr>
<tr>
<td>1999</td>
<td>16.1</td>
<td>32</td>
<td>6</td>
<td>26.7</td>
</tr>
<tr>
<td>2000</td>
<td>18.2</td>
<td>33</td>
<td>6</td>
<td>19.4</td>
</tr>
<tr>
<td>2001</td>
<td>14.7</td>
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<td>6</td>
<td>22.3</td>
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<td>32</td>
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<td>18.5</td>
</tr>
<tr>
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<td>6</td>
<td>22.1</td>
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<td>22</td>
<td>7</td>
<td>25.8</td>
</tr>
<tr>
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<td>17.1</td>
<td>26</td>
<td>8</td>
<td>28.5</td>
</tr>
<tr>
<td>2006</td>
<td>19.1</td>
<td>32</td>
<td>8</td>
<td>37.4</td>
</tr>
<tr>
<td>2007</td>
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<td>48</td>
<td>2</td>
<td>51.1</td>
</tr>
<tr>
<td>2008</td>
<td>43.4</td>
<td>89</td>
<td>15</td>
<td>54.0</td>
</tr>
<tr>
<td>2009</td>
<td>36.3</td>
<td>56</td>
<td>19</td>
<td>68.6</td>
</tr>
<tr>
<td>2010</td>
<td>31.6</td>
<td>54</td>
<td>3</td>
<td>70.7</td>
</tr>
<tr>
<td>2011</td>
<td>31.1</td>
<td>53</td>
<td>6</td>
<td>73.2</td>
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<tr>
<td>2012</td>
<td>24.0</td>
<td>46</td>
<td>13</td>
<td>55.0</td>
</tr>
<tr>
<td>2013</td>
<td>19.8</td>
<td>29</td>
<td>10</td>
<td>45.2</td>
</tr>
</tbody>
</table>
Table 2: Summary of Range of OTM Strike Prices
Summary statistics of strike price ranges of Out-of-the-Money (OTM) SPX options in VIX calculation. Our daily SPX option quotes are obtained from the Ivy DB database of OptionMetrics. This sample covers the period from January 4, 1996 to August 30, 2013. Bottom is defined as a ratio of the lowest strike price of OTM put options over the SPX index level on one day. Top is defined as a ratio of the highest strike price of OTM call options over the SPX index level on that day. Near term represents the first series of SPX options with maturity closer to today but longer than 7 days. Next term represents the second series of SPX options with maturity longer than that of the near term. We exclude any option with zero bid price, as well as ones beyond two consecutive options with zero bid price. More details on data filtrations are described in CBOE (2010).

<table>
<thead>
<tr>
<th>Year</th>
<th>Bottom Mean</th>
<th>Bottom Min</th>
<th>Bottom Top</th>
<th>Next Mean</th>
<th>Next Min</th>
<th>Next Top</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>0.84</td>
<td>0.94</td>
<td>0.62</td>
<td>1.07</td>
<td>1.14</td>
<td>1.02</td>
</tr>
<tr>
<td>1997</td>
<td>0.80</td>
<td>0.95</td>
<td>0.42</td>
<td>1.09</td>
<td>1.17</td>
<td>1.02</td>
</tr>
<tr>
<td>1998</td>
<td>0.71</td>
<td>0.91</td>
<td>0.38</td>
<td>1.10</td>
<td>1.23</td>
<td>1.03</td>
</tr>
<tr>
<td>1999</td>
<td>0.72</td>
<td>0.94</td>
<td>0.54</td>
<td>1.11</td>
<td>1.20</td>
<td>1.04</td>
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<tr>
<td>2000</td>
<td>0.77</td>
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<td>0.54</td>
<td>1.11</td>
<td>1.22</td>
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<tr>
<td>2001</td>
<td>0.74</td>
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<td>0.61</td>
<td>1.14</td>
<td>1.29</td>
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<tr>
<td>2002</td>
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<td>0.57</td>
<td>1.14</td>
<td>1.34</td>
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<td>2003</td>
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<td>0.93</td>
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<td>2004</td>
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<tr>
<td>2006</td>
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<td>0.92</td>
<td>0.63</td>
<td>1.08</td>
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<tr>
<td>2007</td>
<td>0.81</td>
<td>0.94</td>
<td>0.57</td>
<td>1.08</td>
<td>1.19</td>
<td>1.02</td>
</tr>
<tr>
<td>2008</td>
<td>0.68</td>
<td>0.90</td>
<td>0.21</td>
<td>1.17</td>
<td>1.59</td>
<td>1.05</td>
</tr>
<tr>
<td>2009</td>
<td>0.61</td>
<td>0.80</td>
<td>0.35</td>
<td>1.19</td>
<td>1.37</td>
<td>1.08</td>
</tr>
<tr>
<td>2010</td>
<td>0.72</td>
<td>0.87</td>
<td>0.44</td>
<td>1.12</td>
<td>1.25</td>
<td>1.05</td>
</tr>
<tr>
<td>2011</td>
<td>0.73</td>
<td>0.93</td>
<td>0.42</td>
<td>1.11</td>
<td>1.23</td>
<td>1.02</td>
</tr>
<tr>
<td>2012</td>
<td>0.81</td>
<td>0.91</td>
<td>0.57</td>
<td>1.08</td>
<td>1.17</td>
<td>1.04</td>
</tr>
<tr>
<td>2013</td>
<td>0.85</td>
<td>0.93</td>
<td>0.80</td>
<td>1.06</td>
<td>1.09</td>
<td>1.04</td>
</tr>
</tbody>
</table>
Table 3: Summary Statistics
Summary Statistics of SIX, TIX, SKEW, SPX and Realized SPX Return Skewness. This table reports the sample average (Mean), median, maximum, minimum, standard deviation (Std. Dev.), skewness, excess kurtosis and the number of observations on the levels and logarithmic returns of SIX, TIX, SKEW, SPX close and the 30-day realized skewness with calendar days convention (RSkew). SKEW is a modified version of CBOE SKEW, where SKEW = (CBOE SKEW - 100)/10. The daily change is defined as, for example, \( \Delta \text{SIX}_t = \ln(\text{SIX}_t/\text{SIX}_{t-1}) \). Exceptions are made for \( \Delta \text{TIX} \) and \( \Delta \text{RSkew} \) due to negative numbers in the series, which is defined as \( \Delta \text{RSkew}_t = \text{RSkew}_t - \text{RSkew}_{t-1} \). Daily observations are obtained from January 4, 1996 to August 30, 2013.

<table>
<thead>
<tr>
<th>Moments</th>
<th>SIX</th>
<th>TIX</th>
<th>SKEW</th>
<th>RSkew</th>
<th>VIX</th>
<th>SPX</th>
<th>ΔSIX</th>
<th>ΔTIX</th>
<th>ΔSKEW</th>
<th>ΔRSkew</th>
<th>ΔVIX</th>
<th>ΔSPX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1.01</td>
<td>0.22</td>
<td>1.75</td>
<td>-0.02</td>
<td>21.74</td>
<td>1170.24</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>4.53</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Median</td>
<td>1.01</td>
<td>0.24</td>
<td>1.72</td>
<td>-0.03</td>
<td>20.31</td>
<td>1187.95</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.07</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Maximum</td>
<td>1.19</td>
<td>3.91</td>
<td>4.62</td>
<td>4.15</td>
<td>80.86</td>
<td>1709.67</td>
<td>0.07</td>
<td>1.58</td>
<td>1.48</td>
<td>18017.36</td>
<td>0.50</td>
<td>0.11</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.85</td>
<td>-3.68</td>
<td>0.41</td>
<td>-3.17</td>
<td>9.89</td>
<td>598.48</td>
<td>-0.05</td>
<td>-1.06</td>
<td>-1.07</td>
<td>-264.63</td>
<td>-0.35</td>
<td>-0.09</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.07</td>
<td>1.68</td>
<td>0.56</td>
<td>-0.48</td>
<td>8.48</td>
<td>233.64</td>
<td>0.01</td>
<td>0.16</td>
<td>0.13</td>
<td>281.45</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.03</td>
<td>-0.10</td>
<td>-1.67</td>
<td>1.56</td>
<td>1.94</td>
<td>-0.36</td>
<td>0.38</td>
<td>0.38</td>
<td>0.35</td>
<td>63.93</td>
<td>0.59</td>
<td>-0.22</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.70</td>
<td>1.74</td>
<td>45.36</td>
<td>14.07</td>
<td>9.63</td>
<td>2.60</td>
<td>9.62</td>
<td>9.59</td>
<td>10.27</td>
<td>4091.87</td>
<td>6.80</td>
<td>10.39</td>
</tr>
</tbody>
</table>
Table 4: Correlations
Cross-Correlations of SIX, TIX, SKEW, VIX, SPX and 30-day Forward Looking Realized SPX Return Skewness. This table reports cross-correlations on the levels and logarithmic returns of SIX, TIX, SKEW, VIX, SPX close and the 30-day realized skewness with calendar days convention (RSkew). SKEW is a modified version of CBOE SKEW, where SKEW = (CBOE SKEW - 100)/10. The logarithmic return is defined as, for example, \( \Delta \text{SIX}_t = \ln(\text{SIX}_t/\text{SIX}_{t-1}) \). Exceptions are made for \( \Delta \text{TIX} \) and \( \Delta \text{RSkew} \) due to negative numbers in the series, which is defined as \( \Delta \text{RSkew}_t = \text{RSkew}_t - \text{RSkew}_{t-1} \). The common sample has 4,446 daily observations from January 4, 1996 to August 30, 2013.

<table>
<thead>
<tr>
<th>Correlations</th>
<th>SIX</th>
<th>TIX</th>
<th>SKEW</th>
<th>RSkew</th>
<th>VIX</th>
<th>SPX</th>
<th>ΔSIX</th>
<th>ΔTIX</th>
<th>ΔSKEW</th>
<th>ΔRSkew</th>
<th>ΔVIX</th>
<th>ΔSPX</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIX</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>TIX</td>
<td>0.999</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SKEW</td>
<td>0.078</td>
<td>0.071</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RSkew</td>
<td>-0.131</td>
<td>-0.131</td>
<td>-0.006</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VIX</td>
<td>0.515</td>
<td>0.512</td>
<td>-0.166</td>
<td>0.048</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>SPX</td>
<td>-0.244</td>
<td>-0.247</td>
<td>0.303</td>
<td>0.144</td>
<td>-0.290</td>
<td>1.000</td>
<td></td>
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<td></td>
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<tr>
<td>ΔSIX</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔTIX</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔSKEW</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔRSkew</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔVIX</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔSPX</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5: Regression Results I
Regression results for daily changes of SIX, SKEW and TIX against the contemporaneous change of VIX. SKEW is a modified version of CBOE SKEW, where SKEW = (CBOE SKEW - 100)/10. There are 4,446 daily observations from January 4, 1996 to August 30, 2013 in each series. The heteroskedasticity-consistent standard errors and covariance matrix is computed according to Newey and West (1994). Regressions equations are presented in equation (22). The daily change is defined as, for example, \( \Delta \text{SPX}_t = \ln(\text{SPX}_t/\text{SIX}_{t-1}) \); except for \( \Delta \text{TIX}_t = \text{TIX}_t - \text{TIX}_{t-1} \), due to negative values in the series.

<table>
<thead>
<tr>
<th></th>
<th>( \Delta \text{SIX}_t )</th>
<th>( \Delta \text{SKEW}_t )</th>
<th>( \Delta \text{TIX}_t )</th>
<th>( \Delta \text{Skew}^{\text{NonPar}}_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \text{VIX}_t )</td>
<td>0.0786***</td>
<td>-0.4652***</td>
<td>1.7873***</td>
<td>1.4260***</td>
</tr>
<tr>
<td></td>
<td>[0.004]</td>
<td>[0.0397]</td>
<td>[0.1015]</td>
<td>[0.0995]</td>
</tr>
<tr>
<td>Constant</td>
<td>0.0000</td>
<td>0.0002</td>
<td>0.0011</td>
<td>0.0005</td>
</tr>
<tr>
<td></td>
<td>[0.0000]</td>
<td>[0.0011]</td>
<td>[0.0012]</td>
<td>0.0021</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>47.62%</td>
<td>4.90%</td>
<td>47.75%</td>
<td>5.32%</td>
</tr>
</tbody>
</table>
### Table 6: Regression Results II
Regression results for daily changes of SIX, SKEW and TIX against the contemporaneous change of SPX and a down indicator ISPX⁻. SKEW is a modified version of CBOE SKEW, where SKEW = (CBOE SKEW - 100)/10. There are 4,446 daily observations from January 4, 1996 to August 30, 2013 in each series. The heteroskedasticity-consistent standard errors and covariance matrix is computed according to Newey and West (1994). Regressions equations are presented in equation (23). The daily change is defined as, for example, ΔSPXₜ = ln(SPXₜ/SIXₜ₋₁); except for ΔTIXₜ = TIXₜ – TIXₜ₋₁, due to negative values in the series.

<table>
<thead>
<tr>
<th></th>
<th>ΔSIXₜ</th>
<th>ΔSKEWₜ</th>
<th>ΔTIXₜ</th>
<th>ΔSkewₜ\textsuperscript{NonPar}</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔSPXₜ</td>
<td>-0.3100***</td>
<td>3.1132***</td>
<td>-7.0379***</td>
<td>-3.1993***</td>
</tr>
<tr>
<td></td>
<td>[0.0178]</td>
<td>[0.3395]</td>
<td>[0.4038]</td>
<td>[0.7447]</td>
</tr>
<tr>
<td>ΔSPXₜ⁻</td>
<td>-0.0493**</td>
<td>-0.5276</td>
<td>0.0154**</td>
<td>-1.3844</td>
</tr>
<tr>
<td></td>
<td>[0.0248]</td>
<td>[0.4611]</td>
<td>[0.5638]</td>
<td>[1.0442]</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0000</td>
<td>-0.0029</td>
<td>-0.0021</td>
<td>-0.0047</td>
</tr>
<tr>
<td></td>
<td>[0.0001]</td>
<td>[0.0021]</td>
<td>[0.0023]</td>
<td>[0.0049]</td>
</tr>
<tr>
<td>Adj. R\textsuperscript{2}</td>
<td>36.90%</td>
<td>7.73%</td>
<td>36.92%</td>
<td>1.67%</td>
</tr>
</tbody>
</table>
Table 7: Regression Results III
Regression results for daily changes of SIX and SKEW against the 1-day ahead SPX returns. SKEW is a modified version of CBOE SKEW, where SKEW = (CBOE SKEW - 100)/10. There are 4,446 daily observations from January 4, 1996 to August 30, 2013 in each series. The heteroskedasticity-consistent standard errors and covariance matrix is computed according to Newey and West (1994). Regressions equations are presented in equation (24). The daily change is defined as, for example, $\Delta \text{SPX}_t = \ln(\text{SPX}_t/\text{SIX}_{t-1})$; except for $\Delta \text{TIX}_t = \text{TIX}_t - \text{TIX}_{t-1}$, due to negative values in the series.

<table>
<thead>
<tr>
<th>$\Delta \text{SPX}_{t+1}$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \text{SIX}_t$</td>
<td>-0.0695**</td>
<td>-0.0644**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.0322]</td>
<td>[0.0324]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \text{SKEW}_t$</td>
<td>-0.0012</td>
<td>-0.0019</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.0018]</td>
<td>[0.0017]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \text{TIX}_t$</td>
<td></td>
<td>-0.0078**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.0034]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \text{Skew}^{\text{NonPar}}_t$</td>
<td>-0.0011***</td>
<td>-0.0014***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.0003]</td>
<td>[0.0005]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \text{SPX}_t$</td>
<td>-0.0765***</td>
<td>-0.0656***</td>
<td>-0.0768***</td>
<td>-0.0703***</td>
<td>-0.0699***</td>
</tr>
<tr>
<td></td>
<td>[0.0202]</td>
<td>[0.0203]</td>
<td>[0.0203]</td>
<td>[0.0200]</td>
<td>[0.0217]</td>
</tr>
<tr>
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<td>0.0002</td>
<td>0.0002</td>
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<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>[0.0002]</td>
<td>[0.0002]</td>
<td>[0.0002]</td>
<td>[0.0002]</td>
<td>[0.0002]</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.52%</td>
<td>0.45%</td>
<td>0.51%</td>
<td>0.54%</td>
<td>0.58%</td>
</tr>
</tbody>
</table>
### Table 8: Regression Results IV

Regression Results for SIX and SKEW against 30-day Forward Looking Realized SPX Return Skewness. SKEW is a modified version of CBOE SKEW, where $SKEW = (CBOE \text{ SKEW} - 100)/10$. There are 4,107 daily observations from January 4, 1996 to May 25, 2012 in each series. The heteroskedasticity-consistent standard errors and covariance matrix is computed according to Newey and West (1994). Regressions equations are presented in equation (25).

<table>
<thead>
<tr>
<th>RSkew_{30}</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIX</td>
<td>-0.6461***</td>
<td>-1.3138***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.2683]</td>
<td>[0.3118]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SKEW</td>
<td>-0.0408</td>
<td>-0.0496</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.0360]</td>
<td>[0.0328]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TIX</td>
<td>-0.0280***</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>[0.0118]</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Skew^{NonPar}</td>
<td>2.5192***</td>
<td>3.9329***</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>[0.5222]</td>
<td>[0.6596]</td>
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</tr>
<tr>
<td>Constant</td>
<td>0.6297***</td>
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<td>-0.0180</td>
<td>1.1775***</td>
<td>1.4173***</td>
</tr>
<tr>
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<td>[0.2752]</td>
<td>[0.0686]</td>
<td>[0.0212]</td>
<td>[0.0321]</td>
<td>[0.3225]</td>
</tr>
<tr>
<td>Adj. R^2</td>
<td>0.90%</td>
<td>0.21%</td>
<td>0.88%</td>
<td>2.39%</td>
<td>5.76%</td>
</tr>
</tbody>
</table>