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SUBSIDIES IN AN ECONOMY WITH ENDOGENOUS CYCLES OVER NEOCLASSICAL INVESTMENT AND NEO-SCHUMPETERIAN INNOVATION REGIMES

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ABSTRACT

We explore the roles of subsidies in the Matsuyama model (1999) of growth through cycles alternating perpetually between two phases featuring neoclassical investment and neo-Schumpeterian innovation respectively. Subsidies to R&D investment or to the purchase of newly invented intermediate goods can arbitrarily reduce the threshold level of capital per type of intermediate good, beyond which the economy moves from the investment phase to the innovation phase. More importantly, such subsidies can mitigate and eventually eliminate cycles for significant welfare gains that can be equivalent to as much as 10% rises in consumption at all times.

Keywords: Subsidization; Innovation; Capital accumulation; Cycles; Growth

JEL Classifications: E3, E6, H2, H3, O4

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1. Introduction

One theme of the macroeconomic aspect of public policy analysis is to explore whether and how government policies can promote output growth or mitigate output fluctuations for welfare gains. In this regard, different macroeconomic models have different answers. The neoclassical growth model, pioneered by Solow (1956) and Swan (1956), captures how capital accumulation contributes to growth and why growth eventually halts at a stable steady-state output level per capita under diminishing returns to investment. It may need no role of government intervention so long as consumers choose their consumption path optimally as in Cass (1965) and Koopmans (1965) according to the Welfare Theorems. This policy implication remains valid even when exogenous shocks are introduced into the neoclassical growth model for the creation of cycles as in the real business cycle models led by Kydland and Prescott (1982) and Long and Plosser (1983).

The neo-Schumpeterian models developed in the last two decades generate sustainable growth through costly innovations that create new varieties of intermediate goods or improve the quality of existing intermediate goods (see, e.g. Romer, 1990; Aghion and Howitt, 1992). In these R&D endogenous growth models, R&D activities intended for a new variety or a quality improvement incur a fixed cost, whereas the production of each intermediate good incurs a constant marginal cost; and both the innovation and the intermediate goods production use current final output. Monopoly rights are granted to innovators in order to allow them to recoup their innovational costs. The consequent monopoly pricing reduces the demand for new intermediate goods, causing lower final output and slower growth than their socially optimal levels (static and dynamic losses in
efficiency, respectively). The efficiency losses of monopoly pricing justify government subsidization either to R&D investment or to the purchase of newly created intermediate goods, as shown in Barro and Sala-i-Martin (1995) with inelastic labor, and in Zeng and Zhang (2007) with elastic labor, among others. In contrast to the neoclassical growth and the real business cycle models, however, the economy is always on the balanced growth path in such neo-Schumpeterian growth models that are known as the AK model in essence.

Matsuyama (1996, 1999) unifies the neoclassical and the neo-Schumpeterian growth models by assuming that both R&D activities and intermediate goods production can only use available capital from previous savings, among various approaches generating sustainable growth through endogenous cycles.\(^1\) Under this neoclassical-style assumption, innovation can break even to recover the fixed cost when capital per type of intermediate good exceeds a critical level for a profitable scale of the demand for newly invented intermediates. Once currently available capital exceeds this level and induces innovation, however, part of the capital stock must be used for the fixed innovation cost and the remaining amount of capital for manufacturing intermediate goods declines. Consequently, current innovation, if responding more elastically than capital investment to initial abundance in capital per variety, can reduce capital per variety to the extent such that future innovation becomes unprofitable until enough capital is formed again through a neoclassical

\(^1\) Freeman, Hong and Peled (1999), Maliar and Maliar (2004), and Walde (2002, 2005) also attain sustainable growth through endogenous cycles in models with capital accumulation and innovation. In their models, innovations advance non-rivaled general technology for final goods production, rather than rivaled intermediate goods sold under exclusive monopoly rights in the R&D growth models. Francois and Lloyd-Ellis (2003) acknowledge the relevance of such major breakthroughs in general technology in explaining "long waves" of fluctuations but cast doubt on the relevance in explaining high-frequency business cycles. To focus on high-frequency cycles, they assume multi-sectors and argue for endogenous clustering of innovation and cycle implementation. In Bental and Peled (1996), firms producing final goods engage in costly search of better technology from a known and fixed pool of technologies. We follow the approach of Matsuyama (1999) assuming that costly innovations lead to rivaled intermediate goods sold monopolistically, because monopoly pricing creates inefficiency and thus calls for government intervention.
investment phase. It argues that this is an empirically plausible scenario: The balanced
growth path with innovation is unstable and therefore the economy may fluctuate between a
Solow investment phase and a Romer innovation phase perpetually, causing cyclical
movements in consumption, investment, innovation and growth.

In a different type of neo-Schumpeterian growth model by Francois and Shi (1999) and
Haruyama (2009), labor rather than capital is the sole input for innovations and for
intermediate goods production.\(^2\) In their models, cycles can also arise endogenously from
contemporaneous complementarities between investors devoting labor to innovation for
temporary profits. Haruyama (2009) demonstrates that optimal steady-state R&D
subsidization fails to eliminate cycles and should be state-dependent in a fluctuating
economy when labor is the only input for innovation and intermediate goods production.

The cyclical fluctuations in consumption, investment, innovation and growth represent
another source of efficiency loss, given diminishing marginal utility of consumption and
diminishing marginal product of factors, particularly when innovation and intermediate
goods production compete with each other for limited available capital. This adds to the
efficiency losses of monopoly pricing analyzed in the literature and thus differs from the
Pareto optimal variations in consumption across time and sectors facing exogenous shocks
found in Long and Plosser (1983). Therefore, important macroeconomic questions arise as
follows. Can government policies mitigate or eliminate such cyclical fluctuations and
promote innovation, investment and growth at the same time when innovation and
intermediate goods production require capital accumulated from previous savings? Can

\(^2\) This assumption is made in one line of the R&D literature, e.g. Grossman and Helpman (1991), Segerstrom (1991),
such government policies enhance social welfare?

When attempting to answer these questions, it is natural to focus on subsidization associated with R&D activities or with the purchase of new R&D products. Although such subsidization has been considered in the literature with innovation and growth as mentioned earlier, the existing studies have mainly focused on how the subsidies mitigates the losses in efficiency of monopoly pricing on a stable balanced growth path.

To our knowledge, the only work on subsidization in the unified model of Matsuyama (1999), where innovation and intermediate goods production cost available capital, is Aloi and Lasselle (2007). They use a lump-sum subsidy to innovators financed by a lump-sum tax on consumers and find that it can promote growth, stabilize the innovation cycles and increase welfare. However, their subsidy adds directly to capital in the resource constraint for innovation and intermediate goods production in the current period. In the Matsuyama model, by contrast, the constraint on available capital from previous saving for current innovation and intermediate goods production is essential for the creation of cycles. It thus remains to show whether and how more realistic subsidization, which does not relax this assumption, can induce changes in individuals' behavior so as to stabilize the balanced growth path for welfare gains.

In this paper, we explore whether flat-rate subsidization associated with R&D activities can mitigate or eliminate cycles for welfare gains in the Matsuyama model. In doing so, the subsidy provides additional awards to innovators without relaxing the constraint on available capital for innovation and intermediate goods production. We find that subsidies to R&D investment or to the purchase of newly invented intermediate goods, financed by a
consumption tax, can arbitrarily reduce the threshold level of capital per variety, beyond which the economy moves from the investment phase to the innovation phase. Moreover, sufficient subsidization can change the balanced growth path from an unstable one to a stable one and thus eventually eliminate the cycles. In numerical examples for plausible parameterizations, optimal subsidy rates can achieve substantial welfare gains (equivalent to as much as about a 10% increase in consumption in every period) and lie in the range that leads to convergence toward a stable balanced growth path.

Our results appear consistent with the postwar experiences in some industrial nations such as the United States where substantial subsidies are provided to R&D activities and to the purchase of new equipment. For example, there is substantial subsidization in the US tax system: a 50% immediate writing-off of equipment investment, expensing of R&D expenditures, and accelerated depreciation allowances, according to Gordon, Kalambokidis, Rohaly and Slemrod (2004), Gordon, Kalambokidis and Slemrod (2004), and others. At the same time these countries observe much more innovations but dampened recessions compared to previous times on average.

The rest of the paper proceeds as follows. Section 2 introduces the building blocks of the model. Section 3 characterizes the steady states in different regimes and analyzes the global dynamics for different levels of subsidization. Section 4 deals with optimal subsidy rates and presents numerical simulation results. Section 5 concludes.

2. The model

The model is an extension of Matsuyama (1999, 2001) by considering subsidies to R&D
spending and to the purchase of newly invented intermediate goods, financed by a consumption tax. Time is discrete, extending from period one to infinity \( t = 1, 2, \ldots, \infty \).

### 2.1. Production and innovation

There is a single consumption-investment good taken as a numeraire and produced by using capital and labor. Labor is supplied inelastically at an amount \( L \) that also stands for the size of the population of identical, infinitely-lived consumers or workers. Let \( K_{t-1} \) denote the capital stock available for the creation and production of intermediate goods in period \( t \). In the first period, there is an initial capital stock \( K_0 > 0 \).

Capital is first converted into a composite of intermediate goods by a symmetric CES function. Let \( x_i(z) \) denotes the \( z \)th type of available intermediate good in the range \([0, N_i] \) in period \( t \). Labor and the composite of intermediates are combined through a Cobb-Douglas technology for final goods production:

\[
Y_t = A(L)^{1/\sigma} \left\{ \int_0^{N_i} \left[ x_i(z) \right]^{1-1/\sigma} dz \right\}.
\]

where \( A > 0 \) is the total factor productivity parameter and \( \sigma > 1 \) is the direct partial elasticity of substitution between every pair of intermediate goods.

One unit of each type of intermediate good is manufactured by converting \( a \) units of capital. In each period \( t \), old intermediate goods in the range \( z \in [0, N_{t-1}] \) starting with \( N_0 > 0 \) are sold competitively, while new intermediate goods in the range \( z \in [N_{t-1}, N_t] \) may be introduced and sold exclusively by their innovators under a one-period patent protection. Creating a new intermediate requires \( F \) fixed units of capital. By symmetry, we have \( x_i(z) \equiv x_i^c \) for competitively supplied intermediate goods \( z \in [0, N_{t-1}] \), and
For monopolistically supplied intermediate goods $z \in \left[ N_{t-1}, N_t \right]$. The profit function for firms in the final goods sector can be expressed as:

$$\Pi_t = A(L)^{1/\sigma} \left[ N_{t-1} \left(x_t^c\right)^{1-1/\sigma} + \left(N_t - N_{t-1}\right) \left(x_t^m\right)^{1-1/\sigma} \right] - N_{t-1} p_t^c x_t^c - (1 - s_x) \left(N_t - N_{t-1}\right) p_t^m x_t^m - w_t L, \quad 0 \leq s_x < 1,$$

(2)

where $p_t^c$ and $p_t^m$ are the prices of old and new intermediate goods, respectively; $s_x$ is the time-invariant subsidy rate to the purchase of new intermediate goods used in Barro and Sala-i-Martin (1995) and Zeng and Zhang (2007) but not considered in Aloi and Lasselle (2007); and $w_t$ is the wage rate per unit of labor. According to Zeng and Zhang (2007), the subsidies to final output or to the purchase of intermediate goods are equivalent concerning their effects on growth and welfare. Thus, we only consider the latter.

In the final goods sector, factors are paid by their marginal products:

$$p_t^c = (1 - 1/\sigma) A(L)^{1/\sigma} \left(x_t^c\right)^{-1/\sigma},$$

(3)

$$p_t^m (1 - s_x) = (1 - 1/\sigma) A(L)^{1/\sigma} \left(x_t^m\right)^{-1/\sigma},$$

(4)

$$w_t = \frac{1}{\sigma L}.$$

(5)

Let $r_t$ denote the price of capital. Then the marginal cost of manufacturing intermediate goods in period $t$ is equal to $ar_t$. All old intermediate goods are supplied competitively at a price level equal to the marginal cost, $p_t(z) \equiv p_t^c = ar_t$ for $z \in [0, N_{t-1}]$, while all new intermediate goods, once introduced, are sold monopolistically at a higher price level, $p_t(z) \equiv p_t^m = \left[\sigma/(\sigma - 1)\right] ar_t > p_t^c$, for $z \in \left[ N_{t-1}, N_t \right]$, following from (4). From equations (3) and (4), the relationship between $x_t^c$ and $x_t^m$ must satisfy:

$$\frac{x_t^c}{x_t^m} = \left[\frac{p_t^c}{p_t^m (1 - s_x)}\right]^{\sigma} = \left[1 - \frac{1}{\sigma}\right]^{-\sigma} \left(1 - s_x\right)^{\sigma}.$$

(6)
Absent subsidies, the higher price of new intermediates than that of old intermediates yields a smaller equilibrium quantity of each new intermediate than that of each old intermediate, \( x_{t}^{m} < x_{t}^{c} \). This asymmetry in the equilibrium quantities of new versus old intermediates must lead to static and dynamic efficiency losses. The lower demand for new than for old intermediate goods leads to a dynamic efficiency loss in terms of decelerating the rate of innovation because it reduces the profitability scale for innovators to recover the fixed R&D cost. At the same time, the lower demand for new than for old intermediate goods leads to a static efficiency loss in terms of decreasing final output because all intermediates enter final goods production symmetrically and have diminishing marginal contributions to final output.

Subsidies on the purchase of new intermediate goods strengthen final goods producers' demand for new intermediate inputs relative to old ones by reducing the user cost of new intermediates. According to (6), when \( s_{s} < 1 / \sigma \), \( x_{t} > x_{m} \); when \( s_{s} = 1 / \sigma \), \( x_{t} = x_{m} \); when \( s_{s} > 1 / \sigma \), \( x_{t} < x_{m} \). Thus, the subsidy on the purchase of new intermediates may affect the dynamic system significantly in this model.

The one period monopoly enjoyed by the innovator makes it possible to recover the fixed R&D cost. There is free entry for innovative activities. The monopoly profit is equal to the sales revenue net of the fixed R&D cost and the variable manufacturing cost:

\[
\pi_{t} = p_{t}^{m} x_{t}^{m} - r \left[ a x_{t}^{m} + (1 - s_{n}) F \right].
\]

Here, \( s_{n} \) is the time-invariant subsidy rate to the fixed R&D cost \( r F \) used in Barro and Sala-i-Martin (1995) and Zeng and Zhang (2007), as opposed to the lump-sum subsidy used in Aloi and Lasselle (2007). The free entry ensures the following in equilibrium
\[ ax_i^m \leq (\sigma - 1)(1-s_n)F, \quad N_i \geq N_{t-1}, \quad \left[ ax_i^m - (\sigma - 1)(1-s_n)F \right] (N_i - N_{t-1}) = 0. \] (7)

That is, when potential innovators expect the sale of a new intermediate good to be smaller than the break-even level (i.e. \( x_i^m < (\sigma - 1)(1-s_n)F/a \)), there is no incentive for innovation at all, thereby \( N_i = N_{t-1} \). In equilibrium with free entry, when innovation occurs (i.e. \( N_i > N_{t-1} \)), the innovator must just break even such that \( ax_i^m = (\sigma - 1)(1-s_n)F \). By lowering the cost of innovation virtually to any level, the subsidy on the R&D cost can reduce this break-even level of the demand for a new intermediate virtually to any level, and may therefore have significant effects on the dynamic system of the model.

Regardless of the value of the subsidy rates, the resource constraint on the use of available capital for intermediate goods production and innovation in period \( t \) is:

\[ K_{t-1} = N_{t-1}ax_i^c + (N_i - N_{t-1})(ax_i^m + F). \] (8)

Without this assumption, the economy would always be on the unique balanced growth path as in the earlier R&D growth models. This constraint differs from the counterpart in Aloi and Lasselle (2007) that regards the subsidy as an addition to available capital.

Substituting equations (6) and (7) into the above constraint leads to

\[ ax_i^c = a \left[ 1 - \frac{1}{\sigma} \right]^{\sigma} (1-s_n)^{\sigma} x_i^m = \theta \sigma F \min \left\{ k_{t-1}, (1-s_n)^{\sigma} (1-s_n) \right\}, \] (9)

\[ N_i = N_{t-1} + \max \left\{ 0, \frac{K_{t-1}}{\left[ (\sigma - s_n)(\sigma - 1) \right]} - \left( 1-s_n \right)^{\sigma} (1-s_n) \frac{\theta \sigma}{\sigma - s_n (\sigma - 1)} N_{t-1} \right\}, \] (10)

where

\[ k_i = \frac{K_i}{(\theta \sigma F) N_i}, \quad \theta = \left[ 1 - \frac{1}{\sigma} \right]^{1-\sigma}, \quad \theta \in [1,e], \quad e = 2.71828 \ldots. \]

Here, \( \theta \) is increasing with \( \sigma \). Clearly, for innovators to break even in period \( t \), the amount of available capital stock \( K_{t-1} \) must be abundant enough relative to available variety...
According to (9), increasing the subsidy rate on the fixed R&D cost will reduce the demand for both new and old intermediates, while increasing the subsidy rate on purchasing new intermediates will reduce the demand for old intermediates, given any initial state \((N_{t-1}, K_{t-1})\) such that \(k_{t-1}\) is large enough for innovators to break even. According to (10), increasing either of the two subsidy rates will increase the rate of innovation, given any initial state such that \(k_{t-1}\) is large enough for innovators to break even.

We can now rewrite equation (1) as

\[
Y_t = A\left(\frac{K_{t-1}}{a}\right)^{\sigma} \left[ N_{t-1} \left( x_t^c \right)^{1-\frac{1}{\sigma}} + \left( N_t - N_{t-1} \right) \left( x_t^m \right)^{1-\frac{1}{\sigma}} \right].
\]  

(11)

Given any initial state that allows innovators to break even, the subsidies can increase final output by promoting innovation for faster variety expansion, but reduce final output by reducing the demand for each intermediate good. However, the subsidy on the purchase of new intermediates can increase final output by increasing the demand for each new intermediate unless it is too large. To detail such effects further, we rewrite equation (11) by using equations (7), (8), (9) and (10):

\[
Y_t = A\left(\frac{LN_{t-1}}{a}\right)^{\sigma} \left[ \frac{K_{t-1}}{\sigma - s_n (\sigma - 1)} F \left( \frac{(\sigma - 1) \left( 1 - s_n \right)^{1-\frac{1}{\sigma}}}{\theta a} \right) \right]
+ N_{t-1} \left( 1 - s_x \right)^{\sigma-1} \left[ s_n + s_x (\sigma - 1) \right] \left[ \frac{\theta (1 - s_n)}{\sigma - s_n (\sigma - 1)} \right]^{1-\frac{1}{\sigma}}, \text{ if } k_{t-1} \geq k_c.
\]

(12)

The critical value of the capital-variety ratio, \(k_c = (1-s_n)^\sigma (1-s_n)\), below which there is no innovation and hence no subsidization by construction, divides government action in this model into policy-dormant and policy-active regions, respectively. In fact, increasing the rate of either subsidy can reduce the threshold level of the capital-variety ratio, \(k_c\).
virtually to anywhere above zero, enhancing the chance for the economy to stay in the policy-active region with R&D activities. Thus, the subsidization may significantly change the dynamic path of the model.

According to (12), given an initial state \((N_{t-1}, K_{t-1})\) such that \(k_{t-1} \geq k_c\), subsidizing either the fixed R&D cost or the purchase of new intermediates can increase final output if the subsidy rates are sufficiently low such that their positive impact on variety expansion dominates. The opposite occurs for further increases in the subsidy rates if the subsidy rates are already sufficiently high such that their negative impact on the demand for intermediates dominates. To see this clearly, we differentiate final output with respect to one subsidy rate at a time for any initial state \((N_{t-1}, K_{t-1})\) such that \(k_{t-1} \geq k_c\). Focusing first on how \(s_n\) affects \(Y_t\) at \(s_x = 0\) and \(k_{t-1} \geq k_c\), \(dY_t/ds_n\) is signed by two parts additively. One part containing the derivative of \((1-s_n)^{1-1/\sigma}/[\sigma-s_n(\sigma-1)]\) with respect to \(s_n\) is signed by \(-s_n\), while the other part containing the derivative of \(s_n(1-s_n)^{1-1/\sigma}/[\sigma-s_n(\sigma-1)]\) is signed by \(1-s_n(2-1/\sigma)\). Combining the two parts together, \(dY_t/ds_n\) must be positive for very small \(s_n\) but becomes negative when \(s_n\) becomes larger, at least when \(s_n > 1/(2-1/\sigma)\).

When focusing on \(s_x\) at \(s_n = 0\), \(dY_t/ds_x\) is signed by \(1-\sigma s_x\) through signing the derivative of \((1-s_x)^{\sigma-1}s_x\) with respect to \(s_x\). Thus, final output increases with \(s_x\) when \(s_x < 1/\sigma\) under which \(x_c > x_m\); final output peaks at \(s_x = 1/\sigma\) whereby \(x_c = x_m\); any further increase in \(s_x\) leads to \(x_c < x_m\) and thus reduces final output. Therefore, the effects of the subsidies on final output also alter the static efficiency of the model.

Equations (10) and (12) are simplified to
\[
\frac{N_t}{N_{t-1}} \equiv \psi(k_{t-1}, s_x, s_n) = \max \left\{ 1, 1 + \frac{\theta \sigma}{\sigma - s_n(\sigma - 1)} \left[ k_{t-1} - (1 - s_x)^\sigma (1 - s_n) \right] \right\}, \tag{13} \]

\[
\frac{Y_t}{K_{t-1}} \equiv \phi(k_{t-1}, s_x, s_n) = A \left( \frac{L}{\theta \sigma F} \right)^{\frac{1}{\sigma}} (k_{t-1})^{-\frac{1}{\sigma}}, \text{ if } k_{t-1} \leq k_c; \text{ otherwise,} \]

\[
\phi(k_{t-1}, s_x, s_n) = A \left( \frac{L}{\theta \sigma F} \right)^{\frac{1}{\sigma}} \left( 1 - s_n \right)^{1-\frac{1}{\sigma}} \left[ 1 + \left( \frac{1 - s_x}{1 - s_n} \right)^{\frac{\sigma - 1}{\sigma}} \right] \left[ \frac{s_n + s_x (1 - s_n)}{\sigma} \right]. \tag{14} \]

To economize on notations, we assume \( L = 1, a = 1, \) and \( F = 1/\theta \sigma \) without changing the essence of the results.

### 2.2. Households and government

The infinitely lived representative agent derives utility from consumption (with \( L = 1 \)) according to the following preference:

\[
U = \sum_{t=1}^{\infty} \beta^t \ln(C_t), \quad 0 < \beta < 1, \tag{15} \]

where \( \beta \) is the discount factor. The logarithmic utility renders tractability.

In each period \( t \), the agent receives capital income, \( r_t K_{t-1} \), and earns labor income, \( w_t L \). He consumes \( C_t \), facing a proportional consumption tax \( \tau_{c,t} \), and carries over \( K_t \) units of final goods to the next period. The flow budget constraint for the agent is

\[
K_t = r_t K_{t-1} + w_t L - (1 + \tau_{c,t})C_t. \tag{16} \]

Also, the consumer faces an intertemporal solvency restriction:

\[
\lim_{t \to \infty} \frac{K_t}{\prod_{t=1}^{t'} r_s} \geq 0. \tag{17} \]

Taking initial \( K_0 > 0 \) and the sequences \( (\tau_{c,t}, r_t, w_t) \) as given, the agent chooses the sequence \( (C_t, K_t) \) to maximize utility in (15) subject to (16) and (17). The optimal
intertemporal condition is
\[
\frac{1}{(1+\tau_{c,t})}C_t = \frac{\beta r_{t+1}}{(1+\tau_{c,t+1})}C_{t+1},
\]  
(18)
and the binding solvency condition can be written as
\[
\lim_{t \to \infty} \frac{K_t}{\prod_{s=1}^{t} r_s} = \lim_{t \to \infty} \frac{\beta^t}{(1+\tau_{c,t})}C_t = 0.
\]  
(19)

Production factors are compensated competitively according to
\[
w_tL = \frac{1}{\sigma}Y_t
\]
and
\[r_tK_{t-1} = (1-\frac{1}{\sigma})Y_t\]
which, together with (16) to (18), yield
\[
K_t = \beta \left(1-\frac{1}{\sigma}\right)Y_t,
\]  
(20)
\[
C_t = \frac{\left[1-\beta(1-1/\sigma)\right]Y_t}{1+\tau_{c,t}}.
\]  
(21)

From (20) and (21), the agent carries a constant fraction of income as capital into next period and spends the remaining fraction on consumption in the current period. Here, a higher tax rate on consumption spending reduces consumption (for greater subsidies).

From equations (13), (14) and (20), the dynamics of the economy can be uniquely determined by the following system of first-order difference equations in $K$ and $N$:
\[
K_t = \beta (1-1/\sigma) \phi(k_{t-1}, s_x, s_n) K_{t-1},
\]  
(22)
\[
N_t = N_{t-1} + \max \left\{ 0, \frac{\theta}{1-s_n (1-1/\sigma)} \left[ K_{t-1} - (1-s_x)^{\sigma} (1-s_n) N_{t-1} \right] \right\},
\]  
(23)

starting from an initial state $(K_0, N_0)$, given time-invariant subsidy rates, $s_x$ and $s_n$.

Observe in (23) that sufficient subsidization of either type can lead to the introduction of new intermediates, $N_t - N_{t-1} > 0$, for any initial state at time $t$, $(N_{t-1}, K_{t-1})$.

The government runs a balanced budget in every period between taxes and subsidies:
\[
\tau_{c,t}C_t = s_n Fr_t (N_t - N_{t-1}) + s_x ar_t \sigma / (\sigma - 1) (N_t - N_{t-1}) x^w.
\]  
(24)
Without innovation occurring \((N_t = N_{t-1})\), the tax equals zero (the policy-dormant regime).

Combining (20), (21) and (24) together with (7) and (10) yields

\[
\tau_{c,t} = \max \left\{ 0, \frac{(k_{t-1} - k_c)[s_n + \sigma s_x (1-s_n)]}{\left(\frac{\sigma}{\sigma - 1} - \beta\right)[\sigma - s_n (\sigma - 1)]} \right\}. \tag{25}
\]

From equations (22) and (23), the law of motion for the capital variety ratio, \(k_t\), is
governed by the following one-dimensional mapping, \(\Phi : R_+ \rightarrow R_+\),

\[
k_t = \Phi(k_{t-1})
= \begin{cases} 
\beta (1 - 1/\sigma) A (k_{t-1})^{1-1/\sigma} & \text{if } k_{t-1} \leq k_c \\
\beta (1 - 1/\sigma) A (1-s_n)^{1 - 1/\sigma} \left\{ k_{t-1} - (1-s_x)^{\sigma-1} \left[ s_n / (1-s_x) \right] \right\} + \theta \left( k_{t-1} - (1-s_x)^{\sigma-1} \left( 1-s_n \right) \right) & \text{if } k_{t-1} \geq k_c 
\end{cases} \tag{26}
\]

where \(k_c = (1-s_x)^{\sigma} \left( 1-s_n \right)\).

3. The steady state and global dynamic analysis

The steady state of the economy is defined as an equilibrium path on which \(k_t = K_t / N_t\) stays constant over time for any time-invariant subsidy rates, \(s_x\) and \(s_n\). According to (26), the steady state \(k\) of the dynamic system is uniquely determined. Whether new intermediates are introduced or not at the steady state depends on the relationship between the steady state \(k\) and the critical value \(k_c\).

First, if \(k_t = k^* \leq k_c\) in a steady state, then according to (13) and (14), \(N_t = N_{t-1}\) and \(K_t = K_{t-1}\). In this steady state, there is no innovation; all the intermediate goods are competitively supplied; and the economy does not grow in the long run. From (22), on this neoclassical stationary path, \(k^* = \left[ \beta (1 - 1/\sigma) A \right]^\sigma\). The existence of such a stationary path
requires that \( \left[ \beta \left( 1 - 1/\sigma \right) A \right] \leq (k_1)^{\nu/\sigma} \).

Now, suppose that \( k_i = k^{**} > k_c \) holds in a steady state. From (22) and (23), the balanced growth path satisfies the following:

\[
\beta \left( 1 - 1/\sigma \right) \phi(k_{t-1}, s_x, s_n) = \frac{K}{K_{t-1}} = \frac{N}{N_{t-1}} = 1 + \frac{\theta \sigma}{\sigma - s_n (\sigma - 1)} [k_{t-1} - k_c] > 1.
\]

In this steady state, the available capital stock of the economy is large enough relative to the number of existing intermediates such that new intermediates are introduced and that \( K_t \) and \( N_t \) share the same growth rate. The existence of such a balanced growth path requires \( \beta (1 - 1/\sigma) \phi(k_{t-1}, s_x, s_n) > 1 \).

These results concerning the steady state of the dynamic system in (26) are given below.

**Proposition 1.** Let \( G \equiv \beta (1 - 1/\sigma) A \left( k^{**} \right) \), where \( k_c = (1 - s_x)^\sigma (1 - s_n) \), with \( 0 \leq s_x < 1 \) and \( 0 \leq s_n < 1 \).

1. If \( G \leq 1 \), the dynamic system has a unique steady state \( k^* = \Phi(k^*) \) where

\[
k^* = \left[ \beta \left( 1 - 1/\sigma \right) A \right]^{\nu/\sigma} \leq k_c. \text{ At this steady state, the economy has no innovation and does not grow.}
\]

2. If \( G > 1 \), the dynamic system has a unique steady state \( k^{**} = \Phi(k^{**}) \) where

\[
k^{**} = \left[ \theta k_c + s_n (1 - 1/\sigma) - 1 + \beta (1 - 1/\sigma) A (1 - s_n)^{1-1/\sigma} + \Delta^{1/2} \right] / (2 \theta) > k_c,
\]

\[
\Delta = \left[ 1 - s_n (1 - 1/\sigma) - \theta k_c - \beta (1 - 1/\sigma) A (1 - s_n)^{1-1/\sigma} \right]^2
\]

\[
+ 4 \theta \beta (1 - 1/\sigma) A (1 - s_n)^{1-1/\sigma} (1 - s_x)^{\nu - 1} \left[ s_n / \sigma + s_x (1 - s_n) \right].
\]

At this steady state, the economy grows in \( (N_t, K_t) \) at the same constant rate

\[
g = \beta (1 - 1/\sigma) \phi(k^{**}, s_x, s_n) = 1 + \frac{\theta \sigma}{\sigma - s_n (\sigma - 1)} [k^{**} - k_c].
\]
Proof. The solutions for the steady state $k^*$ or $k^{**}$ follow the respective scenarios in (26).

What remains to show is $k^{**} > k_c$ for $G > 1$. First, note the following implication of $G > 1$:

$$\Delta = [1 - s_n (1 - 1/\sigma) - \theta k_c - \beta (1 - 1/\sigma) A(1 - s_n)^{-1/\sigma}]^2 + 4\theta \beta (1 - 1/\sigma) A \cdot k_c (1 - s_n)^{-1/\sigma} - \left\{(1 - s_n)^{-1} [s_n (1 - s_n) + s_n / \sigma]\right.$$  
$$= \theta^2 k_c^2 + [1 - s_n (1 - 1/\sigma) - \beta (1 - 1/\sigma) A(1 - s_n)^{-1/\sigma}]^2 - 2\theta k_c [1 - s_n (1 - 1/\sigma) - \beta (1 - 1/\sigma) A(1 - s_n)^{-1/\sigma}] + 4\theta \beta (1 - 1/\sigma) A \cdot k_c (1 - s_n)^{-1/\sigma} - \left\{(1 - s_n)^{-1} [s_n (1 - s_n) + s_n / \sigma]\right].$$

Note that $1 - s_n (1 - 1/\sigma) - (1 - s_n)(1 - s_n) = s_n (1 - s_n) + s_n / \sigma \geq 0$ and that $G > 1$ implies

$$\beta (1 - 1/\sigma) A(1 - s_n)^{-1/\sigma} > (1 - s_n)(1 - s_n).$$

We can now rewrite the expression of $\Delta$ below:

$$\Delta = \theta^2 k_c^2 + [1 - s_n (1 - 1/\sigma) - \beta (1 - 1/\sigma) A(1 - s_n)^{-1/\sigma}]^2 + 2\theta k_c [2\beta (1 - 1/\sigma) A(1 - s_n)^{-1/\sigma}] - \left\{(1 - s_n)^{-1} [s_n (1 - s_n) + s_n / \sigma] - 1 + s_n (1 - 1/\sigma) + \beta (1 - 1/\sigma) A(1 - s_n)^{-1/\sigma}\right\}$$

$$> \theta^2 k_c^2 + [1 - s_n (1 - 1/\sigma) - \beta (1 - 1/\sigma) A(1 - s_n)^{-1/\sigma}]^2 + 2\theta k_c [1 - s_n (1 - 1/\sigma) - (1 - s_n)(1 - s_n)]$$

$$> \theta^2 k_c^2 + [1 - s_n (1 - 1/\sigma) - \beta (1 - 1/\sigma) A(1 - s_n)^{-1/\sigma}]^2 + 2\theta k_c [1 - s_n (1 - 1/\sigma) - (1 - s_n) (1 - s_n)]$$

$$= \left\{\theta k_c + 1 - s_n (1 - 1/\sigma) - \beta (1 - 1/\sigma) A(1 - s_n)^{-1/\sigma}\right\}^2.$$  

So $k^{**} = [\theta k_c - 1 + s_n (1 - 1/\sigma) + \beta (1 - 1/\sigma) A(1 - s_n)^{-1/\sigma} + \Delta^{1/2}] / (2\theta) > 2\theta k_c / (2\theta) = k_c$. The other root, $k^{**} = [\theta (1 - s_n)^{1/\sigma} (1 - s_n) + s_n (1 - 1/\sigma) - 1 + \beta (1 - 1/\sigma) A(1 - s_n)^{-1/\sigma} - \Delta^{1/2}] / (2\theta)$, is dropped for being inconsistent with $k^{**} > k_c$. Q.E.D.
subsidy rates. Given the fundamentals, the higher the rates of both subsidies, the more likely the economy moves beyond the critical $k_c$ toward the balanced growth path. In fact, given $k^* = \left[ \beta (1-1/\sigma) A \right]^\sigma$, sufficient subsidization ensures $k^* > k_c = (1-s_s)^\sigma (1-s_n)$, while $k^{**} > k_c = (1-s_s)^\sigma (1-s_n)$ remains valid for all permissible rates of subsidies in the full range of $[0,1)$. That is, sufficient subsidization can rule out the neoclassical steady state in the long run and replace it by the steady state with balanced growth in capital and the variety of intermediates.

Now, we investigate the stability of the steady state by examining the asymptotic behavior of $k_t = K_t/N_t$, from any initial state $k_0 = K_0/N_0 > 0$. The mapping $k_t = \Phi(k_{t-1})$ in equation (26) is continuous: It is increasing in the range of $(0,k_c)$ and it may be increasing or decreasing in the range of $(k_c,\infty)$.

When $k_{t-1} \leq k_c = (1-s_s)^\sigma (1-s_n)$, there is no innovation in period $t$, that is, $N_t = N_{t-1}$. Consequently, the two kinds of innovation-oriented subsidies are non-operative in this neoclassical growth regime without innovation. On the other hand, when $k_{t-1} > k_c = (1-s_s)^\sigma (1-s_n)$, new intermediates are introduced and both subsidies can be operative. Economic growth in this region is driven by both the accumulation of capital and the innovation of new varieties of intermediates. Intuitively, when the growth rate of capital accumulation exceeds the growth rate of the variety of intermediates, the resultant ratio of capital per variety $k_t = K_t/N_t$ will increase; conversely, it will decrease. The slope of the transition curve of $k_t = \Phi(k_{t-1})$ in this region with innovation plays a crucial role in determining the asymptotic behavior of $k_t$ and thus deserves careful investigation.

Without the use of subsidies at $s_s = s_n = 0$, the dynamics of $k_t = \Phi(k_{t-1})$ in (26) will
become exactly the same as in Matsuyama (1999), where  $k_c = 1$, and the mapping for $k_i$ is always decreasing in the policy-active region with innovation. Under the empirically plausible conditions $1 < G < \theta - 1$ in his benchmark model, period-2 cycles are prevalent when $k_i$ alternates between the two regions forever. With the subsidies, it is important to ask whether the subsidies can change the slope of the transition equation $k_i = \Phi(k_{i-1})$ so as to mitigate or even eliminate the cycles.

**Proposition 2.** Suppose $\theta > 2$. Define $G_0 \equiv \beta(1-1/\sigma)A$ and $G \equiv \beta(1-1/\sigma)A/(k_c^{1/\sigma})$, where $k_c = (1-s_x)^{\theta}(1-s_n)$, with $0 \leq s_n, s_x < 1$.

1. If $G \leq 1$, then, for any given $k_0 > 0$, the economy will eventually converge toward a stable neoclassical stationary path with $\lim_{t \to \infty} k_i = k^*$ and settle down in the policy-dormant region.

2. If $G > 1$ and $s_x \leq 1/\sigma$, for any given $k_0 > 0$, there may exist cycles forever if the subsidy rates are low enough; if the subsidy rates are high enough (e.g. $s_n > \sigma(\theta - 2)/(1 + \sigma(\theta - 2))$ at $s_x = 0$ or $s_x \to 1/\sigma$ at $s_n = 0$), then $|dk_i/dk_{i-1}| < 1$ and the economy will converge toward a stable balanced growth path oscillatorily with $\lim_{t \to \infty} k_i = k^{**}$.

3. If $G > 1$ and $s_x > 1/\sigma$, for any given $k_0 > 0$, the economy will converge toward a stable balanced growth path monotonically with $\lim_{t \to \infty} k_i = k^{**}$.

**Proof.** In case (1) with $k^* = \beta(1-1/\sigma)A^{1/\sigma} < k_c = (1-s_x)^{\theta}(1-s_n)$ and $k_{i-1} < k_c$, the slope of $k_i = \Phi(k_{i-1}) = \beta(1-1/\sigma)A(k_{i-1})^{1-1/\sigma}$ in (26) is always positive, exceeding 1 at the origin.
(\(k_{t+1} \to 0\)) and falling below 1 at the steady state \((k_s = k^* = \left[\beta (1-1/\sigma) A\right]^\sigma\)) according to:

\[
\frac{dk_t}{dk_{t-1}} = (1-1/\sigma)\beta (1-1/\sigma)Ak_{t-1}^{-1/\sigma}
\]

because \(\sigma > 1\). The steady state level \(k^* < k_s\) is thus stable and the sequence \(\{k_t\}_{t=0}^\infty\) converges toward \(k^*\) for any \(k_0 > 0\) as in the standard neoclassical growth model. We illustrate case (1) in Figure 1.

In cases (2) and (3) with \(k^* = \left[\beta (1-1/\sigma) A\right]^\sigma > k_s = (1-s_x)\sigma (1-s_n)\), the slope of the transition equation \(k_t = \Phi(k_{t-1})\) in (26) for \(k_{t-1} > k_s\) is derived as

\[
\frac{dk_t}{dk_{t-1}} = \frac{\beta (1-1/\sigma)A(1-s_n)^{1-1/\sigma}[1-s_n(1-1/\sigma)][1-\theta(1-s_x)\sigma^{-1}]}{(1-s_n(1-1/\sigma)) + \theta[k_{t-1} - (1-s_x)\sigma (1-s_n)]^2}.
\]

Here, \(1-s_n(1-1/\sigma) > 0\) because \(s_n \in [0,1)\) and \(\sigma > 1\). Also, \(k_{t-1} - (1-s_x)\sigma (1-s_n) > 0\) for \(k_{t-1} > k_s\). So the sign of \(dk_t / dk_{t-1}\) is the same as the sign of \(1-\theta(1-s_x)\sigma^{-1}\). Recalling \(\theta = (1-1/\sigma)^{1-\sigma} > 1\) under \(\sigma > 1\), we have: sign \(dk_t / dk_{t-1}\) > 0 if and only if \(1 > s_x > 1/\sigma\) because with \(s_x \in [0,1)\), \(1-\theta(1-s_x)\sigma^{-1} > 0\) corresponds to \(1 > s_x > 1/\sigma\). Accordingly, \(dk_t / dk_{t-1} \leq 0\) if and only if \(0 \leq s_x \leq 1/\sigma\) under which \(1-\theta(1-s_x)\sigma^{-1} \leq 0\). Also, the absolute value of \(dk_t / dk_{t-1}\) is monotonically decreasing in \(k_{t-1}\), and approaches zero when \(k_{t-1}\) approaches infinity, implying that the dynamic system in (26) cannot converge outward to infinity.

For \(k_{t-1} > k_s\), there are thus two possibilities with either \(dk_t / dk_{t-1} \leq 0\) or \(dk_t / dk_{t-1} > 0\). If \(0 \leq s_x \leq 1/\sigma\) and thus \(dk_t / dk_{t-1} \leq 0\) beyond \(k_s\), then the economy may either oscillate forever with cycles or eventually converge toward the steady state of the balanced growth path such that \(\lim_{t \to \infty} k_t = k^*\), depending on whether \(|dk_t / dk_{t-1}| \geq 1\) or \(< 1\). Specifically, if the subsidy rates are low enough (say zero), then
\[|dk_i/dk_{i-1}| > 1 \] prevails under \( \theta > 2 \) and the economy behaves as in the original model of Matsuyama (1999) with endogenous cycles forever. If the subsidy rates are high enough, then we show \( |dk_i/dk_{i-1}| < 1 \) for \( dk_i/dk_{i-1} \leq 0 \) as follows. First, the slope \( dk_i/dk_{i-1} \) of \( k_i = \Phi(k_{i-1}) \) at \( k_{i-1} = k^{**} \) can be rewritten as:

\[
\frac{dk_i}{dk_{i-1}} \bigg|_{k_{i-1}=k^{**}} = \frac{\beta (1-1/\sigma) A (1-s_n)^{1-1/\sigma} [1-s_n (1-1/\sigma)][1-\theta (1-s_n)^{\sigma-1}]}{[1-s_n (1-1/\sigma) + \theta (k^{**} - (1-s_n)^{\sigma} (1-s_n))]^2} \\
= \frac{4 \beta (1-1/\sigma) A (1-s_n)^{1-1/\sigma} [1-s_n (1-1/\sigma)][1-\theta (1-s_n)^{\sigma-1}]}{\{1-s_n (1-1/\sigma) - \theta (1-s_n)^{\sigma} (1-s_n) + \beta (1-1/\sigma) A (1-s_n)^{1-1/\sigma} + \Delta^{1/2}\}^2},
\]

using the expression of \( k^{**} \) given in Proposition 1 for substitution.

For the special case without any subsidization, we have \( G = G_0 = \beta (1-1/\sigma) A \) and the following

\[
\frac{dk_i}{dk_{i-1}} \bigg|_{k_{i-1}=k^{**}} = \frac{1-\theta}{G} < 0, \text{ at } s_n = s_n = 0.
\]

The absolute value of this slope exceeds one (unstable \( k^{**} \)) if and only if \( 1 < G < \theta - 1 \) as in the original model of Matsuyama (1999). This condition applies under \( \theta > 2 \).

For \( 0 \leq s_n \leq 1/\sigma \), showing \( |dk_i/dk_{i-1}|_{k_{i-1}=k^{**}} < 1 \) is equivalent to showing

\[
F \equiv \{1-s_n (1-1/\sigma) - \theta (1-s_n)^{\sigma} (1-s_n) + \beta (1-1/\sigma) A (1-s_n)^{1-1/\sigma} + \Delta^{1/2}\}^2 \\
+ 4 \beta (1-1/\sigma) A (1-s_n)^{1-1/\sigma} [1-s_n (1-1/\sigma)][1-\theta (1-s_n)^{\sigma-1}] > 0
\]

whereby \( 1-\theta (1-s_n)^{\sigma-1} \leq 0 \). Using the expression for \( \Delta \) in \( F \) leads to

\[
F = [1-s_n (1-1/\sigma) - \theta (1-s_n)^{\sigma} (1-s_n) + \beta (1-1/\sigma) A (1-s_n)^{1-1/\sigma} ]^2 + \Delta + \\
2 \Delta^{1/2} [1-s_n (1-1/\sigma) - \theta (1-s_n)^{\sigma} (1-s_n) + \beta (1-1/\sigma) A (1-s_n)^{1-1/\sigma}] + \\
4 \beta (1-1/\sigma) A (1-s_n)^{1-1/\sigma} [1-s_n (1-1/\sigma)][1-\theta (1-s_n)^{\sigma-1}] \\
= 2 [1-s_n (1-1/\sigma) - \theta (1-s_n)^{\sigma} (1-s_n)]^2 + 2 \beta (1-1/\sigma) A (1-s_n)^{1-1/\sigma}]^2 + \\
4 \beta (1-1/\sigma) A (1-s_n)^{1-1/\sigma} \theta (1-s_n)^{\sigma-1}[s_n (1-s_n) + s_n/\sigma] + [1-s_n (1-1/\sigma)] \\
[1-\theta (1-s_n)^{\sigma-1}] + 2 \Delta^{1/2} [1-s_n (1-1/\sigma) - \theta (1-s_n)^{\sigma} (1-s_n) + \\
\beta (1-1/\sigma) A (1-s_n)^{1-1/\sigma}].
\]
Here, \( \{ \theta(1-s_x)^\sigma [s_x(1-s_n)+s_n/\sigma]+[1-s_n(1-1/\sigma)][1-\theta(1-s_x)^{\sigma-1}] \} \) can be shown to be equal to \([1-s_n(1-1/\sigma)-\theta(1-s_x)\sigma(1-s_n)]\). Thus, we have

\[
F = 2[1-s_n(1-1/\sigma)-\theta(1-s_x)\sigma(1-s_n)]^2 + 2[\beta (1-1/\sigma) A(1-s_n)^{1-1/\sigma}] + 4\beta (1-1/\sigma) A(1-s_n)^{1-1/\sigma} [1-s_n(1-1/\sigma)-\theta(1-s_x)\sigma(1-s_n)] + 2\Delta^{1/2} [1-s_n(1-1/\sigma)-\theta(1-s_x)^\sigma(1-s_n)+\beta (1-1/\sigma) A(1-s_n)^{1-1/\sigma}] + 2\Delta^{1/2} [1-s_n(1-1/\sigma)-\theta(1-s_x)\sigma(1-s_n)+\beta (1-1/\sigma) A(1-s_n)^{1-1/\sigma}]^2 + 2\Delta^{1/2} [1-s_n(1-1/\sigma)-\theta(1-s_x)\sigma(1-s_n)+\beta (1-1/\sigma) A(1-s_n)^{1-1/\sigma}]^2.
\]

A sufficient yet unnecessary condition for \( F > 0 \) is

\[
[1-s_n(1-1/\sigma)-\theta(1-s_x)\sigma(1-s_n)+\beta (1-1/\sigma) A(1-s_n)^{1-1/\sigma}] > [1-s_n(1-1/\sigma)-\theta(1-s_x)^\sigma(1-s_n)+(1-s_x)(1-s_n)] \quad \text{(under } G > 1) .
\]

This condition is satisfied by the stated conditions on the subsidy rates:

\[
s_n > \sigma (\theta - 2) / [1+\sigma (\theta - 2)] \in (0,1) \text{ under } \theta > 2 \text{ at } s_x = 0; \text{ or } s_x \rightarrow 1/\sigma \text{ at } s_n = 0.
\]

Namely, if the subsidy rates are sufficiently high such that \( |dk_x/dk_{t-1}|_{k_{t-1}=k^*} < 1 \), then the economy will eventually converge toward the stable steady state, or the stable balanced growth path. We depict case (2) in Figures 2 and 3.

Finally, if \( s_x > 1/\sigma \), showing \( dk_x/dk_{t-1}|_{k_{t-1}=k^*} < 1 \) for a stable steady state \( k^* \) is equivalent to showing the following,

\[
\left\{1-s_n(1-1/\sigma)-\theta(1-s_x)\sigma(1-s_n)+\beta (1-1/\sigma) A(1-s_n)^{1-1/\sigma} + \Delta^{1/2} \right\}^2 -4\beta (1-1/\sigma) A(1-s_n)^{1-1/\sigma} [1-s_n(1-1/\sigma)][1-\theta(1-s_x)^{\sigma-1}] > 0
\]

whereby \( 1-\theta(1-s_x)^{\sigma-1} > 0 \). The left-hand side of this inequality can be further decomposed into

\[
\left[1-s_n(1-1/\sigma)-\beta (1-1/\sigma) A(1-s_n)^{1-1/\sigma}\right]^2 + \theta(1-s_x)\sigma(1-s_n)
+ 2\Delta^{1/2} \left[1-s_n(1-1/\sigma)-\theta(1-s_x)^\sigma(1-s_n)+\beta (1-1/\sigma) A(1-s_n)^{1-1/\sigma}\right]
+ 2\theta[1-s_n(1-1/\sigma)](1-s_n)^{1-1/\sigma} [\beta (1-1/\sigma) A(1-s_x)(1-s_n)^{1/\sigma}] \\
+ 2\theta \beta (1-1/\sigma) A(1-s_x)^{\sigma-1} (1-s_n)^{1-1/\sigma} [s_n/\sigma + s_x(1-s_n)]
\]

which is strictly positive under \( G > 1 \), \( 0 \leq s_n < 1 \) and \( 1/\sigma < s_x < 1 \). We illustrate case (3).
in Figure 4. Q.E.D.

According to Proposition 2, both types of subsidies can eventually eliminate cycles once their rates are set high enough such that $|dk_t / dk_{t-1}| < 1$ under which the balanced growth path with innovation becomes stable. This is achieved either through strengthening the demand for new intermediates (via a higher $s_s$) or through reducing the innovation cost (via a higher $s_n$) such that R&D activities are profitable even at a low capital-variety ratio. By increasing varieties, the subsidization can exert different impacts on final output and thus on capital investment with a constant saving rate. First, it can directly increase final output by increasing the number of varieties according to equation (11). Second, it can indirectly reduce final output by reducing the equilibrium quantity of each type of intermediate input, as the subsequent increase in the total fixed innovation cost competes for the given amount of existing capital. For $0 \leq s_s < 1/\sigma$, a higher $s_s$ increases the equilibrium quantity of each new intermediate relative to each old intermediate, thereby leading to a smaller indirect effect on final output as opposed to the indirect effect of a higher $s_n$.

It is convenient to look at how these effects work for the stability of the dynamic system by using a general expression for the slope of the transition curve $k_t = \Phi(k_{t-1})$:

$$\frac{dk_t}{dk_{t-1}} = \frac{d(K_t / N_t)}{dk_{t-1}} = \frac{[N_t(dK_t / dk_{t-1}) - K_t(dN_t / dk_{t-1})]}{N_t^2}. $$

Rewrite it as the following ways that may help our interpretation:
The sign of \( \frac{dk_i}{dk_{t-1}} \) is determined by the terms in the bracket on the right-hand side. It is positive (negative) if the ratio of the derivative of capital investment to the derivative of variety expansion with respect to the initial abundance of capital, \( \frac{(dK_i / dk_{t-1})}{(dN_i / dk_{t-1})} \), is greater (smaller) than the resultant capital-variety ratio, \( k_i \).

The absolute value of \( \frac{dk_i}{dk_{t-1}} \) depends positively on the difference between the two responses, as fractions of their new stocks, \( \frac{[(dK_i / dk_{t-1}) / K_i - (dN_i / dk_{t-1}) / N_i]}{\sigma} \), as well as on the new capital-variety ratio, \( K_i / N_i \). In the steady state with \( k_{t-1} = k_i > k_c \), both the sign and the magnitude of \( \frac{dk_i}{dk_{t-1}} \) will depend solely on the gap in the respective elasticity of \( K_i \) and \( N_i \) with respect to \( k_{t-1} \).

Define \( G_0 = \beta (1 - 1/\sigma) \) that serves as the growth factor in the absence of subsidization. Holding \( N_{t-1} \) constant, if \( k_{t-1} > k_c \), from (14) and (22) we have \( dK_i / dk_{t-1} = G_0(1 - s_n)^{1-\sigma} N_{t-1} / [1 - s_n(1 - 1/\sigma)] > 0 \), while from (23) we obtain \( dN_i / dk_{t-1} = \theta N_{t-1} / [1 - s_n(1 - 1/\sigma)] > 0 \). Here, both capital investment and variety expansion respond positively to the initial abundance of capital per variety.

Under the assumption \( 1 < G_0 < \theta - 1 \), however, in the absence of subsidization the variety response to the initial abundance of capital is more elastic than the investment response, causing instability of the balanced growth path in the original Matsuyama model. Without subsidization, at the steady state \( k^* \) on the balanced growth path,
Proposition 1 and equation (23) lead to

\[ k^{\ast} = \frac{G_0 - 1 + \theta}{\theta} > 1 \text{ for } G_0 > 1 \text{ and } s_x = s_\eta = 0; N_t / N_{t-1} = G_0. \]

So \( dk_t / dk_{t-1} = [(dK_t / dk_{t-1}) - k_t(dN_t / dk_{t-1})] / N_t \) at the steady state \( k^{\ast} \) on the balanced growth path equals \( dk_t / dk_{t-1} \big|_{k_t=k^{\ast}} = (G_0 - k^{\ast} \theta)N_{t-1} / N_t = (1 - \theta) / G_0 \), which is negative under \( 1 < \theta \) and smaller than \(-1\) under \( G_0 < \theta - 1 \) and \( 2 < \theta \). In this case, period-two cycles prevail and persist forever.

Subsidizing the fixed innovation cost strengthens the response of variety expansion to the initial abundance of capital, \( dN_t / dk_{t-1} = \Theta N_{t-1} / [1 - s_n (1 - 1 / \sigma)] > 0 \), but weakens the response of investment, \( dK_t / dk_{t-1} = G_0 (1 - s_n)^{1-\sigma} N_{t-1} / [1 - s_n (1 - 1 / \sigma)] > 0 \), when setting \( s_x = 0 \). From (26), if this subsidy is large enough, at least for \( s_n > 1 / (2 - 1 / \sigma) \in (0,1) \), a further increase in \( s_n \) will also lead to lower capital per variety \( k_t \) as long as \( k_t > k_c \) and \( k_{t-1} > k_c \), because beyond the level \( s_n = 1 / (2 - 1 / \sigma) \in (0,1) \) the numerator of \( k_t \) starts to decrease with \( s_n \). Combining them together, the sign of \( dk_t / dk_{t-1} = [G_0 (1 - s_n)^{1-\sigma} - k_t \theta] (N_{t-1} / N_t) / [1 - s_n (1 - 1 / \sigma)] \) is only determined by the factor \( [G_0 (1 - s_n)^{1-\sigma} - k_t \theta] \) whereby both terms, \( G_0 (1 - s_n)^{1-\sigma} \) and \( k_t \theta \), eventually decline with the subsidy rate on the innovation cost when the subsidy rate becomes large enough. This helps to explain why the sign of \( dk_t / dk_{t-1} \) remains negative for all levels of the subsidy rate in the regime with innovation. The remaining factors that only determine the magnitude, not the sign, of \( dk_t / dk_{t-1} \) are decreasing with \( s_n \) as well:

\[ (N_{t-1} / N_t) / [1 - s_n (1 - 1 / \sigma)] = [1 - s_n (1 - 1 / \sigma)] + \theta [k_{t-1} - (1 - s_n)]^{-1} \text{ for } \theta > 1. \]

This explains why the absolute value of \( dk_t / dk_{t-1} \) becomes smaller when the subsidy rate \( s_n \) becomes larger.
Subsidizing the purchase of new intermediates does not affect the responses of capital investment and variety expansion to the initial abundance of capital, when setting \( s_n = 0 \). The sign of \( \frac{dk_t}{dk_{t-1}} \) is merely determined by \( [G_0 - k_t \theta] \) which is initially negative when the subsidy rate is equal to zero. It follows from (26) that a higher subsidy rate for the purchase of new intermediates will reduce the amount of capital per variety \( k_t \) as long as \( k_t > k_c \) and \( k_{t-1} > k_c \), because \( \frac{dk_t}{ds_t} \) is signed by \( [1 - \sigma s_t - \theta(1 - s_t)^\sigma - \theta k_{t-1}(\sigma - 1)] < 0 \) in which \( 1 - \sigma s_t - \theta(1 - s_t)^\sigma < 0 \) attains a maximum \( 1/\sigma - 1 < 0 \) at \( s_x = 1/\sigma \). Consequently, a higher subsidy rate for the purchase of new intermediates will increase the value of \( [G_0 - k_t \theta] \) in general or reduce the absolute value \( |G_0 - k_t \theta| \) when \( G_0 - k_t \theta < 0 \) in particular. At \( G_0 = k^{*\theta} \) as a special case on the balanced growth path, the corresponding subsidy rate is \( s_x = 1/\sigma \) that leads to \( \frac{dk_t}{dk_{t-1}} \bigg|_{k_t = k^*} = 0 \). When the subsidy rate is increased further for \( s_x > 1/\sigma \), \( [G_0 - k^{*\theta}] > 0 \) must hold, leading to \( \frac{dk_t}{dk_{t-1}} > 0 \) on the balanced growth path. Recall that subsidizing the purchase of new intermediates at a rate \( s_x > 1/\sigma \) will lead to \( x_c < x_m \), thereby creating a loss in final output. The loss in final output, due to a higher \( s_x \) beyond \( s_x = 1/\sigma \), will in turn lead to a decline in capital investment for a constant saving rate given any initial \( k_{t-1} \), while variety expansion accelerates at a higher \( s_x \). Consequently, a higher \( s_x \) with \( s_x > 1/\sigma \) will reduce \( k^{*\theta} \) to a level such that \( [G_0 - k^{*\theta}] > 0 \) and thus \( \frac{dk_t}{dk_{t-1}} > 0 \) at the steady state. For \( s_n = 0 \), the absolute value of \( \frac{dk_t}{dk_{t-1}} \) on the balanced growth path is derived below:

\[
\frac{dk_t}{dk_{t-1}} \bigg|_{k_t = k^*} = \frac{G_0 + 1 - \theta(1 - s_x)^\sigma - \Delta^{1/2}}{G_0 + 1 - \theta(1 - s_x)^\sigma + \Delta^{1/2}} < 1.
\]

Therefore, the balanced growth path becomes stable once the subsidy rate on the purchase of new intermediates is set high enough such that \( |\frac{dk_t}{dk_{t-1}}|_{k_t = k^*} < 1 \).
The analysis so far has little quantitative implications of the subsidies and sheds no light on the welfare consequences of the subsidies and on the optimal subsidy rates due to the model's complexity. To gain insights in these directions, we now turn to numerical simulations.

4. Numerical simulation results

In this section, we will gauge the quantitative implications and the welfare gains from the subsidies and find optimal subsidy rates by numerical simulations for plausible parameterizations.

Since the two kinds of subsidies can promote growth, investment and innovation and since they can eliminate cyclical fluctuations, they have potential for enhancing social welfare compared with the suboptimal benchmark model without any government subsidization. Starting with lower final output and slower innovation and growth under monopoly pricing compared to their socially optimal levels in the Romer regime (as shown in Barro and Sala-i-Martin, 1995), increasing the subsidies may have different impacts on final output on the one hand and on innovation and growth on the other. Thus, increasing the subsidy rates tends to have opposing impacts on welfare. The positive effect of subsidies on innovation and growth tends to enhance welfare when the innovation rate and the growth rate are lower than their socially optimal levels. The effect of subsidies on final output is initially positive at low subsidy rates and eventually negative at sufficiently high subsidy rates, as shown earlier in our model.

Moreover, when subsidies mitigate cyclical fluctuations in consumption, investment
and innovation, there are possible efficiency gains due to diminishing marginal utility and diminishing marginal product. Thus, the overall welfare effect is expected to be initially positive, when the subsidy rates are low, but eventually negative, when the subsidy rates become high enough. Unlike existing studies of R&D subsidization that focus on the balanced growth path only, the model here allows us to explore whether the optimal rates of subsidies lie in the range where they eventually eliminate cycles with oscillatory or monotonic convergence toward the steady state on the balanced growth path.

We set a benchmark parameterization as $A = 2.5$, $\beta = 0.63$, $\theta = 2.4414$, and $\sigma = 5$. The value of $\sigma = 5$ is in line with that in Matsuyama (1999) where it plays dual roles: $1 - 1/\sigma = 0.8$ is the share of capital (interpreted broadly as both physical and human capital); $1/(\sigma - 1) = 0.25$ is the monopoly mark-up enjoyed by the innovator. When one period (the length of patent protection) corresponds to 15 years\(^3\), an annual discount factor of 0.97 leads to $\beta = (0.97)^{15} = 0.63$.

The value of $A$ is chosen such that the calibrated economy starts with period-2 cycles in the absence of subsidization. According to Proposition 2, the benchmark parameterization without subsidies, $s_x = 0$ and $s_n = 0$, satisfies $1 < G < \theta - 1$, indicating that the economy grows through period-2 cycles, perpetually moving back and forth between the two regimes. This is an empirically plausible case as argued in Matsuyama (1999). Moreover, we choose the initial states as $K_0 = 0.4$ and $N_0 = 1$.

We track down the equilibrium dynamics and calculate the infinitely lived

\(^3\) According to WTO’s Agreement on Trade-Related Aspects of Intellectual Property Rights, the term of patent is 20 years from the filing date of the application. However, many countries have laws with shorter terms of 6 to 10 years. In our simulation, we take the average term of patent as 15 years. In fact, the shorter the term of patents, the larger the discount factor, thereby the stronger the positive results from subsidization.
representative agent’s welfare in (15) by using the first-order difference equations in (22) and (23) to update the states $K_t$ and $N_t$ and then finding final output, consumption, the tax rate and utility in every period. In order to better comprehend the change in welfare, we use its equivalent variation in consumption in every period. Define the equivalent variation in consumption in each period by $\Delta$ that allows the benchmark case without subsidization to reach the same welfare level as that in a case with subsidization type $i$:

$$U_{\text{no-subsidization}} + \frac{\beta}{1-\beta} \ln(1+\Delta) = U_i.$$  

This corresponds to adding $\sum_{t=1}^{\infty} \beta^t \ln(1+\Delta) = \beta [\ln(1+\Delta)]/(1-\beta)$ to the welfare level in the benchmark case without subsidization. We work out a large number of periods in each case (say 1000) such that any further increase in the number of periods has no impact on welfare within ten decimal digits.

In Table 1 and Table 2, we report the simulation results when increasing $s_s$ and $s_n$ from zero to reach and go beyond a peak of the welfare level, one at a time, respectively. In Figure 5 and Figure 6, we also depict the capital variety ratios in the two regimes $(k^H, k^L)$ in the scenario of period-2 cycles or the steady state capital variety ratio, $k^\ast$, on the balanced growth path when increasing $s_s$ and $s_n$ from zero to sufficiently high rates. The subsequent welfare levels are plotted in the same figure to better illustrate the range in which the optimal rates of subsidies can maximize welfare.

In Table 1, we set $s_n = 0$ and examine the dynamic behavior, the balanced growth rate, the consumption tax rate, the welfare level, and the consumption equivalent variation when varying $s_s$ from 0 in the benchmark case to 40% gradually (by 0.001 each step). When $s_s$ is sufficiently small (e.g., $s_s = 0.01$) the economy still alternates between the policy-dormant
and the policy-active regions, and the social welfare is higher than that in the case without subsidization. When $s_s$ is increased further but still below $1/\sigma = 0.2$, the steady state $k''$ becomes stable and the economy achieves oscillatory convergence toward $k''$ as predicted in case 2 of Proposition 2. As $s_s$ is increased beyond $1/\sigma = 0.2$, the condition for case 3 of Proposition 2 is satisfied for $k_s$ to converge toward $k''$ monotonically. For a better view of the welfare effect, in Figure 5 we vary $s_s$ from 0 to 0.6 ($s_n = 0$) and find the subsequent welfare levels. It is worth noting that the welfare level is a concave and smooth curve, peaking at a unique point when $s_s = 0.16$, due to a balance between the various gains and losses in efficiency mentioned earlier. This optimal rate lies in the range where the subsidy eliminates the cycles with oscillatory convergence toward the steady state. The resultant welfare level at the optimal subsidy rate is 0.0643 compared to -0.0811 in the benchmark case without subsidization. The change in welfare is equivalent to a significant 8.91% rise in consumption in every period from the benchmark case without subsidization.

In Table 2 and Figure 6, we fix $s_s = 0$ and focus on the effects of the subsidy on the fixed R&D cost, $s_n$. Period-2 cycles persist for a relatively wide range of $s_n$. After that, when it is high enough to satisfy the condition for case 2 of Proposition 2, period-2 cycles are replaced by oscillatory convergence to $k''$ in the policy-active region. In Figure 6, we vary $s_n$ from 0 to 90% ($s_s = 0$) and find the subsequent welfare level. It is worth noting that the welfare curve is concave and smooth as well, peaking at a unique point when $s_n^* = 0.61$ that achieves oscillatory convergence toward the steady state with balanced growth in capital and variety. The welfare curve is flatter before peaking and takes longer to reach the optimal level of the subsidy rate than in Figure 5, because this subsidy $s_n$ does
not change the price gap and therefore does not create the additional gain or loss in static efficiency as those created by \( s_x \) on either side of \( s_x = 1/\sigma \). So the efficiency gain from faster variety expansion and from convergence to the steady state at a higher subsidy rate to the fixed R&D cost is gradually offset by a loss from the subsequent decline in the equilibrium quantity of each intermediate. Beyond this optimal subsidy rate, the welfare level declines rapidly, because the efficiency loss from the declined use of each intermediate is increasing at the margin. The maximized welfare level is 0.0668 against -0.0811 in the benchmark case without subsidization. The welfare change is equivalent to a significant 9.07% rise in consumption in every period.

In Figure 7, we jointly use the subsidy to the new intermediate goods and the subsidy to R&D investment at the same time and plot the welfare surface when varying both \( s_x \) and \( s_n \) from zero to sufficiently high rates. It turns out the welfare surface is concave and smooth, peaking at a unique point when \( s_x = 0.1 \) and \( s_n = 0.38 \). The capital variety ratio will converge to the steady state, \( k^* = 0.4972 \), oscillatorily in the policy-active regime when the subsidies are financed by a flat-rate consumption tax of 9.21%. The maximized welfare level is 0.0866, higher than those in cases with one subsidy at a time as expected. The welfare change from the benchmark case is equivalent to a substantial 10.35% rise in consumption in each period.

Note that the respective optimal rates of subsidies mitigate and eventually eliminate the cycles in all the reported cases. So part of the welfare gain must have come from smoothing consumption, which is new compared to the results in existing analysis of subsidization on the balanced growth path alone.
5. Conclusion

We have examined the implications of two types of subsidies, one type to the purchase of new intermediate products and the other to R&D investment, in the model of Matsuyama (1999) with growth through endogenous cycles. One contribution of doing so is that the subsidization can reduce the critical level of the capital-variety ratio substantially, enhancing the possibility for the economy to stay at the policy-active region with sustainable innovation and growth. Sufficient subsidization can rule out the neoclassical regime without innovation from the steady state in the long run.

Another contribution is that we have characterized several possible scenarios of the asymptotic paths of the representative agent economy from any initial state depending on the values of the economic fundamentals and subsidy rates. In a novel yet empirically plausible scenario, sufficient subsidization stabilizes the balanced growth path with innovation and thus leads to oscillatory or monotonic convergence towards it.

Also, we have used numerical examples to gauge the welfare gains from the two types of subsidies starting from an empirically plausible parameterization for a period-2 cycle economy in the absence of subsidization. It turns out that both types of subsidies can help enhance the welfare level significantly in terms of a 9-10% rise in consumption for each period; and the optimal subsidy rates maximizing social welfare are calculated in their plausible ranges that eventually eliminate cycles for consumption smoothing.

Our results in this paper appear consistent not only with substantial subsidization to new investment and R&D spending but also with the combination of intensified innovation
and dampened cyclical fluctuations in many industrial nations in the postwar era.

Our results about R&D subsidies are different from those in Haruyama (2009): optimal R&D subsidies cannot eliminate cycles in their model but can do so in our model. As mentioned earlier, the models are very different (intermediates production and innovation must use accumulated capital in the present model but only use labor in their model). So our results concerning the R&D subsidies are complementary to their results. In particular, our results about the subsidies to the purchase of new intermediates are new in models with endogenous cycles and sustainable growth.
References


Table 1. Results of changing the subsidy to the purchase of new intermediate goods

<table>
<thead>
<tr>
<th>Subsidy rates</th>
<th>Mode of dynamics</th>
<th>( k^{**} ) or ( k^L, k^H )</th>
<th>Growth rate (annual %)</th>
<th>Tax rate</th>
<th>Welfare level</th>
<th>C equivalent variation %</th>
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<tbody>
<tr>
<td>0</td>
<td>Period-2 cycle</td>
<td>(0.983, 1.243)</td>
<td>1.262</td>
<td>0.0, 0.0</td>
<td>-0.0811</td>
<td>0.0</td>
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<tr>
<td>0.01</td>
<td>Period-2 cycle</td>
<td>(0.945, 1.204)</td>
<td>1.272</td>
<td>0.0, 0.34</td>
<td>-0.0716</td>
<td>0.56</td>
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<tr>
<td>0.02</td>
<td>Oscillatory</td>
<td>1.020</td>
<td>1.283</td>
<td>0.37</td>
<td>-0.0626</td>
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<td>0.05</td>
<td>Oscillatory</td>
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<td>1.17</td>
<td>-0.0292</td>
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<td>0.16</td>
<td>Oscillatory</td>
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<td>1.429</td>
<td>8.27</td>
<td>0.0643</td>
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<td>1.493</td>
<td>87.23</td>
<td>-1.0077</td>
<td>-72.32</td>
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</table>

Note: (1) The growth rate in the period-2 cycle economy is calculated as the geometric average of the corresponding growth rates in the two regions following Matsuyama (1999). (2) The values in the brackets beside growth rates indicate the discounted annual rates. (3) The subsidy rates with * indicates the optimal rate maximizing the social welfare.
Table 2. Results of changing the subsidy to the R&D investment

Benchmark parameterization: \( A = 2.5, \beta = 0.63, \theta = 2.4414, \sigma = 5, K_0 = 0.4, N_0 = 1, s_x = s_a = 0 \)

<table>
<thead>
<tr>
<th>Subsidy rates</th>
<th>Mode of dynamics</th>
<th>( k^* ) or ((k^L, k^H))</th>
<th>Growth rate gross (annual %)</th>
<th>Tax rate %</th>
<th>Welfare level</th>
<th>C equivalent variation %</th>
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<tr>
<td>0</td>
<td>Period-2 cycle</td>
<td>(0.983, 1.243)</td>
<td>1.262 (1.56)</td>
<td>0.0, 0.0</td>
<td>-0.0811</td>
<td>0.0</td>
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<td>0.1</td>
<td>Period-2 cycle</td>
<td>(0.888, 1.146)</td>
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<td>0.2</td>
<td>Period-2 cycle</td>
<td>(0.794, 1.047)</td>
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<td>0.0, 1.85</td>
<td>-0.0370</td>
<td>2.62</td>
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<td>0.3</td>
<td>Period-2 cycle</td>
<td>(0.700, 0.947)</td>
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<td>Oscillatory</td>
<td>0.795</td>
<td>1.342 (1.98)</td>
<td>1.79</td>
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<td>0.4</td>
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<td>1.407 (2.30)</td>
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<td>Oscillatory</td>
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<td>1.446 (2.49)</td>
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<td>Oscillatory</td>
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<td>0.9</td>
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<td>1.518 (2.82)</td>
<td>63.02</td>
<td>-1.5241</td>
<td>-133.37</td>
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Note: (1) The growth rate in the period-2 cycle economy is calculated as the geometric average of the corresponding growth rates in the two regions following Matsuyama (1999).
(2) The values in the brackets beside growth rates indicate the discounted annual rates.
(3) The subsidy rates with \* indicates the optimal rate maximizing the social welfare.
Figure 1. $G < 1$

Figure 2. $G > 1, s_x < 1/\sigma$ and $\left| \frac{dk_i}{dk_{i-1}} \right|_{k_{i-1} = k^*} > 1$
Figure 3. $G > 1, s < 1/\sigma$ and $|dk_t/ dk_{t-1}|_{k_{t-1} = k_t} < 1$

Figure 4. $G > 1$ and $s > 1/\sigma$
Figure 5. The dynamics of \( k_i \) and welfare levels by changing \( s_x \).

![Graph showing dynamics of \( k_i \) and welfare levels by changing \( s_x \).]

Figure 6. The dynamics of \( k_i \) and welfare levels by changing \( s_n \).

![Graph showing dynamics of \( k_i \) and welfare levels by changing \( s_n \).]
Figure 7. The welfare by using both $s_x$ and $s_n$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{welfare_graph.png}
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<td>Weber, E.J.</td>
<td>PRE-INDUSTRIAL BIMETALLISM: THE INDEX COIN HYPOTHESIS</td>
</tr>
<tr>
<td>09.13</td>
<td>McLure, M.</td>
<td>PARETO AND PIGOU ON OPHELIMITY, UTILITY AND WELFARE: IMPLICATIONS FOR PUBLIC FINANCE</td>
</tr>
<tr>
<td>09.14</td>
<td>Weber, E.J.</td>
<td>WILFRED EDWARD GRAHAM SALTER: THE MERITS OF A CLASSICAL ECONOMIC EDUCATION</td>
</tr>
<tr>
<td>09.15</td>
<td>Tyers, R. and Huang, L.</td>
<td>COMBATING CHINA’S EXPORT CONTRACTION: FISCAL EXPANSION OR ACCELERATED INDUSTRIAL REFORM</td>
</tr>
<tr>
<td>09.16</td>
<td>Zweifel, P., Plaff, D. and Kühn, J.</td>
<td>IS REGULATING THE SOLVENCY OF BANKS COUNTER-PRODUCTIVE?</td>
</tr>
<tr>
<td>09.17</td>
<td>Clements, K.</td>
<td>THE PHD CONFERENCE REACHES ADULTHOOD</td>
</tr>
<tr>
<td>09.19</td>
<td>Harris, R.G. and Robertson, P.</td>
<td>TRADE, WAGES AND SKILL ACCUMULATION IN THE EMERGING GIANTS</td>
</tr>
<tr>
<td>09.20</td>
<td>Peng, J., Cui, J., Qin, F. and Groenewold, N.</td>
<td>STOCK PRICES AND THE MACRO ECONOMY IN CHINA</td>
</tr>
<tr>
<td>09.21</td>
<td>Chen, A. and Groenewold, N.</td>
<td>REGIONAL EQUALITY AND NATIONAL DEVELOPMENT IN CHINA: IS THERE A TRADE-OFF?</td>
</tr>
<tr>
<td>DP NUMBER</td>
<td>AUTHORS</td>
<td>TITLE</td>
</tr>
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<tr>
<td>10.01</td>
<td>Hendry, D.F.</td>
<td>RESEARCH AND THE ACADEMIC: A TALE OF TWO CULTURES</td>
</tr>
<tr>
<td>10.02</td>
<td>McLure, M., Turkington, D. and Weber, E.J.</td>
<td>A CONVERSATION WITH ARNOLD ZELLNER</td>
</tr>
<tr>
<td>10.03</td>
<td>Butler, D.J., Burbank, V.K. and Chisholm, J.S.</td>
<td>THE FRAMES BEHIND THE GAMES: PLAYER’S PERCEPTIONS OF PRISONER’S DILEMMA, CHICKEN, DICTATOR, AND ULTIMATUM GAMES</td>
</tr>
<tr>
<td>10.04</td>
<td>Harris, R.G., Robertson, P.E. and Xu, J.Y.</td>
<td>THE INTERNATIONAL EFFECTS OF CHINA’S GROWTH, TRADE AND EDUCATION BOOMS</td>
</tr>
<tr>
<td>10.05</td>
<td>Clements, K.W., Mongey, S. and Si, J.</td>
<td>THE DYNAMICS OF NEW RESOURCE PROJECTS A PROGRESS REPORT</td>
</tr>
<tr>
<td>10.06</td>
<td>Costello, G., Fraser, P. and Groenewold, N.</td>
<td>HOUSE PRICES, NON-FUNDAMENTAL COMPONENTS AND INTERSTATE SPILLOVERS: THE AUSTRALIAN EXPERIENCE</td>
</tr>
<tr>
<td>10.07</td>
<td>Clements, K.</td>
<td>REPORT OF THE 2009 PHD CONFERENCE IN ECONOMICS AND BUSINESS</td>
</tr>
<tr>
<td>10.08</td>
<td>Robertson, P.E.</td>
<td>INVESTMENT LED GROWTH IN INDIA: HINDU FACT OR MYTHOLOGY?</td>
</tr>
<tr>
<td>10.09</td>
<td>Fu, D., Wu, Y. and Tang, Y.</td>
<td>THE EFFECTS OF OWNERSHIP STRUCTURE AND INDUSTRY CHARACTERISTICS ON EXPORT PERFORMANCE</td>
</tr>
<tr>
<td>10.10</td>
<td>Wu, Y.</td>
<td>INNOVATION AND ECONOMIC GROWTH IN CHINA</td>
</tr>
<tr>
<td>10.11</td>
<td>Stephens, B.J.</td>
<td>THE DETERMINANTS OF LABOUR FORCE STATUS AMONG INDIGENOUS AUSTRALIANS</td>
</tr>
<tr>
<td>10.12</td>
<td>Davies, M.</td>
<td>FINANCING THE BURRA BURRA MINES, SOUTH AUSTRALIA: LIQUIDITY PROBLEMS AND RESOLUTIONS</td>
</tr>
<tr>
<td>10.13</td>
<td>Tyers, R. and Zhang, Y.</td>
<td>APPRECIATING THE RENMINBI</td>
</tr>
<tr>
<td>10.14</td>
<td>Clements, K.W., Lan, Y. and Seah, S.P.</td>
<td>THE BIG MAC INDEX TWO DECADES ON AN EVALUATION OF BURGERNOMICS</td>
</tr>
<tr>
<td>10.15</td>
<td>Robertson, P.E. and Xu, J.Y.</td>
<td>IN CHINA’S WAKE: HAS ASIA GAINED FROM CHINA’S GROWTH?</td>
</tr>
<tr>
<td>10.17</td>
<td>Gao, G.</td>
<td>WORLD FOOD DEMAND</td>
</tr>
<tr>
<td>10.18</td>
<td>Wu, Y.</td>
<td>INDIGENOUS INNOVATION IN CHINA: IMPLICATIONS FOR SUSTAINABLE GROWTH</td>
</tr>
<tr>
<td>10.19</td>
<td>Robertson, P.E.</td>
<td>DECIPHERING THE HINDU GROWTH EPIC</td>
</tr>
<tr>
<td>10.20</td>
<td>Stevens, G.</td>
<td>RESERVE BANK OF AUSTRALIA-THE ROLE OF FINANCE</td>
</tr>
<tr>
<td>10.21</td>
<td>Widmer, P.K., Zweifel, P. and Farsi, M.</td>
<td>ACCOUNTING FOR HETEROGENEITY IN THE MEASUREMENT OF HOSPITAL PERFORMANCE</td>
</tr>
<tr>
<td>10.22</td>
<td>McLure, M.</td>
<td>ASSESSMENTS OF A. C. PIGOU’S FELLOWSHIP THESSES</td>
</tr>
<tr>
<td>10.23</td>
<td>Poon, A.R.</td>
<td>THE ECONOMICS OF NONLINEAR PRICING: EVIDENCE FROM AIRFARES AND GROCERY PRICES</td>
</tr>
<tr>
<td>10.24</td>
<td>Halperin, D.</td>
<td>FORECASTING METALS RETURNS: A BAYESIAN DECISION THEORETIC APPROACH</td>
</tr>
<tr>
<td>10.26</td>
<td>Chen, A., Groenewold, N. and Hagger, A.J.</td>
<td>THE REGIONAL ECONOMIC EFFECTS OF A REDUCTION IN CARBON EMISSIONS</td>
</tr>
<tr>
<td>10.27</td>
<td>Siddique, A., Selvanathan, E.A. and Selvanathan, S.</td>
<td>REMITTANCES AND ECONOMIC GROWTH: EMPIRICAL EVIDENCE FROM BANGLADESH, INDIA AND SRI LANKA</td>
</tr>
<tr>
<td>DP NUMBER</td>
<td>AUTHORS</td>
<td>TITLE</td>
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<tr>
<td>11.01</td>
<td>Robertson, P.E.</td>
<td>DEEP IMPACT: CHINA AND THE WORLD ECONOMY</td>
</tr>
<tr>
<td>11.02</td>
<td>Kang, C. and Lee, S.H.</td>
<td>BEING KNOWLEDGEABLE OR SOCIABLE? DIFFERENCES IN RELATIVE IMPORTANCE OF COGNITIVE AND NON-COGNITIVE SKILLS</td>
</tr>
<tr>
<td>11.03</td>
<td>Turkington, D.</td>
<td>DIFFERENT CONCEPTS OF MATRIX CALCULUS</td>
</tr>
<tr>
<td>11.04</td>
<td>Golley, J. and Tyers, R.</td>
<td>CONTRASTING GIANTS: DEMOGRAPHIC CHANGE AND ECONOMIC PERFORMANCE IN CHINA AND INDIA</td>
</tr>
<tr>
<td>11.05</td>
<td>Collins, J., Baer, B. and Weber, E.J.</td>
<td>ECONOMIC GROWTH AND EVOLUTION: PARENTAL PREFERENCE FOR QUALITY AND QUANTITY OF OFFSPRING</td>
</tr>
<tr>
<td>11.06</td>
<td>Turkington, D.</td>
<td>ON THE DIFFERENTIATION OF THE LOG LIKELIHOOD FUNCTION USING MATRIX CALCULUS</td>
</tr>
<tr>
<td>11.07</td>
<td>Groenewold, N. and Paterson, J.E.H.</td>
<td>STOCK PRICES AND EXCHANGE RATES IN AUSTRALIA: ARE COMMODITY PRICES THE MISSING LINK?</td>
</tr>
<tr>
<td>11.08</td>
<td>Chen, A. and Groenewold, N.</td>
<td>REDUCING REGIONAL DISPARITIES IN CHINA: IS INVESTMENT ALLOCATION POLICY EFFECTIVE?</td>
</tr>
<tr>
<td>11.09</td>
<td>Williams, A., Birch, E. and Hancock, P.</td>
<td>THE IMPACT OF ON-LINE LECTURE RECORDINGS ON STUDENT PERFORMANCE</td>
</tr>
<tr>
<td>11.10</td>
<td>Pawley, J. and Weber, E.J.</td>
<td>INVESTMENT AND TECHNICAL PROGRESS IN THE G7 COUNTRIES AND AUSTRALIA</td>
</tr>
<tr>
<td>11.11</td>
<td>Tyers, R.</td>
<td>AN ELEMENTARY MACROECONOMIC MODEL FOR APPLIED ANALYSIS AT UNDERGRADUATE LEVEL</td>
</tr>
<tr>
<td>11.12</td>
<td>Clements, K.W. and Gao, G.</td>
<td>QUALITY, QUANTITY, SPENDING AND PRICES</td>
</tr>
<tr>
<td>11.13</td>
<td>Tyers, R. and Zhang, Y.</td>
<td>JAPAN’S ECONOMIC RECOVERY: INSIGHTS FROM MULTI-REGION DYNAMICS</td>
</tr>
<tr>
<td>11.14</td>
<td>McLure, M.</td>
<td>A. C. PIGOU’S REJECTION OF PARETO’S LAW</td>
</tr>
<tr>
<td>11.15</td>
<td>Kristoffersen, I.</td>
<td>THE SUBJECTIVE WELLBEING SCALE: HOW REASONABLE IS THE CARDINALITY ASSUMPTION?</td>
</tr>
<tr>
<td>11.16</td>
<td>Clements, K.W., Izan, H.Y. and Lan, Y.</td>
<td>VOLATILITY AND STOCK PRICE INDEXES</td>
</tr>
<tr>
<td>11.17</td>
<td>Parkinson, M.</td>
<td>SHANN MEMORIAL LECTURE 2011: SUSTAINABLE WELLBEING – AN ECONOMIC FUTURE FOR AUSTRALIA</td>
</tr>
<tr>
<td>11.18</td>
<td>Chen, A. and Groenewold, N.</td>
<td>THE NATIONAL AND REGIONAL EFFECTS OF FISCAL DECENTRALISATION IN CHINA</td>
</tr>
<tr>
<td>11.20</td>
<td>Wu, Y.</td>
<td>GAS MARKET INTEGRATION: GLOBAL TRENDS AND IMPLICATIONS FOR THE EAS REGION</td>
</tr>
<tr>
<td>11.21</td>
<td>Fu, D., Wu, Y. and Tang, Y.</td>
<td>DOES INNOVATION MATTER FOR CHINESE HIGH-TECH EXPORTS? A FIRM-LEVEL ANALYSIS</td>
</tr>
<tr>
<td>Page</td>
<td>Authors</td>
<td>Title</td>
</tr>
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</tr>
<tr>
<td>11.22</td>
<td>Fu, D. and Wu, Y.</td>
<td>EXPORT WAGE PREMIUM IN CHINA’S MANUFACTURING SECTOR: A FIRM LEVEL ANALYSIS</td>
</tr>
<tr>
<td>11.23</td>
<td>Li, B. and Zhang, J.</td>
<td>SUBSIDIES IN AN ECONOMY WITH ENDOGENOUS CYCLES OVER NEOCLASSICAL INVESTMENT AND NEO-SCHUMPERIAN INNOVATION REGIMES</td>
</tr>
<tr>
<td>11.24</td>
<td>Krey, B., Widmer, P.K. and Zweifel, P.</td>
<td>EFFICIENT PROVISION OF ELECTRICITY FOR THE UNITED STATES AND SWITZERLAND</td>
</tr>
<tr>
<td>11.25</td>
<td>Wu, Y.</td>
<td>ENERGY INTENSITY AND ITS DETERMINANTS IN CHINA’S REGIONAL ECONOMIES</td>
</tr>
</tbody>
</table>