CEO Turnover, Earnings Management, and Big Bath

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Abstract: This paper provides theoretical explanations for a ‘big bath’, a phenomenon where an incoming CEO manipulates a company’s income statement to make poor results look even worse. In a game among an outgoing CEO, an incoming CEO, and outside investors, we show that a big bath can be sustained as an equilibrium outcome under some conditions. One central result is that when earnings report issued by the outgoing CEO is sufficiently low, the incoming CEO’s reporting strategy will feature a big bath. A big bath, on one hand, has a direct adverse effect on the incoming CEO’s payoff from reported earnings and stock price. On the other hand, it can save risk premium paid to investors. If the saving on risk premium outweighs the direct drop in payoff from reported earnings and stock price, a big bath will be induced in equilibrium.

The model contains several empirical predictions, including (1) public regulations that increase earnings management cost for the incoming CEO reduces the likelihood of a big bath provided that the CEO compensation is not totally stock-based; however, the effect of public regulations can be muted if the CEO compensation is totally stock-based (2) a big bath is more likely to occur under stock-based incentive schemes than under accounting-based incentive schemes (3) a big bath is most likely to occur when the outgoing CEO is forced to leave and the incoming CEO is promoted internally, and it is least likely to occur when O peacefully leaves the firm and N is recruited externally.

Keywords: CEO Turnover; Earnings Management; Big Bath; Capital Market Price.

JEL Classification Numbers: D86, G34, M41, M51.

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Preface

Thesis title: Managerial Roles in Financial Disclosure, Organizations, and Markets
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The thesis will take the following structure:

Chapter I: Introduction
Chapter II: Hubris, CEO Compensation, and Earnings Manipulation
Chapter III: Managerial Optimism and Market Competition
Chapter IV: CEO Turnover, Earnings Management, and Big Bath
Chapter V: A Survey on Empirical Findings of Earnings Management
Chapter VI: Conclusion

This paper is based on an analysis of CEO strategic reporting and capital market price as posited in Chapter IV.
1 Introduction

‘Big bath’ in accounting is an earnings management strategy that manipulates a company’s income statement to make poor results look even worse. A number of empirical studies have documented big baths used by managers, e.g. Walsh et al. (1991), Beattie et al. (1994), Hwang and Ryan (2000), Christensen et al. (2008), and Riedl and Sri-nivasan (2010). One particularly interesting empirical finding is that big bath behaviour is observed with incoming CEOs, e.g. Pourciau (1993) and Murphy and Zimmerman (1993). This suggests that CEO turnover may affect the motives for CEOs to manipulate earnings reports.

The existing literature on big bath argues that new CEOs takes big baths so they can blame the company’s poor performance on the previous CEO and take credit for the next year’s improvements. Although the ’blame game’ argument makes sense to some extent, it is not without flaw. First, if this argument works for low earnings, it should also work for high earnings. After all, a CEO prefers a lower performance threshold. It would imply that new CEOs will always take a big bath, but this is not what we observe in practice. Moreover, in business, managerial compensation depends heavily on reported earnings and stock price. If new CEOs take a big bath by reporting poorer earnings, it can drive down the stock price of the firm and the payoff for the incoming CEOs. Then, why would CEOs do that?

Few paper provides theoretical foundations to explain this phenomenon. Kirschenheiter and Melumad (2002) is one exception. They consider a situation where one manager issues financial reports and outside investors infer the precision of reported earnings. They show that if true earnings is bad, the manager will take a big bath to introduce additional noise into his report and reduce the perceived precision of earnings report. Another closely related paper is Beyer (2009). She analyses management earnings forecasting and earnings reporting in a setting where both the mean and the variance of firm’s earnings are unknown. Although her focus is not on big baths, her results show how the equilibrium stock price of the firm depends on management forecast, reported earnings, and squared error in the management
earnings forecast.

The innovation of our paper is that we consider CEO turnover and its effects on managers’ reporting strategies and on the capital market.\(^1\) Our basic set-up is similar to Kirschenheiter and Melumad (2002) and Beyer (2009). What differentiates our model is as follows. In those papers, there is only one single CEO, who is in pursuit of maximizing the end-of-game stock price. In contrast, our paper considers an outgoing CEO and an incoming CEO. They have different objectives. The incentive scheme for the CEO in control is a weighted average of his reported earnings and stock price. If accounting-based incentive weights approach zero, then our model converges to theirs.

CEOs in our model features a leader-follower relationship because an outgoing CEO issues the first earnings report before the incoming CEO issues the second earnings report.\(^2\) Beyer (2008) also considers a leader-follower game, where an outside analyst first issues an earnings forecast and then an inside manager issues an earnings report. She argues that when reporting earnings, the manager trades off the dis-utility he obtains from falling short of the analyst’s forecast against the costs of manipulating earnings. While the objectives of analysts and managers in Beyer (2008) are to minimize forecast and reporting errors, the objectives of CEOs in our paper are to maximize a weighted average of reported earnings and stock price net of earnings management cost. Moreover, a big bath will never happen in Beyer (2008) because the manager always chooses to over-report earnings (see her Proposition 1).

One of our main purposes is to provide theoretical explanations for a big bath. We consider a model where an outgoing CEO issues the first earnings report, an incoming CEO issues the second earnings report, and investors in the capital market update their belief about the firm value based on CEOs’ reported earnings. We derive an equilibrium in which the stock price increases with CEOs’ earnings reports but decreases with perceived variation

\(^{1}\)Some empirical studies support the notion that CEOs who are faced with termination engage in income-increasing earnings management in the year prior to termination, e.g. Guan et al. (2005). Hazarika et al. (2012) consider the converse case. They provide empirical evidence that the likelihood of forced CEO turnover increases with the level of earnings management.
in earnings. Furthermore, we show that if the earnings report issued by the outgoing CEO is sufficiently low, the incoming CEO will adopt reporting strategy that features a big bath. There are direct and indirect effects. The direct effect is that a lower report adversely affects the incoming CEO payoff from reported earnings and adversely affect stock price. The indirect effect is that outside risk averse investors will require a lower risk premium since a big bath helps reduce the inferred variation in earnings. If the saving on risk premium outweighs the direct drop in payoff from reported earnings and stock price, a big bath will be induced in equilibrium.

To see this, suppose that the outgoing CEO first issues a strongly negative public earnings report, say, $-100$. Then, the incoming CEO privately receives a mediocre earnings, say, 0. He can choose to report $+50$, 0, or $-50$, to maximize stock price net of his personal earnings management cost. If earnings management is prohibitively high, then the incoming CEO should report 0. If earnings management cost is not too high, it can be better for him to report $-50$ than $+50$. The cost of reporting $-50$ is that it will reduce his payoff from reported earnings and adversely affect stock price as bad signals. However, it reduces the perceived variation in earnings since $-50$ is closer to $-100$, which in turn lower the risk premium required by outside investors. If the saving on risk premium benefit dominates the cost, the incoming CEO will be induce report $-50$.

The rest of this paper is organized as follows. Section 2 presents the basic model. The optimal reporting strategies for incoming CEO and outgoing CEO, and the equilibrium stock price in the capital market are analysed in Section 3. We derive many empirical implications in Section 4. Section 5 concludes. Omitted proof is collected in Appendix.

2 The model

We consider a two-period model adapted from Beyer (2009). The model has two main features: (1) both the mean and the variance of the firm’s earnings are unknown (2) CEO
turnover happens at the end period 1.\footnote{The first feature is shared with Kirschenheiter and Melumad (2002) and Beyer (2009), while the second feature is unique to our paper.}

## 2.1 Players and Production

An all-equity firm hires an outgoing CEO (O) for period 1 and another incoming CEO (N) for period 2.\footnote{The size of equity is normalized to be one.} Both CEOs are risk neutral. CEO turnover occurs at the end period 1 exogenously. The firm stochastically generates earnings \( x_t \) at the end of period \( t, t = 1, 2 \).\footnote{We indicate random variables with ‘tilde’ and their realizations without ‘tilde’.} As in Sankar and Subramanyam (2001) and Liang (2004), all realizations of earnings are retained in the firm until the firm liquidates dividends after period 2. O (N) can privately observe the realization of period 1 (2) earnings and is required to issue a publicly observable report for it. After issuance of period 2 earnings report and before liquidation, there exists a capital market where stock shares of the firm will be sold to outside investors. The next generation of shareholders are entitled to liquidated dividends \( x_1 + x_2 \).\footnote{The results do not rely on the whole company being sold to outside investors. All that is needed is that the managers’ utility is linearly increasing in stock price.}

To be specific, period \( t \) earnings are assumed to be generated as follows:\footnote{We exogenously specify earnings and avoid production issues to focus on managers’ reporting behaviour. This paper does not consider ‘real’ earnings management where managers determine production decision endogenously.}

\[
\tilde{x}_t = \tilde{\mu}_t + \tilde{v}_t
\]  

where \( \tilde{v}_t \sim N(0, \frac{1}{\tilde{\tau}}) \) is the period \( t \) idiosyncratic shock, \( \tilde{\mu}_t \sim N(0, k_1 \frac{1}{\tilde{\tau}}) \) is the firm’s period \( t \) expected gain, and \( \tilde{\tau} \) is the random precision of \( \tilde{v}_t \).\footnote{Unlike Beyer (2009), we allow the variance of \( \tilde{\mu}_t \) to be random and assume it to be a multiple of variance of \( \tilde{v}_t \). Without such assumption, it will be difficult to derive the equilibrium stock price.} The realization of \( \tilde{\tau} \) is unknown to all players throughout the game, its prior distribution is commonly known to be a Gamma distribution with shape parameter \( \alpha > 1/2 \) and scale parameter \( \beta > 0 \).\footnote{That is, the density \( g(\tau) = \frac{\alpha^{\alpha-1} \exp(-\beta \tau)}{\beta^{\alpha} \Gamma(\alpha)} \). As in Beyer (2009), it can be shown that the equilibrium in Proposition 1 would remain the same if managers privately observed \( \tau \) before issuing earnings report. The key is that outside investors are uncertain about the precision. Parameters \( \alpha \) and \( \beta \) may depend on the}
assumed that $\tilde{\mu}_t$ and $\tilde{\nu}_t$, $t = 1, 2$, are independent, and $\tilde{\nu}_2$ and $\tilde{\nu}_1$ are independent. Moreover, since CEOs work in the same firm, earnings may be linked. To reflect possible persistence of earnings, we assume $\rho = \text{corr}(\tilde{x}_1, \tilde{x}_2)$, where $\rho \in [0, 1]$. It captures to what extent period 1 earnings predicts period 2 earnings. Our distributional assumptions imply that $(\tilde{x}_1, \tilde{x}_2)$ follows bivariate normal with mean vector $(0, 0)$ and covariance matrix

$$
\begin{pmatrix}
(1 + k_1)\frac{1}{\tau} & \rho(1 + k_1)\frac{1}{\tau} \\
\rho(1 + k_1)\frac{1}{\tau} & (1 + k_1)\frac{1}{\tau}
\end{pmatrix}
$$

Following prior theoretical work (Bushman and Indjejikian, 1993; Kim and Suh, 1993; Sankar and Subramanyam, 2001) and empirical evidence (Aggarwal, 2008; Jacque, 2008), we assume that an executive compensation scheme consists of an accounting-based part and a stock-based part, and compensation schemes are exogenously given. That is, O’s incentive scheme is $(1 - s)R_1 + sP$, and N’s incentive scheme is $(1 - w)R_2 + wP$, where $R_1$ is period 1 earnings report, $R_2$ is period 2 earnings report, $P$ is stock price, $s$ ($w$) is stock-based incentive weight measuring how much O’s (N’s) payoff depends on stock price (e.g. vesting of stock option), and $1 - s$ ($1 - w$) is accounting-based incentive weight measuring how much O’s (N’s) payoff depends on reported earnings. With accounting-based incentive, the CEO will be tempted to inflate earnings. With stock-based incentive, the CEO will be tempted to manipulate his earnings report in a way to boost the stock price. However, stock price is determined in the rational capital market, which cannot be easily fooled, so the role of stock-based incentive is more subtle.

Investors in the capital market are rational and risk averse. It means that the risk-

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9 It implies that $\text{corr}(\tilde{\mu}_1, \tilde{\mu}_2) = \frac{1 + k_1}{k_1} \rho$ since $\text{Cov}(\tilde{\mu}_1, \tilde{\mu}_2) = \text{Cov}(\tilde{x}_1 - \tilde{\nu}_1, \tilde{x}_2 - \tilde{\nu}_2) = \text{Cov}(\tilde{x}_1, \tilde{x}_2) = \rho \text{Var}(\tilde{x}_1) = \rho(1 + k_1)\frac{1}{\tau}$. On the other hand, $\text{Cov}(\tilde{\mu}_1, \tilde{\mu}_2) = \text{corr}(\tilde{\mu}_1, \tilde{\mu}_2) \text{Var}(\tilde{\mu}_1) = \text{corr}(\tilde{\mu}_1, \tilde{\mu}_2) k_1 \frac{1}{\tau}$. Setting these two terms equal yields the desired result. Similar argument shows that $\text{corr}(\tilde{y}_1, \tilde{y}_2) = \frac{1 + k_1}{1 + k_1 + k_2} \rho$. This is unlike Beyer (2009). If $\rho = 0$, innovations are transitory and current earnings provide no information about future earnings. However, because CEOs’ earnings reports together affect stock price, there is still connection between O’s and N’s reporting. We will discuss more about this in Section 5.

10 As in Kirschenheiter and Melumad (2002) and Beyer (2009), we abstract from contracting issues for simplicity.
adjusted market price of the firm is \( P(\Omega) = E[\bar{x}_1 + \bar{x}_2 | \Omega] - \gamma Var[\bar{x}_1 + \bar{x}_2 | \Omega] \), where \( \Omega \) represents all information available to investors and \( \gamma \) measures investors’ risk aversion.\(^{11}\) In the model, the information available to investors are CEOs’ earnings reports, so rational investors value the firm based on earnings reports and are aware of potential reporting bias.\(^{12}\) With stock-based incentive schemes, CEOs will be induced to manipulate their earnings reports to increases perceived mean and decrease perceived variance.

Because it is crucial for our analysis, we explain more on \( \tilde{\tau} \) being unknown to outside investors. Outside investors are rational and risk-averse, so they update their belief about variance of earnings (risk) after they observe earnings reports and require a risk premium to compensate risks borne by them. If the precision were known to them, the variance of earnings and the associated risk premium would be a commonly known constant, and earnings reports issued by CEOs would not change the perceived risk. However, an attempts to reduce risk perceived by outside investors is the key driving force for a big bath in our model. The assumption of \( \tilde{\tau} \) being unknown to outside investors is thus essential for our results.

2.2 Earnings Report

At the end of period 1, O privately observes the realization of period 1 earnings \( x_1 \), and then he is required to issue an earnings report \( R_1 \). However, O is not confined to tell the truth and can issue an earnings report the differs from the true period 1 earnings at some personal cost. Following Dye and Sridhar (2004), earnings management cost (EMC) for O is assumed to be: \( \frac{C_0}{2} (R_1 - x_1 - \epsilon_1)^2 \), where \( C_0 \) is unit EMC for O and \( \epsilon_1 \) is a noise. Similarly, at the end of period 2, N privately observes the realization of period 2 earnings \( x_2 \), and then he is required to issue an earnings report \( R_2 \). N can bias his earnings report at some personal cost: \( \frac{C_N}{2} (R_2 - x_2 - \epsilon_2)^2 \), where \( C_N \) is unit EMC for N and \( \epsilon_2 \) is another noise.

\(^{11}\)Verrecchia (1983) and Melumad et al. (1999) make similar assumptions about firms’ stock price
\(^{12}\)If a stock price is formed at the end of period 1, then \( \Omega \) contains \( P_1 \). However, we find that \( P_1 \) will be quadratic in \( y_1 \) and is not normally distribution. It will be difficult to derive stock price at the end of period 2. Thus, we only allow trading at the end of period 2 for tractability.
The realization of $\epsilon_t$ is privately observed by CEO in control at the end of period $t$. It is a priori unknown to everyone (even CEO himself) yet commonly known to be identically and independently normally distributed: $\tilde{\epsilon}_t \sim N (0, k_2^{1/2})$. Moreover, N must take O’s earnings report into account when choosing his own earnings report since outside investors value the firm based on both earnings reports.

The existence of noises in EMC prevents outside investors from perfectly infer true earnings. To see this, note that noises in EMC break down the direct mapping between actual earnings and reported earnings, obscuring the true performance under CEO management. The reporting technology places restrictions on the ability of the CEO to communicate the truth. That is, the CEO in control privately observes two dimensions of information, including realization of earnings and realization of noise in his EMC. However, he is only permitted to communicate by a single-dimensional signal, that is, his earnings report. Communication is restricted in that the CEO cannot communicate the full dimensionality of his private information due to a limited message space, and hence the Revelation Principle is not applicable here. As a result, the reporting function is not invertible, and true earnings cannot be unambiguously backed out from earnings reports.\(^\text{13}\)

Dye and Sridhar (2004) gives two interpretations of $\epsilon_t$. First, it is a random variable reflecting idiosyncratic circumstances that influence the CEO’s misreporting costs. For example, if litigation arises related to this report, courts may see the firm’s actual circumstances with error; that is, they may see only $x_1 + \epsilon_1$ and base their assessment of damages on this distorted image of $R_1$. Second, the CEO observes $x_1$ with error, i.e., the CEO sees only $x_1^e = x_1 + \epsilon_1$, and the CEO’s expected liability takes the form, $\frac{C_0}{2} (R_1 - x_1^e)^2$. We provide an alternative interpretation: $\epsilon_t$ can reflect the CEO’s psychic cost of EM and may depend on the CEO’s identification of with the firm. When $\epsilon_t > 0$, the CEO tends to over-report; when $\epsilon_t < 0$, the CEO tends to under-report.

\(^\text{13}\)See Aeya et al. (1988) for an excellent discussion of Revelation Principle in accounting.
2.3 Information and Time

All players share the same prior beliefs about distributions of random variables in the model. The parameters of the model, \((s, w, C_O, C_N, k_1, k_2, \alpha, \beta, \gamma, \rho)\), and the sequence of events are common knowledge.\(^{14}\) The realizations of \(x_1\) and \(\epsilon_1\) are private information of O, and the realizations of \(x_2\) and \(\epsilon_2\) are private information of N. The choice of reporting bias is hidden action of the CEO in control.

The time sequence of events is as follows. During period 1, O is in control. At the end of period 1, O privately observes \(x_1\) and \(\epsilon_1\) and issues a public report \(R_1\). Then CEO turnover happens: O leaves the firm and N takes control. During period 2, N is in control. At the end of period 2, N privately observes \(x_2\) and \(\epsilon_2\) and issues a public report \(R_2\). After that, stock shares are sold to outside investors at a market price based on \(R_1\) and \(R_2\). The next generation of shareholders are entitled to liquidated dividends of \(x_1\) and \(x_2\). Figure 1 summarizes the sequence of events, and Table 1 lists notations.

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\(^{14}\)See Sunder (2002) for a discussion of how common knowledge can be closely related to accounting and capital market research.
Table 1: Notation Table.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>period t earnings</td>
</tr>
<tr>
<td>$R_t$</td>
<td>period t earnings report</td>
</tr>
<tr>
<td>$\mu_t$</td>
<td>expected value of $x_t$</td>
</tr>
<tr>
<td>$v_t$</td>
<td>period t idiosyncratic shock to earnings</td>
</tr>
<tr>
<td>$\epsilon_t$</td>
<td>CEOs’ earnings management cost</td>
</tr>
<tr>
<td>$y_t$</td>
<td>sum of $x_t$ and $\epsilon_t$</td>
</tr>
<tr>
<td>$\frac{1}{\tau}$</td>
<td>variance of $v_t$</td>
</tr>
<tr>
<td>$\alpha$, $\beta$</td>
<td>shape and scale parameter of $\tau$</td>
</tr>
<tr>
<td>$k_1\frac{1}{\tau}$</td>
<td>variance of $\mu_t$</td>
</tr>
<tr>
<td>$k_2\frac{1}{\tau}$</td>
<td>variance of $\epsilon_t$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>correlation between earnings</td>
</tr>
<tr>
<td>$C_O$</td>
<td>unit earnings management cost for O</td>
</tr>
<tr>
<td>$C_N$</td>
<td>unit earnings management cost for N</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>information available to investors</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>measure of risk aversion of investors</td>
</tr>
<tr>
<td>$s$</td>
<td>stock share offered to O</td>
</tr>
<tr>
<td>$w$</td>
<td>stock share offered to N</td>
</tr>
</tbody>
</table>

2.4 Definition of Equilibrium

An equilibrium in this model consists of a reporting rule by O, $R_1(.)$, a reporting rule by N, $R_2(.)$, and a capital market pricing rule, $P(.)$, such that:15

(i) Given the pricing function $P(.)$, a report issued at the end of period 1 $R_1$, and N’s private information $(x_2, \epsilon_2)$, N chooses an earnings report (a real number) $R_2 = R_2(R_1, x_2, \epsilon_2)$ to maximize a weighted average of period 2 earnings report and stock price net of his personal EMC:

$$\max_{R_2}(1 - w)R_2 + wP(R_1, R_2) - \frac{C_N}{2}(R_2 - x_2 - \epsilon_2)^2$$ (2.2)

(ii) Given the pricing function $P(.)$, the reporting rule by N $R_2(.)$, and O’s private information $(x_1, \epsilon_1)$, O chooses an earnings report (a real number) $R_1 = R_1(x_1, \epsilon_1)$ to maximize a weighted average of period 1 reported earnings and stock price:

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15The equilibrium concept we use here is noisy rational expectation equilibrium.
\[
\max_{R_1}(1-s)R_1 + sE[\{P(R_1, R_2(R_1, \tilde{x}_2, \tilde{\epsilon}_2)) \mid x_1, \epsilon_1\} - \frac{C_O}{2}(R_1 - x_1 - \epsilon_1)^2 \quad (2.3)
\]

(iii) Given the reporting rule by O \(R_1(.)\) and the reporting rule by N \(R_2(.)\), the capital market price \(P(\Omega)\) satisfy:

\[
P(\Omega) = E[\tilde{x}_1 + \tilde{x}_2 \mid \Omega] - \gamma Var[\tilde{x}_1 + \tilde{x}_2 \mid \Omega] \quad (2.4)
\]

where \(\Omega = \{R_1, R_2\}\).

Based on information available to each player, we conjecture the equilibrium to be:\footnote{Our conjecture differs from Beyer (2009).}

\[
R_2(R_1, y_2) = r_0 + r_1R_1 + r_2y_2 \quad (2.5)
\]

\[
R_1(y_1) = m_0 + m_1y_1 \quad (2.6)
\]

\[
P(R_1, R_2) = p_0 + p_1R_1 + p_2R_2 + p_3 \left[ \left( \frac{R_2 - (r_0 + r_1R_1)}{r_2} \right)^2 + \left( \frac{R_1 - m_0}{m_1} \right)^2 \right] + p_4 \left( \frac{R_2 - r_0 - r_1R_1}{r_2} \right) \left( \frac{R_1 - m_0}{m_1} \right) \quad (2.7)
\]

where \(y_t \equiv x_t + \epsilon_t, \; t = 1, 2\), is the sum of the CEO’s private information, which can be inferred from earnings reports. However, investors are uncertain about the components. That is why investors cannot perfectly back out true earnings from earnings reports.

3 Results

3.1 Equilibrium

The following proposition characterizes the reporting strategy of the outgoing CEO and the incoming CEO and the associated capital market equilibrium price. In the equilibrium,
managers’ reporting strategy is a linear function of his private information and his predecessor’s earnings report (if any), and the stock price is a quadratic function of managers’ earnings report. The quadratic term captures investors updating their beliefs about the variance of the firm’s earnings.

**Proposition 1** There is an equilibrium in the model defined as follows.\(^{17}\)

(i) N’s optimal reporting strategy is:

\[
R_2(R_1, y_2) = r_0^* + r_1^* R_1 + r_2^* y_2
\]

where \(y_2 = x_2 + \epsilon_2\) and the coefficients are given by

\[
\begin{align*}
   r_2^* &= \frac{1}{2} (1 + A_N) \\
   r_1^* &= \frac{\rho(1 + k_1)(1 - A_N)}{(1 + k_1 + k_2)(1 + A_O)} \\
   r_0^* &= \frac{m_0^* p_4^* \left(C_N - \sqrt{C_N^2 + 8w p_3^* C_N}\right) + 4m_1^* p_3^*[1 - (1 - p_2^*)w]}{4C_N m_1^* p_3^*}
\end{align*}
\]

where\(^{18}\)

\[
A_N \equiv \sqrt{1 - \frac{8w\gamma(1 + \rho)(1 + k_1)k_2(1 + k_1 + k_2)}{C_N \alpha[(1 + k_1)(1 + \rho) + k_2]^2[1 + k_1 + k_2 - (1 + k_1)\rho]}}
\]

\[
A_O \equiv \sqrt{1 - \frac{8s\gamma(1 + \rho)(1 + k_1)k_2}{C_O \alpha(1 + k_1 + k_2)[(1 + k_1)(1 + \rho) + k_2]}}
\]

and \(p_3^*\) and \(p_4^*\) are defined as below.

(ii) The O’s optimal reporting strategy is:

\[
R_1(y_1) = m_0^* + m_1^* y_1
\]
where \( y_1 = x_1 + \epsilon_1 \) and the coefficients are given by

\[
\begin{align*}
m^*_1 &= \frac{1}{2} (1 + A_O) \\
m^*_0 &= \frac{1 + k_2 + s(1 - k_2) + \rho(1 + s) + k_1(1 + \rho)(1 + s) + (1 - s)[(1 + k_1)(1 + \rho) + k_2]A_O}{C_O[(1 + k_1)(1 + \rho) + k_2](1 + A_O)}
\end{align*}
\]

(iii) The equilibrium pricing function is:

\[
P(R_1, R_2) = p^*_0 + p^*_1 R_1 + p^*_2 R_2 + p^*_3 \left[ \left( \frac{R_2 - (r^*_0 + r^*_1 R_1)}{r^*_2} \right)^2 + \left( \frac{R_1 - m^*_0}{m^*_1} \right)^2 \right] + p^*_4 \left( \frac{R_2 - r^*_0 - r^*_1 R_1}{r^*_2} \right) \left( \frac{R_1 - m^*_0}{m^*_1} \right)
\]

where the pricing coefficients are given by

\[
\begin{align*}
p^*_4 &= \frac{2\gamma \rho(1 + \rho)(1 + k_1)^2 k_2}{\alpha[(1 + k_1)(1 + \rho) + k_2][(1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2]} \\
p^*_3 &= -\frac{\gamma(1 + \rho)(1 + k_1) k_2(1 + k_1 + k_2)}{\alpha[(1 + k_1)(1 + \rho) + k_2][(1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2]} \\
p^*_2 &= \frac{2(1 + \rho)(1 + k_1)}{(1 + A_N)[(1 + k_1)(1 + \rho) + k_2]} \\
p^*_1 &= \frac{2(1 + \rho)(1 + k_1) [(1 + k_1)(1 - \rho) + k_2] + (1 + k_1)(1 + \rho) + k_2 A_N}{(1 + A_N)(1 + A_O)(1 + k_1 + k_2)[(1 + k_1)(1 + \rho) + k_2]} \\
p^*_0 &= -\frac{2\gamma \beta(1 + k_1) k_2(1 + \rho)}{\alpha[(1 + k_1)(1 + \rho) + k_2]} - p^*_1 m^*_0 - p^*_2 (r^*_0 + r^*_1 m^*_0)
\end{align*}
\]

3.2 Stock Price

The equilibrium stock price has some intuitively appealing properties. First, it indicate that investor use some squared term and product term of earnings reports to measure risk. \( p^*_4 > 0 \) unless \( \rho = 0 \). It means that when investors perceive that \( y_1 \) and \( y_2 \) have different signs, they will require higher risk premium. \( p^*_3 < 0 \). It means that the risk premium will be higher when investors perceive that absolute variation in \( y' \)'s is higher. Note that the less risk averse investors are, the lower \( p^*_4 \) and \( p^*_3 \) (in absolute value), the higher risk premium is
required. Second, it indicates that management earnings reports have an impact on stock price. \( p_1^* > 0 \) and \( p_2^* > 0 \). It means that higher earnings reports can boost stock price if risk premium remains constant. Lastly, \( p_0^* < 0 \). It indicates a lower bound for earnings reports. When earnings reports fall below the lower bound, stock price will be non-positive.\(^{20}\)

### 3.3 O's reporting

For O’s reporting strategy, we first find that if \( C_O \) approaches infinity, then \( m_1^* = 1 \) and \( m_0^* = 0 \). This means that if EMC for O is prohibitively high, O’s main objective is to minimize his EMC. Then his reporting strategy is fully responsive to \( y_1 \). Second, if \( s = 0 \), then \( m_1^* = 1 \) and \( m_0^* = \frac{1}{C_O} > 0 \). This is intuitive: if O only cares about his reported earnings, then he has strong incentive to over-report. As Fig 2 indicates, O will always over-report with a constant mark-up of \( \frac{1}{C_O} \). This mark-up reflects O’s information rent, and it decreases with EMC for O.

On the other hand, if \( s > 0 \), then \( 0 < m_1^* < 1 \) and \( \frac{1}{C_O} > m_0^* \).\(^{21}\) When O cares not only reported earnings but stock price, he will choose a smoother reporting rule with less over-reporting at \( y_1 = 0 \) to reduce the perceived variation in earnings and the associated risk premium. As Fig 3 suggests, if O’s payoff also depends on stock price, O is willing to give up some information rent by choosing a reporting function that is flatter than 45 degree line with a smaller intercept. In this case, he benefits more from a higher stock price although he gives up some information rent he could have gained from reported earnings, .

\(^{19}\)If investors are risk-neutral, then \( p_1^* = p_3^* = 0 \).

\(^{20}\)We do not need to impose non-negative constraints on stock price for it is positive in equilibrium. Note that the scale parameter \( \beta \) affects \( p_0^* \) but not \( p_t^* \), \( t = 1, 2, 3 \). This is because \( \beta \) only changes the scale (numeraire) of the model and thus affects the capital market prices only through the constant term, \( p_0^* \). Also note that when \( C_O \) and \( C_N \) approach infinity, \( A_O \) and \( A_N \) will be 1, making \( p_1^* \) and \( p_2^* \) independent of \( \alpha \).

\(^{21}\)\( \frac{1}{C_O} > m_0^* \) because \( \partial m_0^*/\partial s < 0 \). For the sign of \( m_0^* \), we find that \( m_0^* > 0 \) if \( k_2 \leq 1 \) and \( 0 < s < 1 \).
3.4 N’s reporting

We first find that if $C_N$ approaches infinity, then $r_2^* = 1$, $r_1^* = r_0^* = 0$. This means that if EMC for N is prohibitively high, N’s main objective is to minimize his EMC and N cares little about stock price and reported earnings. Hence, his reporting strategy is fully responsive to $y_2$ and not responsive to O’s earnings report. Second, if $\rho = 0$, then $r_1^* = 0$. Recall that if $\rho = 0$, it also implies that $p_1^* = 0$. The direct and indirect link between $R_1$ and $R_2$ break down.\textsuperscript{22} Third, if $w = 0$, then $r_2^* = 1$, $r_1^* = 0$ and $r_0^* = \frac{1}{C_N}$. It states that\textsuperscript{22}When $\rho = 0$, $0 < r_2^* < 1$ and $0 < r_0^* < 1$. 

\textsuperscript{22}
when N’s incentive weight on stock price is zero, N’s report will fully reveal \( y_2 \) and will be unresponsive to O’s report. N’s reporting rule becomes a straight line with a slope of 1 and a intercept of \( \frac{1}{C_N} \) on \((y_2, R_2)\) plot. This intercept reflects information rent of N and decreases with N’s EMC.

On the other hand, if \( w > 0 \), then \( 0 < r^*_2 < 1 \) and \( 0 < r^*_1 < 1 \). Also, \( r^*_0 > 0 \) for sufficiently low \( \rho \).\(^{23}\) It says that when N’s incentive weight on stock price is positive, N’s earnings report will be increasing in \( y_2 \) and O’s earnings report yet with less proportion than 1. The intuition is similar to O’s case. Since N has incentive to boost stock price, he chooses a smoother reporting rule to reduce the perceived variation in earnings and the associated risk premium (See Fig 4). We will elaborate on this in the next section.

### 3.5 Big Bath

For ease of exposition, we call the situation where \( R_2 < x_2 \) for some \( x_2 \leq 0 \) a big bath. It is a situation where N issues a poorer earnings report when the actual earnings is already poor. Clearly, a poorer earnings report reduces N’s payoff from reported earnings and has an adverse impact on stock price. Then why would N take a big bath? The next proposition indicates some possible driving forces behind a big bath.

**Proposition 2** Whether a big bath can occur in equilibrium depends on EMC for N, the noise in EMC for N, the correlation between earnings, N’s incentive weight on stock price, and O’s earnings report:

(i) When \( C_N \) approaches infinity, or \( \rho = 0 \), or \( w = 0 \), a big bath will not occur in equilibrium unless the noise in EMC for N is sufficiently low.\(^{24}\)

(ii) When \( w > 0 \), \( \rho > 0 \), and \( C_N < \infty \), a big bath will occur in equilibrium if (a) the noise in N’s EMC is sufficiently low, or/and if (b) O’s earnings report is sufficiently low.\(^{25}\)

\(^{23}\)This can be shown by taking limits: \( \lim_{\rho \to 0} r^*_0 = \) because \( \partial r^*_0 / \partial w < 0 \). Also, \( r^*_0 > 0 \).

\(^{24}\)See Appendix for the conditions.

\(^{25}\)See Appendix for the conditions.
Proof. See Appendix.

Intuitively, when $C_N$ is prohibitively high, N’s objective is to minimize his EMC, so his earnings report will be responsive only to $y_2$. Also, when $\rho = 0$, $x_1$ and $x_2$ are not correlated, and then pricing coefficient on the cross product term becomes zero. The link between $R_1$ and $R_2$ breaks down. When $w = 0$, N’s payoff depends only on his reported earnings, making his not concerned about stock price. In these three cases, N’s reporting will not be responsive to O’s earnings report. The only driving force for a big bath will be $\epsilon_2$ being sufficiently low.

When $w > 0$, $\rho > 0$, and $C_N < \infty$, N’s earnings report only partially reveals $y_2$ and depends on O’s earnings report. Strategic concern between the outgoing CEO and the incoming CEOt thus plays a vital role. In particular, we find that if earnings report issued by O is strongly negative, then N will be induced to take a big bath.

To better illustrate how interaction between O and N can lead to a big bath, we suppress the effect of noise in EMC by assuming $\epsilon_2 = 0$ for the moment. According to Proposition 1, when $w > 0$, $\rho > 0$, and $C_N < \infty$, N’s reporting function will be a line flatter than 45 degree line with an intercept of $r^*_0 + r^*_1 R_1$ on the plot ($y_2$, $R_2$), as in Fig 4. It says that if O issues $R_1 \geq 0$, N will issue a $R_2 > x_2$ for all $x_2 \leq 0$. Thus, there is no big bath in this case.\(^{26}\) The intuition is that if N were to take a big bath, he would first experience a drop in his payoff from reported earnings ($1 - w > 0$) and from stock price ($w > 0$ and $p_2^* > 0$). Secondly he would experience a higher risk premium paid to investors because of a change of sign in perceived $x_1$ and $x_2$ ($p_4^* > 0$). N thus has no incentive to take a big bath in this case. This result is consistent with empirical finding of Abarbanell and Lehavy (2003). They suggest that firms that perform poorly may not take a bath when the firm’s stock price remains sensitive to the level and/or composition of reported earnings.

However, if O issues a strongly negative $R_1$ and ceteris paribus, it will shift the reporting function of N downwards. There will thus exist an interval for a big bath to occur, $[\bar{x}_2, 0]$.\(^{26}\) $r^*_0$ can be negative. Nevertheless we find that $\lim_{\gamma \to 0} r^*_0 > 0$ and $\lim_{\gamma \to 0} r^*_0 > 0$. Without loss of generality, we focus on the case where $r^*_0 > 0$ for illustration. Moreover, for any $R_1 \geq 0$, there will be no big bath. This can be seen by noting that the risk premium part in the stock price can only be minimized by a positive $R_2$ when $R_1 \geq 0$.\(^{26}\)
To see this, note that if N takes a big bath ($R_2 < x_2 \leq 0$), there will be direct and indirect effects. The direct effect is a drop in payoff from reported earnings ($1 - w > 0$) and from stock price ($p_2^* > 0$ and $w > 0$). The indirect effect is more subtle. If O issues a strongly negative $R_1$, outside investors will infer that $y_1$ is also strongly negative due to O’s reporting rule ($m_1^* > 0$ and $m_0^* > 0$). Suppose now that N receives medium earnings, say $x_2 = 0$, at the end of period 2. Even if N knows that issuing a $R_2 < 0$ will adversely affect the stock price, N may choose to issue a $R_2 < 0$ to make investors believe that $x_2 < 0$. Because by doing so, N can reduce the perceived variation in earnings and the associated risk premium. Putting direct and indirect effects together, if the indirect effect outweighs the direct effect, N will be induced to take a big bath. Note that in contrast to Kirschenheiter and Melumad (2002), a big bath will not happen for extremely low (strongly negative) $x_2$ in our model. The reason is that for extremely low $x_2$, the saving on risk premium is not enough to compensate the direct drop in reported earnings and stock price, so N chooses not to take a big bath.

Figure 4: Reporting Function of N
4 Empirical Implications

This section examines how EMC for O, EMC for N, O’s incentive weight on stock price, and N’s incentive weight on stock price affect equilibrium reporting strategies, equilibrium stock price, and the likelihood of a big bath.

4.1 Effects of $C_O$

We summarize main results in this section by the following proposition.

**Proposition 3** The effects of O’s EMC are:

(i) When $C_O$ gets higher, O’s reporting strategy is more responsive to $y_1$ and has a smaller intercept at $y_1 = 0$. If $C_O$ approaches infinity, O’s optimal reporting rule coincides with the 45 degree line on $(y_1, R_1)$ plots.

(ii) When $C_O$ gets higher, the responsiveness of N’s reporting strategy toward N’s private information is unaffected, yet N’s reporting strategy becomes less responsive toward O’s report unless $\rho = 0$ and has a larger intercept at $R_1 = y_2 = 0$ if $\frac{s(1-s)}{C_O}$ is relatively small.

(iii) When $C_O$ gets higher, the sensitivity of stock price to N’s report and to risks are unaffected, yet the sensitivity to O’s report decreases.

(iv) The likelihood of a big bath is unaffected by $C_O$.

**Proof.** See Appendix.

The intuition is explained as follows. When $C_O$ increases, O will be more induced to minimize his EMC, so his reporting rule becomes more responsive to $y_1$, and O over-reports less at $y_1 = 0$ because his information rent shrinks as EMC for him rises. When $C_O$ approaches infinity, O will only try to minimize his EMC. His reporting function will be fully responsive to $y_1$ and coincide with the 45 degree line on the plots $(y_1, R_1)$, as in Fig 5.

When $C_O$ increases, the responsiveness of N’s reporting rule to $y_2$ will not change but the responsiveness to $R_1$ will be lower. This is because N, as the second-mover, will adapt
to any change in O’s reporting rule. N knows now that $m_1^*$ gets larger and $m_0^*$ gets smaller, so N will try to back out the change in $R_1$ by lowering $r_1^*$ and raising $r_0^*$.

When EMC for O gets higher, the capital market will decrease its sensitivity to O’s report, while sensitivities to N’s report and to risk are not affected. This seemingly counter-intuitive result makes good economics sense after some reflections. When $C_O$ rises, $m_1^*$ gets closer to 1. On one hand, it indicates that O reports more truthfully about $x_1$. On the other hand, it also implies that O’s reporting rule is more sensitive to the noise in his EMC since an increase in $y_1$ can be caused not only by an increase in $x_1$ but also by an increase in $\epsilon_1$. This introduces more noises into O’s earnings report, so investors is less sensitive to O’s earnings report.

As Fig 4 indicates, $P(\text{big bath}) = P(r_0^* + r_1^* R_1 < 0, \bar{x}_2 < x_2 < 0)$. Since $C_O$ is borne by O, N will try to back out any change in $R_1$ caused by $C_O$ by adapting his reporting rule such that $r_0^* + r_1^* R_1$ and $\bar{x}_2$ remain intact. The likelihood of a big bath is thus unaffected by $C_O$.

### 4.2 Effects of $C_N$

Using the same argument as the case of $C_O$, we have the following proposition.
Proposition 4 The effects of $N$’s EMC are:

(i) When $C_N$ gets higher, $O$’s reporting strategy is unaffected.

(ii) When $C_N$ gets higher, $N$’s reporting strategy become more responsive toward $N$’s private information and less responsive toward $O$’s report, and it has a smaller intercept at $R_1 = y_2 = 0$ if $\frac{(1+\rho)}{C_N}$ is relatively small. When $C_N$ approaches infinity, $N$’s reporting rule coincides with the 45 degree line on $(y_2, R_2)$ plots.

(iii) When $C_N$ gets higher, the sensitivities of stock price to risks are unaffected, yet the sensitivity to $N$’s report decreases, and the sensitivity to $O$’s report increases.

(iv) The likelihood of a big bath decreases with $C_N$ unless $w = 1$.

Proof. See Appendix □

The intuition here is the same as Proposition 3. One additional interesting finding is part (iv). It implies that public regulations that increase earnings management cost for the incoming CEO reduces the likelihood of a big bath for $w < 1$. However, if $N$’s payoff is totally stock-based, i.e. $w = 1$, then the effect of public regulation will be muted. It tell us that when designing policies to regulate earnings management, the regulatory body should be aware of this potential substitution relationship between incentive schemes and public regulations. Otherwise, all attempts will be in vain.

4.3 Effects of $s$

The main results in this section are summarized in the following proposition.

Proposition 5 The effects of $O$’s incentive weight on stock price are:

(i) When $s$ increases, $O$’s reporting strategy is less sensitive to $y_1$ and has a smaller intercept at $y_1 = 0$ if $\frac{s}{CO}$ is relatively small.

(ii) When $s$ increases, the sensitivity of $N$’s reporting strategy to $y_2$ is unaffected, yet $N$’s reporting strategy becomes more sensitive to $O$’s earnings report and has a larger intercept at $R_1 = y_2 = 0$ if $\frac{(1+\rho)}{CO}$ is relatively small.
(iii) When \( s \) rises, the sensitivities of stock price to risks and to \( N \)'s earnings report are unaffected, and the sensitivity of stock price to \( O \)'s earnings report increases.

(iv) The likelihood of a big bath is unaffected by \( s \).

**Proof.** See Appendix.  ■

The intuition is explained as follows. When \( O \)'s incentive weight on stock price is higher, \( O \)'s reporting function will become flatter and have a smaller intercept at \( y_1 = 0 \). This is because when \( O \) cares more about stock price, \( O \) smooths his earnings report and over-reports less at \( y_1 = 0 \) in order to reduce perceived variation in earnings.

When \( O \)'s incentive weight on stock price is higher, \( N \)'s reporting rule is more sensitive to \( O \)'s report and has a larger intercept at \( R_1 = y_2 = 0 \). This is because \( N \) knows now that \( m_1^* \) is lower and \( m_0^* \) is higher. \( N \) will adapt his reporting strategy by raising \( r_1^* \) and lowering \( r_0^* \) to back out the change in \( R_1 \) caused by \( s \) as \( N \) is the second-mover and does not benefit from \( s \).

When \( O \)'s incentive weight on stock price rises, capital market will be more sensitive to \( O \)'s earnings report, while the sensitivities of stock price to risks and to \( N \)'s earnings report is unaffected. To see this, note that when \( s \) increases, \( m_1^* \) decreases. It indicates that \( O \)'s earnings report is less sensitive to noises, so investors put more weight on \( O \)'s earnings report.

Since only \( O \) benefits from \( s \), \( N \) will try to back out any change in \( R_1 \) caused by \( s \) by adapting his reporting rule such that \( r_0^* + r_1^* R_1 \) and \( \bar{x}_2 \) remain intact. The likelihood of a big bath is thus unaffected by \( s \).

**4.4 Effect of \( w \)**

The main results are summarized in the following proposition.

**Proposition 6** The effects of \( N \)'s incentive weight on stock price are:

(i) When \( w \) gets larger, \( O \)'s reporting strategy is unaffected.
(ii) When \( w \) gets larger, \( N \)'s reporting strategy becomes less responsive toward \( N \)'s private information and more responsive toward \( O \)'s report, and it has a smaller intercept at \( R_1 = y_2 = 0 \) if \( \frac{(1+\rho)}{C_O} \) is relatively small.

(iii) When \( w \) gets larger, the pricing coefficients on risks are unaffected, the pricing coefficient on \( N \)'s report increases, and the pricing coefficient on \( O \)'s report decreases.

(iv) A big bath is more likely to occur under stock-based incentive schemes \( (w > 0) \) than under accounting-based incentive schemes \( (w = 0) \).

**Proof.** See Appendix. ■

The intuition here is the same as Proposition 5. One additional interesting finding is that a big bath is more likely to be induced under stock-based incentive schemes than under accounting-based incentive schemes. The effect of accounting-based incentive schemes is straightforward. It induces the CEO to inflate his earnings report, and thus a big bath cannot occur in equilibrium. However, the effect of stock-based incentive schemes is subtle when investors are risk averse and uncertain about both mean and variance of earnings. Inflating earnings, on one hand, can boost the perceived mean of earnings. On the other hand, it may raise the perceived variance of earnings and adversely affect the stock price. If that is the case, a big bath will be induced in equilibrium. Thus, a big bath is more likely to occur under stock-based incentive schemes than under accounting-based incentives schemes.

## 5 Discussion

In this section, we consider the following 2x2 CEO turnover types: the outgoing CEO is forced to leave or peacefully leave the company and the incoming CEO is promoted internally or recruited externally. We then discuss how the types of CEO turnover may affect earnings management cost \( (C_N) \), and the correlation between earnings \( (\rho) \). These factors in turn affect CEO’s reporting behaviour and the likelihood of a big bath. Recall first that when \( C_N \) gets higher, the the likelihood of a big bath becomes smaller. When \( \rho \) gets closer to zero, \( r_1^* \)
approaches zero and the link between $R_2$ and $R_1$ is cut. Thus, it is unlikely to have a big bath. When $\epsilon_1$ gets lower, $R_2$ will shift downwards and hence the likelihood of a big bath becomes larger.

Case 1: O peacefully leaves the firm and N is recruited externally. In this case, $C_N$ can be high since O can sit in the board to monitor N and N is not familiar with the accounting system of the firm yet. $\rho$ can be close to zero since N is recruited from outside. By our previous discussion, a big bath is least likely to occur in this case.

Case 2: O is forced to leave and N is recruited externally. As in case 1, $\rho$ can be close to zero. However, unlike case 1, O cannot sit in the board to monitor N, so $C_N$ here is smaller than case 1. Thus, the likelihood of a big bath in case 2 is higher than case 1.

Case 3: O peacefully leaves the firm and N is promoted internally. In this case, $C_N$ can be high since O can sit in the board to monitor N. Also, $\epsilon_1 > 0$ as O peacefully leaves the firm. However, N is promoted from inside, so $\rho$ is far from zero. This makes the likelihood of a big bath higher than the cases where N is recruited externally. It is hard to compare the likelihood of a big bath in case 3 with that in case 2. Having said that, the likelihood in case 3 shall be higher than case 1.

Case 4: O is forced to leave and N is promoted internally. In this case, $C_N$ can be low since O does not stay in the board to monitor N, and N can have a good knowledge about its accounting system since N has worked for the firm for a while. Furthermore, since N is promoted internally, $\rho$ can be highly positive. This implies $r_1^*$ is also highly positive. As Fig 4 indicates, it is more likely for a given negative $R_1$ to draw $R_2$ downwards. Putting all factors together, a big bath is most likely to occur in this case.

We sum up our predictions about how CEO turnover types can affect the likelihood of a big bath in the following proposition.

**Proposition 7** A big bath is most likely to occur when O is forced to leave and N is promoted internally and least likely to occur when O peacefully leaves and N is recruited externally. The likelihoods of other cases lie between them.
6 Conclusion

We consider a reporting game played by an outgoing CEO, an incoming CEO, and rational investors in the capital market. CEOs’ reporting strategies are inextricably linked together because their incentive schemes depend on stock prices in the capital market. Complementary to prior literature, we provide different economics rationales for a big bath to occur as an equilibrium outcome. In particular, we show that if earnings report by the outgoing CEO is strongly negative, the incoming CEO will choose a reporting strategy that features a big bath. It has direct and indirect effects. First, it reduces the bonus on reported earnings and has an adverse effect on the stock price. However, if earnings report by O is sufficiently low, then a big bath can reduce the perceived variation in earnings and the associated risk premium required by investors. If the indirect effect outweighs the direct effect, a big bath will be induced in equilibrium.

The model contains several empirical predictions, including (1) public regulations that increase earnings management cost for the incoming CEO reduces the likelihood of a big bath provided that the CEO compensation is not totally stock-based; however, the effect of public regulations can be muted if the CEO compensation is totally stock-based (2) a big bath is more likely to occur under stock-based incentive schemes than under accounting-based incentive schemes (3) a big bath is most likely to occur when the outgoing CEO is forced to leave and the incoming CEO is promoted internally, and it is least likely to occur when O peacefully leaves the firm and N is recruited externally.\footnote{The last prediction is consistent with empirical finding in Choi et al. (2012).}

This paper can be extended in several ways. In this paper, CEO turnover and CEO compensation are exogenously given. How endogeniety of CEO turnover and CEO compensation affects managers’ reporting strategies and the equilibrium stock price in the capital market is left as an avenue for future research. Another possible yet challenging extension is to consider kinky reporting rules as in Guttman et al. (2006).
References


Proof of Proposition 1

We construct an equilibrium to our reporting game by backward induction. Following Fischer and Verrecchia (2000), our analysis consists of two parts. The first part is managers’ problems. We consider N’s reporting strategy for an arbitrarily given earnings report by O. Then, taking the reporting strategy of N into account, we analyse O’s reporting strategy. The second part is market pricing function. We analyse how stock price and reporting rules are jointly determined in equilibrium.

Step 1. N’s Decision Problems

We first analyse the decision problem faced by N. At the end of period 2, given O’s report \(R_1\) and based on N’s private information \(x_2\) and \(\epsilon_2\), N chooses to report an earnings number \(R_2\) to maximize his payoff from incentive scheme \((1 - w)R_2 + wP\) net of his EMC. Using the conjecture of the equilibrium, the decision problem for N is:

\[
\max_{R_2} (1 - w)R_2 + p_0 + p_1 R_1 + p_2 R_2 + p_3 \left( \left( \frac{R_2 - (\hat{r}_0 + \hat{r}_1 R_1)}{\hat{r}_2} \right)^2 + \left( \frac{R_1 - \hat{m}_0}{\hat{m}_1} \right)^2 \right) + p_4 \left( \frac{R_2 - \hat{r}_0 - \hat{r}_1 R_1}{\hat{r}_2} \right) \left( \frac{R_1 - \hat{m}_0}{\hat{m}_1} \right) - \frac{C_N \hat{r}_2^2 y_2}{2} (A.1)
\]

where the hats indicate that these are the conjectured coefficients the capital market uses when valuing the firm.\(^{28}\)

From the FOC for \(R_2\), we obtain:\(^{29}\)

\[
R_2 = \frac{(1 - w)\hat{r}_2^2 + w \left( p_2 \hat{r}_2^2 - 2 p_3 \hat{r}_0 - p_4 \frac{\hat{m}_0 \hat{r}_2}{\hat{m}_1} \right) + w \left( -2 p_3 \hat{r}_1 + p_4 \frac{\hat{r}_2}{\hat{m}_1} \right) R_1 + C_N \hat{r}_2^2 y_2}{C_N \hat{r}_2^2 - 2 w p_3} (A.2)
\]

It shows that N’s optimal reporting strategy is linear in \(R_1\) and \(y_2\), as conjectured. In

\(^{28}\)In a slight abuse of notation, the pricing coefficients here represent the ones conjectured by N, not the true ones, and the hats indicates N’s conjecture of market conjecture about managers’ reporting rules. The appropriate meaning should be clear in each case. We could make distinction between them. However, it would complicate notation and make no difference in equilibrium.

\(^{29}\)The SOC is satisfied if \(p_3 < 0\), which will be the case in equilibrium.
equilibrium, the conjectured reporting function must be true. Equating coefficients yields:

\[
\begin{align*}
    r_2 &= \frac{C_N \hat{r}_2^2}{C_N \hat{r}_2^2 - 2wp_3} \\
    r_1 &= \frac{w \left( -2p_3 \hat{r}_1 + p_4 \hat{\tilde{y}}_2 \right)}{C_N \hat{r}_2^2 - 2wp_3} \\
    r_0 &= \frac{(1 - w) \hat{r}_2^2 + w \left( p_2 \hat{r}_2^2 - 2p_3 \hat{r}_0 - p_4 \frac{\tilde{y}_0 \hat{r}_2}{m_1} \right)}{C_N \hat{r}_2^2 - 2wp_3}
\end{align*}
\]

Moreover, in equilibrium conjectured coefficients must equal true coefficients:\(^30\)

\[
\begin{align*}
    r_2 &= \frac{1}{2} \left( 1 + \frac{\sqrt{C_N + 8wp_3}}{\sqrt{C_N}} \right) \quad (A.3) \\
    r_1 &= \frac{p_4(-1 + \frac{\sqrt{C_N + 8wp_3}}{\sqrt{C_N}})}{4m_1p_3} \quad (A.4) \\
    r_0 &= \frac{C_N m_0 p_4 + 4m_1p_3[1 - (1 - p_2)w] + m_0 p_4 \sqrt{C_N^2 + 8wp_3 C_N}}{4C_N m_1 p_3} \quad (A.5)
\end{align*}
\]

**Step 2. O’ Decision Problems**

Next we solve the decision problem faced by O. At the end of period 1, based his private information \(x_1\) and \(\epsilon_1\) and (correct) anticipation of N’s reporting rule, O chooses to report an earnings number \(R_1\) to maximize his expected payoff from incentive scheme \((1 - s)R_1 + sP\) net of his EMC. The decision problem for O is:

\[
\max_{R_1} (1 - s)R_1 + sE[P(\tilde{R}_1, R_2(\tilde{R}_1, \tilde{y}_2)) \mid y_1] - \frac{C_O}{2}(R_1 - y_1)^2 \quad (A.6)
\]

where O uses \(y_1\) to predict \(\tilde{y}_2\) and anticipates N’s reporting strategy to be \(R_2 = r_0 + r_1 R_1 + r_2 \tilde{y}_2\), where \(r’\)'s are given in (A.3) to (A.5). From the FOC for \(R_1\), we can

\(^{30}\)We do not wait till the decision problem of O is solved to impose this requirement because equilibrium \(r’\)'s are determined in period 2 and O will correctly anticipate the outcome.
solve:

\[ R_1 = \frac{(1 - s) + s \left( p_1 + p_2 r_1 - \frac{2p_3 \hat{m}_0}{\hat{m}_1} \right)}{C_O - \frac{2sp_3}{\hat{m}_1}} + \frac{C_O + \frac{spkp_4}{\hat{m}_1}}{C_O - \frac{2sp_3}{\hat{m}_1} y_1} \]  

(A.7)

The reporting rule is linear in \( y_1 \), as the conjecture.\(^{32}\) In equilibrium, the conjectured function of \( R_1 \) must be true. By equating coefficients, we obtain:

\[ m_1 = \frac{C_O + \frac{spkp_4}{\hat{m}_1}}{C_O - \frac{2sp_3}{\hat{m}_1}} \]

\[ m_0 = \frac{(1 - s) + s \left( p_1 + p_2 r_1 - \frac{2p_3 \hat{m}_0}{\hat{m}_1} \right)}{C_O - \frac{2sp_3}{\hat{m}_1}} \]

Moreover, in equilibrium the conjectured coefficients must equal the true coefficients. Then we have:

\[ m_1 = \frac{1}{2} \left( 1 + \frac{\sqrt{C_O + 8sp_3 + 4ks\rho p_4}}{\sqrt{C_O}} \right) \]  

(A.8)

\[ m_0 = \frac{1 + s(-1 + p_1 + p_2 r_1)}{C_O} \]  

(A.9)

**Step 3. Market Pricing Function**

We now use the optimal reporting strategy of O and N solved in last section to derive the consistent market pricing coefficients. Since the investors are rational, they will use all available information in their calculation about total earnings generated by the project. At the end of period 2, the available information to investors contains earnings reports issued by O and N. The equilibrium price is:

\[ P(R_1, R_2) = E[\tilde{x}_1 + \tilde{x}_2 \mid R_1, R_2] - \gamma Var[\tilde{x}_1 + \tilde{x}_2 \mid R_1, R_2] \]  

(A.10)

For outside investors, observation of \((R_1, R_2)\) is informationally equivalent to that of

\(^{31}\)See other part of Appendix for the derivation. The SOC is satisfied if \( p_3 < 0 \), which is true in equilibrium.

\(^{32}\)In a slight abuse of notation, the hats indicates O’s conjecture of market conjecture about O’s reporting rule.
The stock price is given by:

\[
P = -\frac{2\gamma \beta (1 + k_1) k_2 (1 + \rho)}{\alpha[(1 + k_1)(1 + \rho) + k_2]} + \frac{(1 + k_1)(1 + \rho)}{(1 + k_1)(1 + \rho) + k_2} (y_1 + y_2)
\]

\[
- \frac{\gamma (1 + k_1) k_2 (1 + k_1 + k_2)(1 + \rho)}{\alpha[(1 + k_1)(1 + \rho) + k_2][((1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2)]}(y_1^2 + y_2^2)
\]

\[
+ \frac{2\gamma (1 + k_1)^2 k_2 \rho (1 + \rho)}{\alpha[(1 + k_1)(1 + \rho) + k_2][((1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2)]} y_1 y_2
\]

(A.11)

On the other hand, it follows from the conjecture in (2.7), (2.5), and (2.6):

\[
P(R_1, R_2) = [p_0 + p_1 m_0 + p_2 (r_0 + r_1 m_0)] + (p_1 m_1 + p_2 r_1 m_1) y_1 + p_2 r_2 y_2 + p_3 (y_1^2 + y_2^2) + p_4 y_1 y_2
\]

(A.12)

Equating the pricing coefficients in (A.11) and (A.12) yields:

\[
p_4 = \frac{2\gamma (1 + k_1)^2 k_2 \rho (1 + \rho)}{\alpha[(1 + k_1)(1 + \rho) + k_2][((1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2)]}
\]

(A.13)

\[
p_3 = -\frac{\gamma (1 + k_1) k_2 (1 + k_1 + k_2)(1 + \rho)}{\alpha[(1 + k_1)(1 + \rho) + k_2][((1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2)]}
\]

(A.14)

\[
p_{2r_2} = \frac{(1 + k_1)(1 + \rho)}{(1 + k_1)(1 + \rho) + k_2}
\]

(A.15)

\[
p_{1m_1} + p_{2r_1 m_1} = \frac{(1 + k_1)(1 + \rho)}{(1 + k_1)(1 + \rho) + k_2}
\]

(A.16)

\[
p_0 + p_{1m_0} + p_{2r_0} + p_{2r_1 m_0} = -\frac{2\gamma \beta (1 + k_1) k_2 (1 + \rho)}{\alpha[(1 + k_1)(1 + \rho) + k_2]}
\]

(A.17)

Using (A.3) to (A.5) and (A.8) to (A.9) and simultaneously solving the equations, we

---

33However, investors are not sure about the components of y’s, in which only x’s positively contribute to firm value.

34See other part of Appendix for the derivation.
obtain the equilibrium pricing coefficients: \(^{35}\)

\[
p^*_4 = \frac{2\gamma \rho (1 + \rho)(1 + k_1)^2 k_2}{\alpha[(1 + k_1)(1 + \rho) + k_2][(1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2]} \quad (A.18)
\]

\[
p^*_3 = -\frac{\gamma(1 + \rho)(1 + k_1)k_2(1 + k_1 + k_2)}{\alpha[(1 + k_1)(1 + \rho) + k_2][(1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2]} \quad (A.19)
\]

\[
p^*_2 = \frac{2(1 + \rho)(1 + k_1)}{(1 + A_N)[(1 + k_1)(1 + \rho) + k_2]} \quad (A.20)
\]

\[
p^*_1 = \frac{2(1 + \rho)(1 + k_1)[((1 + k_1)(1 - \rho) + k_2) + ((1 + k_1)(1 + \rho) + k_2)A_N]}{(1 + A_N)(1 + A_O)(1 + k_1 + k_2)[(1 + k_1)(1 + \rho) + k_2]} \quad (A.21)
\]

where \(^{36}\)

\[
A_N \equiv \sqrt{1 - \frac{8w\gamma(1 + \rho)(1 + k_1)k_2(1 + k_1 + k_2)}{C_N\alpha[(1 + k_1)(1 + \rho) + k_2]^2[1 + k_1 + k_2 - (1 + k_1)\rho]}}
\]

\[
A_O \equiv \sqrt{1 - \frac{8s\gamma(1 + \rho)(1 + k_1)k_2}{C_O\alpha(1 + k_1 + k_2)[(1 + k_1)(1 + \rho) + k_2]}}
\]

Step 4. Equilibrium Reporting of O

After solving the equilibrium stock price, we can now solve the equilibrium reporting strategy for O. Substituting the equilibrium pricing coefficients into (A.8) and (A.9) yields the equilibrium reporting rule for O:

\[
m^*_1 = \frac{1}{2}(1 + A_O) \quad (A.22)
\]

\[
m^*_0 = \frac{1 + k_2 + s(1 - k_2) + \rho(1 + s) + k_1(1 + \rho)(1 + s) + (1 - s)[(1 + k_1)(1 + \rho) + k_2]A_O}{C_O[(1 + k_1)(1 + \rho) + k_2](1 + A_O)} \quad (A.23)
\]

Step 5. Equilibrium Reporting of N

We next solve the equilibrium reporting strategy of N. Substituting the equilibrium pricing

---

\(^{35}\)We use * to indicate equilibrium. The expression of \(p^*_0\) is tedious and has no effects on CEOs’ reporting behaviour, so we do not include it here.

\(^{36}\)To prevent \(A_O\) and \(A_N\) from being complex-valued, we focus on the case where \(\gamma\) is properly bounded and relatively small to \(C_O\) and \(C_N\).
coefficients into (A.3) and (A.4) yields the equilibrium reporting rule for N:

\[ r_2^* = \frac{1}{2} (1 + A_N) \]  \hspace{1cm} (A.24)

\[ r_1^* = \frac{\rho(1 + k_1)(1 - A_N)}{(1 + k_1 + k_2)(1 + A_O)} \]  \hspace{1cm} (A.25)

**Step 6.** \( p_0^* \) and \( r_0^* \) are solved by substituting the equilibrium values into (A.17) and (A.5).

**Derivation of (A.7)**

**Step 1.** Claim:

\[ E(\tilde{y}_2 \mid y_1) = \rho ky_1 \]  \hspace{1cm} (A.26)

where \( k \equiv \frac{1 + k_1}{1 + k_1 + k_2} \).

**Proof:** Since \( \tilde{y}_t = \tilde{x}_t + \tilde{\epsilon}_t = \tilde{\mu}_t + \tilde{v}_t + \hat{\epsilon}_t \sim N \left( 0, (1 + k_1 + k_2)\frac{1}{\tau} \right) \), \( t = 1, 2 \), it is well-known that:

\[
E(\tilde{y}_2 \mid y_1) = E(\tilde{y}_2) + \frac{Cov(\tilde{y}_1, \tilde{y}_2)}{Var(\tilde{y}_1)} (y_1 - E(\tilde{y}_1))
\]

\[
= 0 + \frac{Cov(\tilde{x}_1, \tilde{x}_2)}{(1 + k_1 + k_2)\frac{1}{\tau}} y_1
\]

\[
= \frac{\rho(1 + k_1)\frac{1}{\tau}}{(1 + k_1 + k_2)\frac{1}{\tau}} y_1
\]

\[
= \rho ky_1
\]

where \( k \equiv \frac{1 + k_1}{1 + k_1 + k_2} \).

**Step 2.** O conjectures the stock price to be

\[
P = p_0 + p_1 R_1 + p_2 R_2 + p_3 \left[ \left( \frac{R_2 - \hat{r}_0 - \hat{\epsilon}_1 R_1}{r_2} \right)^2 + \left( \frac{R_2 - \hat{r}_0 - \hat{\epsilon}_1 R_1}{r_2} \right)^2 \right] + p_4 \left( \frac{R_2 - \hat{r}_0 - \hat{\epsilon}_1 R_1}{r_2} \right) \left( \frac{R_1 - \hat{m}_0}{m_1} \right). \hspace{1cm} (39)
\]

Since \( P \) and \( R_2 \) are determined in period 2, according to subgame perfect Nash equilibrium, O will correctly anticipate N’s reporting rule and market pricing function with

---

37The expression of \( r_0^* \) is messy, so we do not include it here.


39In a slight abuse of notation, \( p' \)'s indicates pricing coefficients conjectured by O, and hats here indicate O’s conjecture of market conjecture about managers’ reporting rule.
correct coefficients. In addition, O knows that \( \hat{r}' s \) must equal true \( r' s \) in period 2 to reach an equilibrium. Thus, O anticipates that \( \frac{R_2 - (\hat{r}_0 + \hat{r}_1 R_1)}{r_2} = \frac{R_2 - (r_0 + r_1 R_1)}{r_2} = \tilde{y}_2 \). However, when making decision, O does not have reasons to believe that his reporting rule will be correctly conjectured by outside investors. That is why \( m's \) still have hats.

Using these facts, the FOC for \( R_1 \) is:

\[
0 = (1 - s) + s \left[ p_1 + p_2 r_1 + \frac{2 p_3}{m_1} (\frac{R_1 - \hat{m}_0}{m_1}) + \frac{p_4}{m_1} E(\tilde{y}_2 \mid y_1) \right] - C_O(R_1 - y_1) \quad (A.27)
\]

Substituting (A.26) into the FOC for \( R_1 \) and collecting term yields (A.7).

**Derivation of (A.11)**

**Step 1.** Since \( R_1 \) and \( R_2 \) are informationally equivalent to \( y_1 \) and \( y_2 \), it follows that

\[
E[\bar{x}_1 + \bar{x}_2 \mid R_1, R_2] = E[\bar{x}_1 + \bar{x}_2 \mid y_1, y_2] \\
Var[\bar{x}_1 + \bar{x}_2 \mid R_1, R_2] = Var[\bar{x}_1 + \bar{x}_2 \mid y_1, y_2]
\]

**Step 2.** Because \( \bar{x}_t \sim N(0, (1 + k_1)\frac{1}{\tau}) \), for \( t = 1, 2 \), and \( corr(\bar{x}_1, \bar{x}_2) = \rho \), it follows:

\[
\bar{x}_1 + \bar{x}_2 \sim N \left( 0, 2(1 + k_1)(1 + \rho)\frac{1}{\tau} \right) 
\]

Due to normality and recall that \( \tilde{y}_t = \bar{x}_t + \tilde{e}_t \sim N(0, (1 + k_1 + k_2)\frac{1}{\tau}) \), it can be shown that:

\[
E[\bar{x}_1 + \bar{x}_2 \mid y_1, y_2] = \frac{(1 + \rho)(1 + k_1)}{1 + k_1 + k_2 + \rho(1 + k_1)} (y_1 + y_2) \quad (A.29)
\]

**Step 3.** To compute \( Var[\bar{x}_1 + \bar{x}_2 \mid y_1, y_2] \), we first fix \( \tilde{\tau} \) at an arbitrary level \( \tau \). Due to normality, it can be shown that:

\[
Var[\bar{x}_1 + \bar{x}_2 \mid y_1, y_2, \tau] = \frac{2(1 + \rho)(1 + k_1)k_2}{(1 + k_1 + k_2 + \rho(1 + k_1))} \frac{1}{\tau} \quad (A.30)
\]

\[40\] See other part of Appendix for the proof.

\[41\] See other part of Appendix for the proof.
where $\tau$ is a fixed constant.

**Step 4.** We now consider $\tilde{\tau} \sim \text{Gamma}(\alpha, \beta)$. Using standard techniques in Bayesian statistics, we have:

$$g(\tau | y_1, y_2) \propto g(\tau) f(y_1, y_2 | \tau)$$

$$\propto \tau \exp \left( -\tau \left( \beta + \frac{(1 + k_1 + k_2)(y_1^2 + y_2^2)}{2[(1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2]} - \frac{(1 + k_1)\rho y_1 y_2}{(1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2} \right) \right)$$

This shows that given $y_1$ and $y_2$, $\tilde{\tau}$ is Gamma distributed with shape parameter $\hat{\alpha} = \alpha + 1$ and scale parameter $\hat{\beta} = \beta + \frac{(1 + k_1 + k_2)(y_1^2 + y_2^2)}{2[(1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2]} - \frac{(1 + k_1)\rho y_1 y_2}{(1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2}$. Furthermore, it can be shown that if $\tilde{\tau} \sim \text{Gamma}(\hat{\alpha}, \hat{\beta})$ then:

$$E\left( \frac{1}{\tilde{\tau}} \right) = \frac{\hat{\beta}}{\hat{\alpha} - 1}. \quad (A.31)$$

Hence, we have:

$$E\left[ \frac{1}{\tau} | y_1, y_2 \right] = \frac{\beta}{\alpha} + \frac{(1 + k_1 + k_2)(y_1^2 + y_2^2)}{2\alpha[(1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2]} - \frac{(1 + k_1)\rho y_1 y_2}{\alpha[(1 + k_1 + k_2)^2 - (1 + k_1)^2 \rho^2]} \quad (A.32)$$

**Step 5.** Substituting (A.29), (A.30), and (A.32) into (A.10) yields the stock price (A.11). ■

**Derivation of (A.29) and (A.30)**

Since $\tilde{x}_1 + \tilde{x}_2 \sim N\left(0, 2(1 + k_1)(1 + \rho)\frac{1}{\tau}\right)$, and $\tilde{y}_t = \tilde{x}_t + \tilde{\epsilon}_t \sim N(0, (1 + k_1 + k_2)\frac{1}{\tau})$. Due to normality, it is well-known that:

$$E[\tilde{x}_1 + \tilde{x}_2 | y_1, y_2] = E(\tilde{x}_1 + \tilde{x}_2) + \Sigma_{x,y} \Sigma_{y_1,y_2}^{-1} (y - E(y))$$

---

42 See other part of Appendix for the proof.
where

\[
\Sigma_{x,y} \equiv \begin{pmatrix}
\text{Cov}(\tilde{x}_1 + \tilde{x}_2, \tilde{y}_1) & \text{Cov}(\tilde{x}_1 + \tilde{x}_2, \tilde{y}_2) \\
\text{Cov}(\tilde{x}_1, \tilde{x}_1) + \text{Cov}(\tilde{x}_2, \tilde{x}_1) & \text{Cov}(\tilde{x}_1, \tilde{x}_2) + \text{Cov}(\tilde{x}_2, \tilde{x}_2)
\end{pmatrix}
\]

\[
= \begin{pmatrix}
(1 + \rho)(1 + k_1) & (1 + \rho)(1 + k_1) \\
(1 + \rho)(1 + k_1)^{\frac{1}{\bar{\tau}}} & (1 + \rho)(1 + k_1)^{\frac{1}{\bar{\tau}}}
\end{pmatrix}
\]

\[
\Sigma_{y_1,y_2} \equiv \begin{pmatrix}
\text{Var}(\tilde{y}_1) & \text{Cov}(\tilde{y}_1, \tilde{y}_2) \\
\text{Cov}(\tilde{y}_2, \tilde{y}_1) & \text{Var}(\tilde{y}_2)
\end{pmatrix}
\]

\[
= \begin{pmatrix}
\text{Var}(\tilde{x}_1) + \text{Var}(\tilde{\epsilon}_1) & \text{Cov}(\tilde{x}_1, \tilde{x}_2) \\
\text{Cov}(\tilde{x}_2, \tilde{x}_1) & \text{Var}(\tilde{x}_2) + \text{Var}(\tilde{\epsilon}_2)
\end{pmatrix}
\]

\[
= \begin{pmatrix}
(1 + k_1 + k_2)^{\frac{1}{\bar{\tau}}} & \rho(1 + k_1)^{\frac{1}{\bar{\tau}}} \\
\rho(1 + k_1)^{\frac{1}{\bar{\tau}}} & (1 + k_1 + k_2)^{\frac{1}{\bar{\tau}}}
\end{pmatrix}
\]

\[
y \equiv (y_1, y_2)^T
\]

\[
E(y) \equiv (E(y_1), E(y_2))^T = (0, 0)^T
\]

Using the results above, we obtain:

\[
E[\tilde{x}_1 + \tilde{x}_2 \mid y_1, y_2] = \frac{(1 + \rho)(1 + k_1)}{1 + k_1 + k_2 + \rho(1 + k_1)} (y_1 + y_2)
\]

Due to normality, it is well-known that:\footnote{The standard reference is again De Groot (1970).}

\[
\text{Var}[\tilde{x}_1 + \tilde{x}_2 \mid y_1, y_2, \tau] = \text{Var}(\tilde{x}_1 + \tilde{x}_2) - \Sigma_{x,y}^{-1} \Sigma_{y,x}
\]

where \(\Sigma_{y,x}\) is the transpose of \(\Sigma_{x,y}\) and \(\bar{\tau} = \tau\). Using the results above and after some
algebra, we obtain:

\[ \text{Var}[\tilde{x}_1 + \tilde{x}_2 \mid y_1, y_2, \tau] = \frac{2(1 + \rho)(1 + k_1)k_2}{(1 + k_1 + k_2 + \rho(1 + k_1))} \frac{1}{\tau} \]

\[ \blacksquare \]

**Derivation of (A.31)**

If \( \tilde{\tau} \sim \text{gamma}(\hat{\alpha}, \hat{\beta}) \), then

\[
E(\frac{1}{\tilde{\tau}}) = \int_0^\infty \frac{1}{\tau} \frac{\tau^{\hat{\alpha} - 1} e^{-\hat{\beta} \tau}}{\Gamma(\hat{\alpha})\hat{\beta}^{-\hat{\alpha}}} \ d\tau
\]

\[ = \frac{1}{\Gamma(\hat{\alpha})\hat{\beta}^{-\hat{\alpha}}} \int_0^\infty \tau^{\hat{\alpha} - 2} e^{-\hat{\beta} \tau} \ d\tau \]

\[ = \frac{1}{\Gamma(\hat{\alpha})\hat{\beta}^{-\hat{\alpha}}} \Gamma(\hat{\alpha} - 1)\hat{\beta}^{-(\hat{\alpha} - 1)} \]

\[ = \frac{\hat{\beta}}{\hat{\alpha} - 1} \]

where the second last equality follows from: \( \int_0^\infty \frac{\tau^{\hat{\alpha} - 2} e^{-\hat{\beta} \tau}}{\Gamma(\hat{\alpha} - 1)\hat{\beta}^{-(\hat{\alpha} - 1)}} \ d\tau = 1 \), and the last equality follows from: \( \Gamma(\hat{\alpha}) = (\alpha - 1)\Gamma(\hat{\alpha} - 1) \). \( \blacksquare \)

**Proof of Proposition 2.**

(i) By Proposition 1, \( R_2 - x_2 = r_0^* + r_1^* R_1 + (r_2^* - 1) x_2 + r_2^* \epsilon_2 \). If \( C_N \rightarrow \infty \), then \( r_2^* = 1 \), \( r_1^* = r_0^* = 0 \). It further implies that \( R_2 - x_2 = \epsilon_2 \). If \( \rho = 0 \), then \( r_1^* = 0 \), \( 0 < r_2^* < 1 \), and \( 0 < r_0^* < 1 \). It further implies that \( R_2 - x_2 = r_0^* + (r_2^* - 1) x_2 + r_2^* \epsilon_2 \) and that for \( x_2 \leq 0 \), \( R_2 - x_2 < 0 \) if \( \epsilon_2 < -(r_0^* + (r_2^* - 1) x_2) / r_2^* \). If \( w = 0 \), then \( r_2^* = 1 \), \( r_1^* = 0 \) and \( r_0^* = \frac{1}{C_N} \). It further implies that \( R_2 - x_2 = \frac{1}{C_N} + \epsilon_2 \) and that for \( x_2 \leq 0 \), \( R_2 - x_2 < 0 \) if \( \epsilon_2 < -\frac{1}{C_N} \).

(ii) If \( w > 0 \), \( \rho > 0 \), and \( C_N < \infty \), then then \( 0 < r_2^* < 1 \), \( 0 < r_1^* < 1 \). It further implies that for \( x_2 \leq 0 \), \( R_2 < x_2 \) if \( \epsilon_2 < -(r_0^* + r_1^* R_1 + (r_2^* - 1) x_2) / r_2^* \) or if \( R_1 < (-r_0^* + (1 - r_2^* x_2 - r_2^* \epsilon_2) / r_1^* \). \( \blacksquare \)

**Proof of Proposition 3.**
(i). Taking partial derivatives yields:

\[
\frac{\partial m_1^*}{\partial C_O} = \frac{2s\gamma(1 + \rho)(1 - k\rho)(1 + k_1)k_2}{C_O\alpha(1 + k_1 + k_2)[(1 + k_1)(1 + \rho) + k_2]A_O} > 0 \tag{A.33}
\]

\[
\frac{\partial m_0^*}{\partial C_O} = -s(1 + \rho)(1 + k_1) - (1 - s)[(1 + k_1)(1 + \rho) + k_2]A_O < 0 \tag{A.34}
\]

and

\[
\lim_{C_O \to \infty} m_1^* = 1 \tag{A.35}
\]

\[
\lim_{C_O \to \infty} m_0^* = 0 \tag{A.36}
\]

(ii). Taking partial derivatives yields:

\[
\frac{\partial r_2^*}{\partial C_O} = 0 \tag{A.37}
\]

\[
\frac{\partial r_1^*}{\partial C_O} = \frac{\rho(1 + k_1)(1 - A_N)(1 - A_O)}{2(1 + k_1 + k_2)C_OA_O(1 + A_O)} < 0 \tag{A.38}
\]

The last inequality follows from the fact that \(A_N < 1\) and \(A_O < 1\) for \(0 < s < 1\), and \(0 < w < 1\). Also, \(\frac{\partial r_1^*}{\partial C_O} > 0\) if \(s(1-s)\) is approximately zero.

(iii). It follows from \(\frac{\partial p_4^*}{\partial C_O} = \frac{\partial p_3^*}{\partial C_O} = \frac{\partial p_2^*}{\partial C_O} = 0\), and

\[
\frac{\partial p_1^*}{\partial C_O} = -\frac{8(1 + k_1)^2k_2s\gamma(1 + \rho)^2[1 + k_1 + k_2 - k(1 + k_1)\rho^2][1 + k_1 + k_2 - (1 + k_1)\rho] + [(1 + k_1)(1 + \rho) + k_2]A_N}{C_O^2\alpha(1 + k_1 + k_2)[(1 + k_1)(1 + \rho) + k_2]^2[1 + k_1 + k_2 - (1 + k_1)\rho]A_O(1 + A_O)^2(1 + A_N)} < 0 \tag{A.39}
\]

(iv). As in Fig 4,

\[
P(big\ bath) = P(r_0^* + r_1^*R_1 < 0, \bar{x}_2 < x_2 < 0)
\]

\[
= P(y_1 < \bar{y}_1, \bar{x}_2 < x_2 < 0) \tag{A.40}
\]

where \(\bar{y}_1 \equiv -\frac{r_0^*}{r_1^*m_1} - \frac{m_0^*}{m_1}\) and \(\bar{x}_2 \equiv \frac{r_0^* + r_1^*m_0^*}{1-r_2^*} + \frac{r_1^*m_1}{1-r_2^*}y_1\). This probability is a double integral over a specific area on \((y_1,x_2)\) plots, with a bivariate normal pdf as its integrand.
Since $C_O$ has no effect on its integrand, and $\partial y_1/\partial C_O = \partial x_2/\partial C_O = 0$, the likelihood of a big bath is unaffected by $C_O$. ■

**Proof of Proposition 4.**

Proof of (i) to (iii) is similar to Proposition 3, so we omit it.

(iv). $\partial y_1/\partial C_N = \frac{(1-w)(1+k_1+k_2)}{\rho(1+k_1)A_N C_N^2} < (=)0$ if $w < (=)1$. Also, $\partial x_2/\partial C_N = \frac{1-w}{C_N A_N} > (=)0$ if $w < (=)1$. It means that provided that $w < 1$, if $C_N$ gets higher, the area of the double integral shrinks and thus the likelihood of a big bath becomes smaller. ■

**Proof of Proposition 5.**

(i). This part follows from $\partial m_1^*/\partial s = < 0$, and $\partial m_0^*/\partial s < 0$ if $\frac{s}{C_O}$ is small enough.

(ii). This part follows from $\partial r_2^*/\partial s = 0$, and $\partial r_1^*/\partial s > 0$. Moreover, we obtain: $\partial r_0^*/\partial s > 0$ if $\frac{1+w}{C_N}$. 

(iii). It is due to $\partial p_1^* / \partial s = \partial p_2^* / \partial s = \partial p_3^* / \partial s = 0$, and $\partial p_4^* / \partial s > 0$.

(iv) It is due to $\partial y_1 / \partial s = 0$ and $\partial x_2 / \partial s = 0$. ■

**Proof of Proposition 6.**

Proof of (i) to (iii) is similar to Proposition 5, so we omit it.

(iv) The expressions of $\partial y_1 / \partial w$ and $\partial x_2 / \partial w$ are too complicated to determined their signs. Nevertheless, we have shown that when $w = 0$, then $r_2^* = 1$, $r_1^* = 0$ and $r_0^* = \frac{1}{C_N}$, so the likelihood of a big bath is zero. On the contrary, when $w > 0$, the likelihood becomes positive. Thus, a big is bath more likely to occur under stock-based incentive schemes than under accounting-based incentive schemes. ■