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BARGAINING DELEGATION IN MONOPOLY

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Bargaining Delegation in Monopoly

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Abstract

We study efficiency and distributional implications of bargaining in a monopoly, where the shareholders and the workers delegate the task of bargaining to a manager and a union leader respectively. Bargaining delegation leads to underproduction causing the organizational pie to contract severely rendering mutual gains from delegation impossible. With an increase in the union's bargaining power profit may rise and the union's utility may fall. This suggests that delegation can compensate for weaker bargaining power. Further, numerical examples confirm that if the shareholders and the workers had a choice over delegation, they would indeed choose to delegate, on some occasions giving rise to a Prisoners' Dilemma problem.

***JEL* Classification:** L12, L14, D43. **Key Words:** Managerial incentives, efficient bargaining, bilateral delegation

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1 Introduction

The strategic delegation approach developed by Vickers (1985), Fershtman (1985), Fershtman and Judd (1987) and Sklivas (1987) has shown that in Cournot oligopoly firms will resort to sales-oriented managerial incentives to gain strategic advantage over their rivals. Szymanski (1994) examined this argument in the context of (right-to-manage) wage bargaining with labor union and showed that sales orientation occurs only when the union's bargaining power is very low. When the union's bargaining power is sufficiently high, firms would orient their managers to profit maximization.

The sales-orientation result helped to explore many issues[1]. For example, Basu (1995) offered a new explanation of Stackelberg equilibrium; Zhang and Zhang (1997) studied R&D incentives, Lambertini and Trombetta (2002) stability of collusion, and Mujumdar and Pal (2007) endogenous market structure in a two-period duopoly. Apart from their reliance on sales-orientation, these articles commonly assumed oligopolistic environments to justify delegation in the first place.

Despite the influence of the strategic delegation literature, there are some limitations of the delegation models that cannot be overlooked. First, the incentive to delegate has not been examined in the context of monopoly (with the exception of Zabojnik (1998)).[2] Second, though Szymanski (1994) introduced wage bargaining, delegation in his model is one-sided; that is, only the shareholders delegate (in each firm), but not the union members. In reality, unions are always represented by union leaders. Third, his use of right-to-manage bargaining (which is known to be an inefficient protocol) is not helpful in separating efficiency from distribution. The oligopolistic setup provides further hindrance by keeping the bargaining motive inseparable from the strategic motive.

In this paper we aim to address these limitations by studying a monopoly, which is free from strategic interactions, and introducing bilateral delegation in the context of wage and employment bargaining. The bargaining protocol we use is efficient bargaining (McDonald and Solow, 1981). We consider fairly general demand and cost conditions; for example, production technology only need to exhibit non-increasing returns to scale.[3] We resort to special cases such as linear demand and constant returns mainly to provide clearer compar-

[1] Its popularity is felt even outside the oligopoly literature. See for instance Basu *et al* (1997) for development policy, Ray (1999) for tenancy choice and Das (1997) for trade policy.

[2] Zabojnik (1998) considered only a monopoly setup, and assumed that production requires both managerial effort and a specific human capital from worker. A sales-oriented managerial incentive contract helps to overcome the problem of underinvestment in specific human capital.

[3] It is worth noting that most previous contributions (such as Fershtman and Judd (1987) and Szymanski (1994)) considered linear demand and constant returns technology.

ative statics.

Our analysis throws up some surprises. In monopoly, where the sole purpose of delegation is to gain bargaining advantage, shareholders will always orient their manager to profit maximization and the workers will always orient their union leader to net wage bill maximization. Consequently delegation will result in underproduction causing the bargaining pie to shrink.[4] Of course, how much the pie will shrink as well as how it will be distributed depends on the relative bargaining powers of the players. Weakening of one's bargaining power directly reduces one's payoff, but it also induces one to use stronger incentives; thus a countervailing effect is unleashed. If the latter effect dominates, a player can experience a *rise* in his payoff after his bargaining power (exogenously) falls – a surprising possibility from the bargaining point of view.

Assuming linear demand and constant returns technology, we can sharpen some of these points. First, the conventional bargaining power-payoff (positive) relationship holds only under unilateral delegations (when delegation is artificially restricted to one party). Under bilateral delegation, the countervailing force (via delegation incentives) is so strong that the direct effect of loss in bargaining power is overturned. In consequence, the weaker the player, the higher the payoff. Second, bilateral delegation is more inefficient than any unilateral delegations. There are also some situations, where both parties are worse off after delegation. We also see that if the game is extended to endogenize the players' decision to delegate, the outcome is clear cut: both will delegate. Therefore, we may observe sometimes a Prisoners' Dilemma in delegation.

In the following section, we present the model followed up by the analysis and discussion of results. A special case of linear demand and constant returns to scale technology is then studied along with its numerical illustration.

2 The model

We consider a monopoly with labor as the only input, which along with wage is subjected to negotiation.[5] The firm's sales revenue is denoted as $s = p(q)q$, where q is the output, and $p(q)$ is the standard inverse demand curve, $p'(q) < 0, p''(q) \leq 0$. Assuming a concave production function $q = q(l)$, we write $s = s(l)$, $s''(l) < 0$.

The shareholders of the firm hire a manager and offer an incentive scheme z, which is,

[4] In our setup, in the base case of no-delegation employment does not depend on the players' bargaining powers, and thus the bargaining pie remains invariant.

[5] If other inputs are considered, they can be regarded as part of fixed cost for simplicity.

as in Fershtman and Judd (1987) (henceforth in short FJ), a convex combination of sales s and profit π as follows:[6]

$$z = \beta\pi + (1-\beta)s = s(l) - \beta wl. \tag{1}$$

Non-trivial delegation arises if $\beta \neq 1$, and there are two types of delegation that can arise: sales oriented delegation (i.e. $\beta < 1$), and profit oriented delegation (i.e. $\beta > 1$). Shareholders maximize profit $\pi = s - wl$.

The workers' union consists of N identical workers whose reservation wage is θ and its objective function is $u = (w-\theta)l$. At the worker selection stage, l members are randomly hired and the remaining $(N-l)$ members receive the reservation wage from outside. Workers appoint a union leader who is asked to maximize:

$$v = \gamma u(.) + (1-\gamma)wl = wl - \gamma\theta l. \tag{2}$$

Delegation is captured by $\gamma \neq 1$. If $\gamma > 1$ the union leader is oriented to net wage bill maximization as opposed to $\gamma < 1$ when he is oriented to gross wage bill maximization.[7] Effectively, the leader is induced to overvalue ($\gamma > 1$) or undervalue ($\gamma < 1$) the opportunity cost of the union.

The firm's wage and employment are an outcome of bargaining between the firm manager and the union leader. This is a scenario of efficient bargaining between two delegates. The bargaining power of the union leader (and also the union) is exogenously given by α, $0 \leq \alpha \leq 1$, and the manager's (and also the shareholders') bargaining power is $(1-\alpha)$. The reservation payoffs of all parties are zero.

In stage 1 of this simple game the shareholders choose β and simultaneously, the union chooses γ. Then in stage 2 wage and employment (and consequently output) are determined through generalized Nash bargaining. We first consider the stage 2 problem, which is solved

[6] Many authors assume a slightly general incentive scheme: $I = A + bz$. For $A = 0$, the assumption of $b \neq 1$ does not change our analysis. For $A > 0$, some of the quantitative results may change, but the qualitative results continue to hold.

[7] It is conceivable that the union leader might be encouraged to negotiate for higher employment. For example, the leader might be assigned an objective function $v = (w-\theta)l + \gamma l = wl - (\theta - \gamma)l$. Formally, the analysis will resemble the approach we have taken.

by maximizing $B = [v^{\alpha} z^{1-\alpha}]$ with respect to (w, l).[8] The solution yields

$$
\begin{aligned}
s'(l) &= \beta\gamma\theta, && (3)\\
w &= (1-\alpha)\gamma\theta + \alpha\frac{s(l)}{\beta l}. && (4)
\end{aligned}
$$

Eqs. (3) and (4) give employment and wage respectively. In particular, note that the output choice is not directly affected by the bargaining powers, and l maximizes $(s(l) - \beta\gamma\theta)$. Moreover, when $\beta = \gamma = 1$, Eq. (3) yields the efficient level of output and Eq. (4) expresses the wage rate as a weighted average of the marginal revenue product and average revenue productivity of labor.[9]

After substituting (4) and rearranging terms we can write firm's profit and the union's utility respectively in the following way:

$$
\begin{aligned}
\pi &= s(l)\left(1 - \frac{\alpha}{\beta}\right) - (1-\alpha)\theta\gamma l\\
&= (1-\alpha)[s(l) - \theta l] + \alpha \underbrace{s(l)\left[1 - \frac{1}{\beta}\right]}_{\text{Shareholders delegation effect}} - (1-\alpha)\underbrace{\theta l[\gamma - 1]}_{\text{Union delegation effect}}, && (5)
\end{aligned}
$$

$$
\begin{aligned}
u &= \alpha\frac{s(l)}{\beta} - \theta l[1 - (1-\alpha)\gamma]\\
&= \alpha[s(l) - \theta l] - \alpha \underbrace{s(l)\left[1 - \frac{1}{\beta}\right]}_{\text{Shareholders delegation effect}} + (1-\alpha)\underbrace{\theta l[\gamma - 1]}_{\text{Union delegation effect}}. && (6)
\end{aligned}
$$

Eq. (5) shows a decomposition of profit. In the absence of any delegation profit is given by the first term, $(1-\alpha)[s(l) - \theta l]$, which is simply the share of the bargaining pie. The next two terms capture the effects of delegation by shareholders and the union respectively. Similarly, Eq. (6) shows the effects of delegation on union's utility (or net wage bill). The first term is simply the union's share of the surplus, and the next two terms are due to delegations. Several key observations now can be made.

[8]The first order condition of this maximization problem can be written as:

$$
\begin{aligned}
B_w &= [v^{\alpha-1} z^{-\alpha}][\alpha z\frac{\partial v}{\partial w} + (1-\alpha)v\frac{\partial z}{\partial w}] = 0\\
B_l &= [v^{\alpha-1} z^{-\alpha}][\alpha z\frac{\partial v}{\partial l} + (1-\alpha)v\frac{\partial z}{\partial l}] = 0.
\end{aligned}
$$

[9]The output level should be regarded as constrained efficient due to the presence of market power.

1. If delegation is (individually) profitable then the shareholders will set $\beta > 1$ (i.e. profit orientation, rather than sales orientation) and the union will set $\gamma > 1$ (i.e net wage bill orientation). This is evident from the fact that for each party its own delegation effect is non-negative if and only if its incentive term exceeds 1.

2. It is also apparent that shareholders delegation will be profitable only if $\alpha > 0$, and the union delegation will be profitable only if $\alpha < 1$. Further, if the shareholders have all the bargaining power (i.e. $\alpha = 0$) by choosing $\beta \neq 1$ the shareholders will only reduce the surplus (for any given γ), and hence they must choose $\beta = 1$. A similar reasoning helps us argue that if $\alpha = 1$ the union must choose $\gamma = 1$.

3. Delegation ensures a strictly positive payoff for either side even when a given side has the least bargaining power. $\pi(\alpha = 1) > 0$ if $\beta > 1$ and $u(\alpha = 0) > 0$ if $\gamma > 1$. Thus, delegation acts as a substitute for bargaining power.

4. Given $\beta > 1$ and $\gamma > 1$ employment will fall strictly below the no-delegation level (evident from Eq. (3)). Hence total revenue will also fall. Therefore, for delegation to be profitable for the shareholders the wage bill must fall sufficiently. By the same logic, the wage rate (or the net wage bill) must rise with union delegation.

We now characterize the optimal incentive schemes. In stage 1, the shareholders and the union perfectly anticipate l and w and choose β and γ from the following equations[10]

$$\pi'(\beta) = \frac{1}{\beta^2}\left[\beta^2(\beta-1)\gamma\theta\frac{\partial l}{\partial \beta} + \alpha s(.)\right] = 0, \tag{7}$$

$$u'(\gamma) = \theta\left[(\gamma-1)\frac{\partial l}{\partial \gamma} + (1-\alpha)l\right] = 0. \tag{8}$$

We assume $s'''(l) \leq 0$ to ensure $\pi''(\beta) < 0$ and $\frac{\partial \pi'(\beta)}{\partial \gamma} < 0$ (for details see Appendix). It can also be established that $u''(\gamma) < 0$ and $\frac{\partial u'(\gamma)}{\partial \beta} < 0$. Therefore, $\frac{\partial \beta}{\partial \gamma} < 0$ and $\frac{\partial \gamma}{\partial \beta} < 0$. That is, β and γ are strategic substitutes to each other.

Definitions: Let (β^*, γ^*) be the Nash equilibrium incentives satisfying Eqs. (7) and (8) and assume that it is unique and stable. Let β_u and γ_u be the optimal incentives under unilateral delegations. That is, β_u solves Eq. (7) for $\gamma = 1$, and γ_u solves Eq. (8) for $\beta = 1$.

[10] Write $l = l(\beta, \gamma)$ and obtain from (3) $\frac{\partial l}{\partial \beta} = \frac{\theta\gamma}{s''(.)} < 0$, $\frac{\partial l}{\partial \gamma} = \frac{\theta\beta}{s''(.)} < 0$. Then differentiate π to get $\pi'(\beta) = (1-\alpha)[s'(l) - \theta]\frac{\partial l}{\partial \beta} + \alpha s'(l)\frac{\partial l}{\partial \beta}\{1 - \frac{1}{\beta}\} - (1-\alpha)\theta(\gamma-1)\frac{\partial l}{\partial \beta} + \alpha\frac{s(.)}{\beta^2} = 0$. Substituting (3) we will get (7). Eq. (8) can be obtained from $u'(\gamma) = \left[\alpha(s'(l) - \theta) - \alpha(1 - \frac{1}{\beta})s'(l) + (1-\alpha)(\gamma-1)\theta\right]\frac{\partial l}{\partial \gamma} + (1-\alpha)\theta l = 0$.

Proposition 1. (***Delegation for bargaining***) *Suppose $0 < \alpha < 1$ and the Nash equilibrium incentives (β^*, γ^*) are unique. Then $\beta^* > 1, \gamma^* > 1$, and $\beta'(\alpha) > 0$, $\gamma'(\alpha) < 0$. Moreover, at $\alpha = 0$, $\beta^* = 1$, $\gamma^* > 1$, and at $\alpha = 1$, $\gamma^* = 1$, $\beta^* > 1$. Thus, employment will be set below the no-delegation level at all $0 \le \alpha \le 1$. Further, since β and γ are strategic substitutes at any $\alpha \in [0, 1]$ $\beta^* \le \beta_u$ and $\gamma^* \le \gamma_u$.*

Now we turn our attention to the bargaining pie, $P = \pi + u = s(l) - \theta l$. Ff α increases, the pie will expand (contract) only if employment expands (contracts). Since employment responds negatively to incentives, what matters most is the effect of α on incentives.

$$P'(\alpha) \quad = \quad \underbrace{[(\beta^*\gamma^* - 1)\theta]}_{>0} \underbrace{\frac{\partial l^*}{\partial \beta\gamma}}_{<0} \left[\underbrace{\gamma^* \frac{\partial \beta^*}{\partial \alpha}}_{>0} + \underbrace{\beta^* \frac{\partial \gamma^*}{\partial \alpha}}_{<0} \right]. \tag{9}$$

As can be seen, in general the sign of $P'(\alpha)$ is ambiguous. However, if delegation is unilateral, then one of the terms inside the bracket is zero, and the sign of $P'(\alpha)$ is clear-cut. Let P^* be the equilibrium bargaining pie under bilateral delegation, and let P_U and P_F be the same respectively when only the union delegates and when only the shareholders delegate.

Proposition 2. (***Bargaining pie***) *(a) $P_U > (\le) P_F$ if and only if $\gamma_u < (\ge) \beta_u$.*
(b) $P^* \le \min[P_U, P_F]$ if and only if $\beta^*\gamma^* \ge \max[\gamma_u, \beta_u]$.
(c) ${P_U}'(\alpha) > 0$, ${P_F}'(\alpha) < 0$. $P^{*\prime}(\alpha) > (<) 0$ if $\partial \beta^*\gamma^*/\partial\alpha < (>) 0$.

Now we would like to see how the individual payoffs change with the bargaining power of the union. Applying the envelope theorem we obtain

$$\pi'(\alpha) \quad = \quad - \left[\underbrace{\frac{s(l^*)}{l^*} - \beta^*\gamma^*\theta}_{>0} \right] \frac{l^*}{\beta^*} - \theta \left[\underbrace{(1-\alpha)l^* - \gamma^*(\beta^* - 1)\frac{\partial l^*}{\partial \gamma}}_{>0} \right] \underbrace{\frac{\partial \gamma^*}{\partial \alpha}}_{<0}, \tag{10}$$

$$u'(\alpha) \quad = \quad \left[\underbrace{\frac{s(l^*)}{l^*} - \beta^*\gamma^*\theta}_{>0} \right] \frac{l^*}{\beta^*} - \left[\underbrace{\frac{\alpha s(l^*)}{\beta^{*2}} - (\gamma^* - 1)\theta \frac{\partial l^*}{\partial \beta}}_{>0} \right] \underbrace{\frac{\partial \beta^*}{\partial \alpha}}_{>0}. \tag{11}$$

The first term of Eq. (10) (inside the bracket) is positive because of concavity of $s(l)$. The expression inside the bracket of the second term is also positive due to the fact that $\partial l^*/\partial\gamma < 0$. But the overall effect is ambiguous for both π and u, and consequently we have a curious possibility – profit can increase and net wage bill can decrease with an increase in the bargaining power of the union. The ambiguity somewhat diminishes (but does not disappear) if only one side delegates. For example, if only the shareholders delegate, $\pi'(\alpha) < 0$ (because γ is held constant at 1), but $u'(\alpha)$ remains ambiguous. Similarly, if only the workers delegate, $u'(\alpha) > 0$, but $\pi'(\alpha)$ is ambiguous.

Proposition 3. (***Payoffs and the bargaining power***)

(a) If only the shareholders delegate, $\pi'(\alpha) < 0$ at all α, but $u'(\alpha) > 0$ if $\partial\beta_u/\partial\alpha < \left[1 - \frac{s'(l)}{s(l)}\right]\frac{\beta}{\alpha}$.

(b) If only the union delegates, $u'(\alpha) > 0$ at all α, but $\pi'(\alpha) < 0$ if $|\frac{\partial\gamma_u}{\partial\alpha}| < \left[1 - \frac{s'(l)}{s(l)}\right]\frac{s(l)}{(1-\alpha)\theta l}$.

(c) When both delegate, $u'(\alpha)$ and $\pi'(\alpha)$ are ambiguous.

What is clear-cut is that delegation will cause severe inefficiency ruling out mutual gains. But whether an individual side will benefit from delegation *ex post* is unclear, for which we consider an example.

2.1 An example

We assume linear demand and constant returns to scale (CRS) technology. Suppose $p = a - q$ and $q = l$. The stage 2 Nash bargaining employment and wage are:

$$l = \frac{a - \beta\gamma\theta}{2}, \qquad w = \frac{\alpha(a + \beta\gamma\theta)}{2\beta} + (1-\alpha)\gamma\theta.$$

The resultant profit and utility are

$$\begin{aligned} \pi &= \left\{a - \frac{(a-\beta\gamma\theta)}{2} - \alpha\frac{(a+\beta\gamma\theta)}{2\beta} - (1-\alpha)\theta\gamma\right\}\frac{(a-\beta\gamma\theta)}{2} \\ u &= \left[\alpha\frac{(a+\beta\gamma\theta)}{2\beta} + (1-\alpha)\theta\gamma - \theta\right]\frac{(a-\beta\gamma\theta)}{2}. \end{aligned}$$

By maximizing these we obtain the optimal β and γ from the following:

$$-\beta^2\gamma^2\theta^2\left[2(\beta-1)+\alpha\right] + \alpha a^2 = 0, \tag{12}$$

$$(1-\alpha)(a - \beta\theta\gamma) + \beta\theta(1-\gamma) = 0. \tag{13}$$

It is clear that since if $\alpha = 0$, then $\beta^* = 1$, and if $\alpha > 0$, then $\beta^* > 1$ (since $a > \beta^*\gamma^*\theta$ for positive output). Similarly if $\alpha = 1$, $\gamma^* = 1$ and at $\alpha < 1$, $\gamma^* > 1$. In addition, we can make the following remark.

Remark 1. (***Linear demand, CRS technology and bargaining pie***) *Suppose the demand curve is linear and production exhibits constant returns to scale. Then for any* $\alpha \in [0,1]$*, the bargaining pie under bilateral delegation,* P^**, is bounded above by the smallest of the bargaining pies under unilateral delegation. That is* $P^* \leq \min[P_F, P_U]$*.*

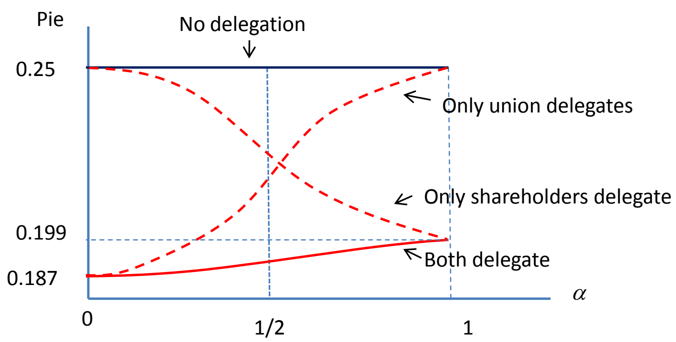

Fig. 1: Bargaining pie

Simulation: We set $a = 2$ and $\theta = 1$ and report the relevant figures in Table 1. Case 0 is the case of no delegation. With $\beta = \gamma = 1$ we obtain the standard results – employment and the bargaining pie $(\pi + u)$ remain invariant to α and $\pi'(\alpha) < 0$.

Case 1 and Case 2 represent the cases of unilateral delegation by shareholders and workers respectively. In both cases employment is lower, and so is the bargaining pie. In Case 1 wage does not grow with α as it does in Case, and consequently profit is much higher. In case 2, both wage and the union's utility are much higher than the no delegation case. In both cases, $u'(\alpha) > 0, \pi'(\alpha) < 0$.

Finally, Case 3 describes bilateral delegation. Here employment is the lowest, and the pie is the smallest though it is increasing in α. A visual illustration of the relationship between the bargaining pie and the union's bargaining power under different scenarios is given in Fig. 1. But most dramatic finding is the reversal of relation between π and α. Here, $\pi'(\alpha) > 0$ and $u'(\alpha) < 0$. The intuition is that with an increase in the union's bargaining power two effects are unleashed. There is a direct effect leading to a loss in profit. But there is also an indirect effect occurring through the reduced union incentive γ which helps to increase profit.[11] In this case, the indirect effect outweighs the direct effect resulting in the reversal of profit and union power relationship.

Optimality of delegation: Since $\pi'(\alpha) > 0$ we see from Table 1 that in equilibrium delegation is profitable for the shareholders (compared to the no delegation case) only after

[11] There is yet another effect occurring through β – adjustment in firm's own incentive term. Since β is optimally adjusted (with any change in α) its first order effect on profit is zero.

α exceeds (approximately) 0.55 and for the union only if α is less than 0.39. Therefore, at all $\alpha \in (0.39, 0.55)$ both parties are worse off after delegation. Moreover, if we examine the parties' decision to delegate or not delegate by comparing the equilibrium payoffs in different scenarios of unilateral or bilateral delegation, we see that at all $\alpha \in (0,1)$ delegation is a dominant strategy for each side. At $\alpha = 0$ delegation will still be the dominant strategy of the union, but shareholders will be indifferent between delegation and no-delegation. At $\alpha = 1$ delegation will be the dominant strategy of the shareholders, but the union will feel indifferent.

Now consider the symmetric case of $\alpha = 0.5$. Here if the shareholders and the workers did not delegate, they would have got 0.125 each; but after delegation they get 0.11 and 0.06 respectively. Both are clearly worse off. Since we know that delegation is their dominant strategy, then we have a Prisoners' Dilemma situation. In fact, the Prisoners' Dilemma occurs over the range of $\alpha \in [0.39, 0.55]$.

Appendix

1. **Proof of Proposition 1.** First we consider the stability conditions for the Nash equilibrium. Consider $\pi''(\beta)$.

   Write $\pi''(\beta) = \frac{1}{\beta^2}\left[\beta^2(\beta-1)\theta\gamma\frac{\partial^2 l}{\partial\beta^2} + [\theta\gamma\beta(3\beta-2)+\alpha s'(l)]\frac{\partial l}{\partial\beta}\right]$. Substituting (3) this can be further reduced to $\frac{\theta\gamma}{\beta^2}\left[\beta(\beta-1)\frac{\partial^2 l}{\partial\beta^2} + [(3\beta-2)+\alpha]\frac{\partial l}{\partial\beta}\right]$. Consider the bracketed term. Since $\beta > 1$ and $\frac{\partial l}{\partial\beta} < 0$, the second term is clearly negative. So we need either $\frac{\partial^2 l}{\partial\beta^2}$ sufficiently small, or non-positive. It is straight-forward to derive $\frac{\partial^2 l}{\partial\beta^2} = -\theta\frac{\partial l}{\partial\beta}\frac{s'''(l)}{(s'')^2}$ which is non-positive if $s'''(l) \leq 0$.

   Next, we show $\frac{\partial\pi'(\beta)}{\partial\gamma} < 0$. $\frac{\partial\pi'(\beta)}{\partial\gamma} = \beta^2(\beta-1)\theta\frac{\partial l}{\partial\beta} + \beta^2(\beta-1)\theta\gamma\left[\frac{\theta}{s''(l)} - \theta\gamma\frac{s'''(l)}{(s''(l))^2}\frac{\partial l}{\partial\gamma}\right]$. Since $s'''(l) \leq 0$, this expression is negative. Hence $\frac{\partial\beta}{\partial\gamma} < 0$.

   Now consider $u''(\gamma)$ and $\frac{\partial u'(\gamma)}{\partial\beta}$. $u''(\gamma) = \theta\left[(\gamma-1)\frac{\partial^2 l}{\partial\gamma^2} + (2-\alpha)\frac{\partial l}{\partial\gamma}\right]$. Following the same procedure as above we can establish that $\frac{\partial^2 l}{\partial\gamma^2} \leq 0$. Hence $u''(\gamma) < 0$. Finally, using the fact $\frac{\partial l}{\partial\gamma} = \frac{\partial l}{\partial\beta}\frac{\beta}{\gamma}$ we write $\frac{\partial u'(\gamma)}{\partial\beta} = \theta\beta\frac{(\gamma-1)}{\gamma}\frac{\partial^2 l}{\partial\beta^2} + \theta\frac{\partial l}{\partial\beta}\left[\frac{(\gamma-1)}{\gamma} + (1-\alpha)\right]$. Since we have already established that $\frac{\partial^2 l}{\partial\beta^2} \leq 0$, it is clear that $\frac{\partial u'(\gamma)}{\partial\beta} < 0$. Therefore, $\frac{\partial\gamma}{\partial\beta} < 0$.

   Further we assume that the stability condition for the Nash equilibrium holds: $\Delta = \pi''(\beta)u''(\gamma) - \frac{\partial\pi'(.)}{\partial\gamma}\frac{\partial u'(.)}{\partial\beta} > 0$. Let $(\beta^*, \gamma*)$ be the unique Nash equilibrium.

Table 1: Simulation results

$a = 2$	$\theta = 1$						
		Case 0:	No delegation	$(\beta = 1$	$\gamma = 1)$		
α	w	l	u	π	$\pi + u$		
1	1.5	0.5	0.25	0	0.25		
0.75	1.375	0.5	0.187	0.063	0.25		
0.55	1.275	0.5	0.137	0.113	0.25		
0.5	1.25	0.5	0.125	0.125	0.25		
0.39	1.195	0.5	0.097	0.153	0.25		
0.25	1.125	0.5	0.063	0.187	0.25		
0	1	0.5	0	0.25	0.25		
			Case 1:	Only shareholders delegate	$(\gamma = 1)$		
α	β	w	l	u	π	$\pi + u$	
1	1.45	1.19	0.275	0.052	0.15	0.199	
0.75	1.395	1.16	0.30	0.049	0.16	0.21	
0.5	1.322	1.13	0.34	0.043	0.18	0.224	
0.25	1.213	1.08	0.39	0.031	0.206	0.24	
0	1	1	0.5	0	0.25	0.25	
			Case 2:	Only union delegates	$(\beta = 1)$		
α	γ	w	l	u	π	$\pi + u$	
1	1	1.5	0.5	0.25	0	0.25	
0.75	1.2	1.5	0.4	0.20	0.04	0.24	
0.5	1.33	1.5	0.335	0.167	0.05	0.22	
0.25	1.42	1.5	0.29	0.145	0.061	0.20	
0	1.5	1.5	0.25	0.125	0.062	0.187	
			Case 3:	Both	delegate		
α	β^*	γ^*	w	l	u	π	$\pi + u$
1	1.45	1	1.19	0.275	0.052	0.15	0.199
0.75	1.327	1.10	1.25	0.27	0.068	0.13	0.196
0.55	1.235	1.192	1.309	0.264	0.082	0.113	0.195
0.5	1.21	1.217	1.33	0.263	0.086	0.11	0.193
0.39	1.15	1.28	1.369	0.264	0.097	0.096	0.193
0.25	1.10	1.349	1.41	0.256	0.105	0.09	0.191
0	1	1.5	1.5	0.25	0.125	0.062	0.187

Differentiating Eqs. (7) and (8) we obtain[12]

$$
\begin{aligned}
\frac{\partial \beta}{\partial \alpha} &= \frac{1}{\Delta}\left[-\frac{\partial \pi'(\beta)}{\partial \alpha}u''(\gamma) + \frac{\partial \pi'(\beta)}{\partial \gamma}\frac{\partial u'(\gamma)}{\partial \alpha}\right] = -\frac{1}{\Delta}\left[\frac{s(l)}{\beta^2}u''(\gamma) + \theta l\frac{\partial \pi'(\beta)}{\partial \gamma}\right] > 0,\\
\frac{\partial \gamma}{\partial \alpha} &= \frac{1}{\Delta}\left[-\frac{\partial u'(\gamma)}{\partial \alpha}\pi''(\beta) + \frac{\partial \pi'(\beta)}{\partial \alpha}\frac{\partial u'(\gamma)}{\partial \beta}\right] = \frac{1}{\Delta}\left[\theta l \pi''(\beta) + \frac{s(l)}{\beta^2}\frac{\partial u'(\gamma)}{\partial \beta}\right] < 0.
\end{aligned}
$$

That $\beta(\alpha = 0) = 1$ and $\gamma(\alpha = 1) = 1$ is obvious from Eq. (7) Eq. (8).

To see $\beta^* \leq \beta_u$ note that β^* is optimal response to $\gamma^* \geq 1$, whereas β_u is an optimal response to $\gamma = 1$. Since β and γ are strategic substitutes, it must be that $\beta^* \leq \beta_u$. Analogous reasoning establishes $\gamma^* \leq \gamma_u$. **Q.E.D.**

2. **Proof of Proposition 2.** Since the proof is straight forward we discuss only the sufficiency part. The necessity part is omitted for economy.

   (a) Let the solution to the equation $s'(l) = \theta$ be written as $l(\theta)$. Given strict concavity of $s(l)$, the bargaining pie $P = s(l) - \theta l$ increases in l at all $l < l(\theta)$, and decreases in l at all $l > l(\theta)$. Now consider the cases of unilateral delegation. When only the firm delegates, employment is given by $s'(l) = \beta_u\theta$, and when only the union delegates employment is given by $s'(l) = \gamma_u\theta$ resulting in bargaining pies P_F and P_U respectively. If $\beta_u > (\leq)\gamma_u$, we have $l(\beta_u\theta) < (\geq)l(\gamma_u\theta)$. Consequently $P_F < (\geq)P_U$.

   (b) In the bilateral case, employment is given by $s'(l) = \beta^*\gamma^*\theta$, and the pie is P^*. If $\beta^*\gamma^* \geq \max[\beta_u, \gamma_u]$, then $l(\beta^*\gamma^*\theta) \leq \min[\beta_u, \gamma_u]$, and hence $P^* \leq \min[P_F, P_U]$.

   (c) Follows from Eq. (9). **Q.E.D.**

3. **Proof of Remark 1.** Suppose $\alpha \in (0, 1)$ and both sides delegate. From Eq. (12) we obtain $\beta^*\gamma^*\theta = a[\alpha/\{2(\beta^* - 1) + \alpha\}]^{1/2}$.

   Contrast this with the case of unilateral delegation by the firm (where $\gamma = 1$ and we write $\beta = \beta_u$): $\beta_u\theta = a[\alpha/\{2(\beta_u - 1) + \alpha\}]^{1/2}$. β_u is the optimal response to $\gamma = 1$, and β^* is the optimal response to $\gamma^* > 1$. Since, β and γ are strategic substitutes, it must be that $\beta_u > \beta^*$. Hence, $\beta^*\gamma^*\theta > \beta_u\theta$.

   Now consider Eq. (13). Rewrite it as $\beta^*\gamma^*\theta = [(1 - \alpha)a + \beta^*\theta]/(2 - \alpha)$. Contrast this with the case of unilateral delegation by the union. Set $\beta = 1$ and $\gamma = \gamma_u$ in the above and it is clearly smaller. Hence, we can say $\beta^*\gamma^*\theta > \gamma_u\theta$. This allows us to conclude that for any $\alpha \in (0, 1)$, $\beta^*\gamma^* > \max[\beta_u, \gamma_u]$ and hence, $P^* < \min[P_U, P_F]$.

[12]Note that $\frac{\partial \pi'(\beta)}{\partial \alpha} = \frac{s(l)}{\beta^2} > 0$ and $\frac{\partial u'(\gamma)}{\partial \alpha} = -\theta l < 0$.

Finally consider $\alpha = 0$ and $\alpha = 1$. At $\alpha = 0$, $\beta^* = \beta_u = 1$ and $\gamma^* = \gamma_u > 1$. Therefore, $P^* = P_U$. Alternatively, when $\alpha = 1$, $\beta_u > 1$ and $\gamma^* = \gamma_u = 1$. Hence, $P^* = P_F$.

Combining all these observations we write, $P^* \leq \min[P_F, P_U]$. **Q.E.D.**

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