A Structural Analysis of the Mining Activities at a South African Vanadium Mine

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Abstract

Natural resource studies have dwindled in recent years with little attention being paid to this industry despite its pivotal role on the world’s financial exchanges and its ability to thrive while the rest of the economy falters. Economics has not developed a model of mining that both captures the economic constraints on mining firms, as well as the geological nature of ore reserves. In this paper I propose a structural model of a firms per period decisions regarding where to mine, how much to produce, and the optimal levels of inventory to maintain. I perform this analysis with the use of detailed firm level data relating to a Vanadium mine in South Africa. This paper is unique in the way that it incorporates both geological and economic uncertainty into the firms decision making process with the use of structural techniques. This paper is the first to develop a framework to consider the inventory behaviour of a firm with endogenous inputs.

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Preface

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My thesis is based on data relating to a Vanadium mine in South Africa over a period of ten years. I have extensive data regarding not only production volumes and costs, but also inventories, and geological exploration. Using these data I aim to analyse the following concepts in my thesis:

1. There is no complete firm-level model of mining. I develop a model of the firm’s per period decisions regarding where to mine, and how much to produce. Through the use of detailed production, cost and geological data I develop of model of how mining occurs. This model is interesting in itself, and also leads to more research opportunities.

2. The upgrade decision of a natural resource firm has not been studied with this level of data. Technological change is generally seen as an exogenous process that is either a smooth continuous process, or a one-off event. I abstract from these assumptions, and determine the factors that drive the firm to change their technology by modelling their monthly decisions.

3. I explore the inventory decisions of a firm with endogenous inputs. This research contributes the inventory literature, as well as the literature on vertically integrated firms. To date, no such study has been done.

4. I lastly include a short chapter on the effect of electricity cost and supply shocks on the firm’s production decisions. In the past three years, South Africa has experienced increased volatility in both electricity costs and supply which provides an interesting background to this analysis.
This paper discusses the structural model of mining that is the base for the rest of my thesis. The importance of this structural model is that it includes both geological and economic factors without placing restrictive assumptions on functional forms and parameter distribution.

The structure of my thesis is as follows:
Chapter 1: Introduction
Chapter 2: Vanadium Industry and Data Discussion
Chapter 3: A Model of Mining and Inventory Decisions
Chapter 4: Technology Upgrade Within a Model of Mining
Chapter 5: Managing Uncertain Electricity Supply in the Production Process
Chapter 6: Conclusion
1 Introduction

Research in natural resource economics is most definitely out of style. Neither the recent large swings in oil and other commodity prices, nor the effect of the recent financial turmoil has encouraged much research into commodity markets. That the mining industry is experiencing a significant boom, while the rest of the world’s economy stagnates, has not tempted economists. The debate in economics regarding exhaustible resources has long been dominated by [Hotelling (1931)]’s paper on the implications of finite reserves for the evolution of commodity prices and consumption under an optimal plan. In the 1970’s this paper defined research in this topic. [Slade and Thille (2009)] have provided a comprehensive review of this literature. At the time, this interest in modelling commodity firms was driven by large swings in commodity prices, especially that of oil. Since this time, interest in resource economics has dwindled.

Throughout this paper I use structural econometric techniques in the vein of [Rust (1987)]’s seminal paper that considers the optimal time for bus replacement. I expand on this framework by considering a firm that has two decision processes — the first relating to production, and the second relating to mining. From this framework, I investigate the monthly decisions of a Vanadium mine in South Africa regarding where to mine, how much to produce, and the volume of inventories to keep on hand.

Structural papers in the resource literature, such as [Young (1992)] as well as [Chermak and Patrick (2001)], have been unable to capture fully the dynamics that exist between mining and processing. The link between mining and production has been poorly explored in the literature. As discussed by [Slade and Thille (2009)], mining is generally modelled as an exogenous input to the production process with no feedback from production back to mining. I believe this is an assumption that needs to be re-examined. The literature on mining models has been mainly theoretical as the availability of data regarding the operation of mines and plants has been a major stumbling block to empirical work. The geological literature has looked at ways to model ore reserves for mine planning, but these models are not considered in the setting of stochastic prices and uncertain demand. Further, these
papers do not consider the firm’s joint decision to mine and operate a production plant. In this paper I develop a structural model of mining that combines both geological and economic principles to develop a complete model of mining.

In this paper I also contribute to the production and inventory literature by considering the firm’s production and inventory decisions based on demand and geological information. I consider the production and inventories of both raw ore and finished output. Inventory studies generally follow the class of models referred to as \((S, s)\) models, first developed by Scarf [1960]. \((S, s)\) models determine two thresholds: the upper threshold \(S\) which indicates the optimal level of inventory holding for a firm, and the lower threshold \(s\) which indicates the inventory level at which the firm will place an order. \((S, s)\) model has been successfully used in macro papers, for example, Caplin [1985], Fisher and Hornstein [2000], as well as Kahn and Thomas [2007]. The \((S, s)\) models have also been used in micro studies such as the influential research of Aguirregabiria [1999] regarding inventory and markup decisions in retail firms. Hall and Rust [2000] used a \((S, s)\) model with stochastically evolving bounds to study price speculation by a commodity intermediary.

In this paper I make two contributions to the inventories literature. Firstly, I develop a model of the inventory behaviour of a resource firm with endogenous inputs. No such model has been developed in the literature. The results from this analysis can be applied beyond the mining industry to any vertically integrated firm. Secondly, I show that one reason for the poor performance of production smoothing models with stock-out avoidance is insufficient data regarding inventory holdings, the full set of cost shocks faced by firms, and the true nature of demand. In the case of a mining and production firm, I illustrate that the production smoothing model is capable of fully explaining the volatility observed, and the affects that various components of the model have on inventory holdings. Of particular interest is the role played by the geological constraints of the property on the firm’s decisions. I shall look at not only inventory of the finished product, but also inventories of the raw material.

Because of data limitations, most researchers in economics who have studied firm be-
haviour have focused on industry-level data as opposed to firm-level data. Research at the industry-level is important to understanding the effects of policy changes, or understanding how firms interact in a competitive market. Research at the firm-level is important to understanding the within-firm processes that drive a firm’s responses to demand and supply shocks. In the following sections, I illustrate how access to a detailed data set enables me to gain a deeper understanding of the dynamics of the firm’s inventory and production decisions. I am also able to perform meaningful counterfactual exercise to understand how the firm responds to production, and cost shocks.

2 Global Vanadium Industry

Vanadium (\(^{23}\text{V}\)) is a silver-grey, soft and ductile metal. It is not found in Nature by itself, but is found as a trace element in a number of different rock forms. Vanadium most commonly occurs along with deposits of titaniferous magnetite, uraniferous sandstone, and siltstone and phosphate rock; of which titaniferous magnetite is the most common deposit. Countries such as China, South Africa, and Russia are the largest producers of Vanadium and its oxide derivatives; together, they currently account for approximately 90 percent of world supply. South Africa is the second largest producer after China, accounting for 32 percent of world supply. To put this in context, Saudi Arabia accounts for five percent of the world’s oil supply.

Because of its physical properties (which include strength and hardness, as well as resistance to fatigue) \(^{23}\text{V}\) is used almost exclusively in the production of both ferrous and non-ferrous alloys. Vanadium is used mainly as a hardening additive to iron and steel products which accounts for approximately 87 percent of its use. Other uses include conductors in magnets, and catalysts in the manufacturing process of sulphuric acid, as well as in the production of glass coatings to block infrared radiation. Apart from its strengthening properties, \(^{23}\text{V}\) also resists corrosion and combats oxidation which makes it a popular alloy. When combined with Aluminium (Al), it can be used in jet engines and
high speed airframes.

Ferrovanadium (Vanadium alloyed with Iron FeV) is the most common alloy and is used in the production of carbon steel, high-strength-low-alloy steel (HSLA), full alloy steel, and tool steel. The alloy’s uses include armour plating for military vehicles, and car engine parts and pistons. FeV is also used in the construction of frames for high-rise buildings and oil drilling platforms.

A potential innovation in the Vanadium industry is the use of Vanadium in green technology. Recently, there has been much research into the use of Vanadium in lithium-vanadium batteries as a replacement for the traditional lithium-cobalt batteries for use in electric cars. Lithium-vanadium batteries are safer and up to six times more efficient at producing power than lithium-cobalt batteries. Most of this research is done in China and the US with large international backers such as Warren Buffet’s Berkshire Hathaway. The battery was recently tested by the automotive industry in Subaru’s G4e electric car.

A further technological innovation is Vanadium Redox Batteries (VRB) that are designed to store large amounts of energy in a safe manner, which can be adjusted to meet variable energy loads. VRB technology was first implemented at the University of New South Wales, Australia, by Skyllas-Kazacos et al. [1986]. This technology can be used in wind and solar plants to help generators cope with large surges in demand. These batteries have a 35–50 year life, and have low maintenance costs which makes them the cheapest option on a kilowatt hour basis. Currently, these batteries are uncommon in commercial applications, but several companies are developing prototype batteries for use at renewable energy plants. More information on these batteries are available in sources such as Skyllas-Kazacos et al. [1991] and Shibata [1991].

Currently-known world reserves of Vanadium are expected to supply Mankind’s needs for about one hundred years. The demand for and supply of Vanadium products over the past twenty to thirty years has been relatively stable. Demand is expected to remain robust due to construction projects in BRICS countries, with the potential to increase if Vanadium’s green potential is realised. Moskalyk and Alfantazi [2003] have written an
in-depth review of the vanadium industry.

3 Data

The data for this article are from a Vanadium mine in South Africa which is currently owned by a large international mining company. I shall refer to this mine as “the firm” throughout. This mine plays an important role in the global vanadium market, accounting for perhaps ten percent of world supply. Globally, only three other mines rival this mine in terms of production capacity.

Opencast mining commenced at the firm in 1989 at which time a concentrate plant was also constructed. Concentrated Vanadium was transported to an off-site processing plant where it was then turned into Vanadium Pentoxide (\(V_2O_5\)). In 1994, the property changed ownership after which the processing plant was constructed so as to turn concentrated Vanadium into \(V_2O_5\) on site. In 2002, the plant was upgraded to further produce FeV which trades at a significantly higher price than \(V_2O_5\). Since 2002, no major changes have occurred at the plant except for small technological upgrades.

The firm’s main activities are the mining and processing of Vanadium into \(V_2O_5\) and FeV. Processing requirements generally dictate how the remainder of operations are organised. Over the life of the mine, six pits have been sunk. At any point in time, no more than two pits are typically active. Once the ore has been mined, it is transported to a crushing facility located on the property.

After the ore has been crushed, it is then transported to a processing plant. At this plant, a sequence of twenty-six different processes convert the crushed ore to the final output. For example, the ore is first reduced using powdered coal in a kiln, after which it is then reduced to a metallic state in an electric furnace. Next, aluminium sulphate and sulphuric acid are added to purify the ore. A roast-leach process is used to produce \(V_2O_5\). The slag from this production process can then be used to produce FeV through further
heating and cooling and by adding aluminium and iron. Moskalyk and Alfantazi (2003) have written a detailed description of the various methods that can be used to produce $V_2O_5$ and FeV. From an economist’s perspective, it is useful to imagine a sequence of Leontief technologies exists where the coefficients depend on ore grades.

The unique and detailed nature of the data defines the structure of my research, and provides an opportunity to explore the dynamics of firm decision-making. I have three types of data: 1) data relating to monthly production and inventory volumes as well as mine output; 2) data relating to average selling prices and volumes, and detailed cost data; and 3) data concerning geological exploration and mineral assays.

3.1 Prices, Production, and Inventories

In this section, I describe the selling prices for FeV as well as the production and inventory decisions for this good. Selling prices in the context of this firm can be seen as predetermined. FeV is sold through contracts that are negotiated by the firm’s majority shareholder. The firm has no influence over the selling prices, and is informed at the start of each month as to the negotiated contract volumes and prices. Vanadium prices are not traded on an exchange, and all Vanadium contracts are organised through contracts between producers and buyers. Prices can vary widely between different producers depending on the ore quality. Market consultants publish monthly estimates of “market” prices which are available from databases such as Bloomberg and Reuters. Contract prices can significantly differ from these “market” prices. In figure 1 I plot the inflation-adjusted average monthly selling price for Rhovan’s contracts that reach maturity for FeV during the period June 2000 to March 2011. The figure further illustrates the time series of production and inventories during the same period.

The price series exhibits several interesting features, especially two distinct peaks, and a trough. The first peak in prices is the period mid-2004 to early-2005, which resulted from the shut-down of two Vanadium mines, and the reduction in production at another
Figure 1: FeV Production and Inventories as well as Average Selling Prices

mine while the demand for Vanadium stayed constant. The second peak in prices is the period mid-2007 to mid-2008. This spike resulted from increased demand for Vanadium in the steel and aerospace industries due to increased economic activity leading up to the global financial crisis (GFC). Further contributing factors were power supply interruptions in South Africa, and bad weather in China causing decreased production. After this period, prices fell sharply due to the GFC and the resulting reduction in the demand of Vanadium from the steel industry.

Unfortunately, I do not observe the detail of the firm’s contracts, or the demand for its product. I only observe sales volumes for FeV. Sales have remained relatively constant
throughout the period, despite large price fluctuations. The firm never experienced zero sales in FeV, even during the shutdown period in 2009, illustrating the robust demand for FeV.

I observe two shutdown periods in 2009. The first period in January 2009 relates to a technology upgrade at the plant that allowed lower ore grades to be accepted into the production process. The second period resulted from the GFC. The shutdown was largely driven by demand factors, such as defaults on current contracts, and the inability to negotiate new contacts. In November 2008, there is a sharp dip in production due to rolling blackouts throughout South Africa which were caused by maintenance problems at several major power plants. Production in figure [1] follows a cyclical pattern, with periods of increased production followed by periods of declining production that are generally related to routine maintenance at the plant. The production data exhibit a clear capacity constraint.

From figure [1] I observe that production and inventory volumes vary as prices change. Inventories and production appear to have a strong link to selling prices. The high level of FeV inventories leading up to the price spike in 2005 was a common feature in the Vanadium industry at that time. Inventory volumes fell sharply during the price spike, and have again increased during the period of low prices precipitated by the GFC (but not to the extent that was previously observed). The movements in the FeV inventories follow trends observed throughout the industry.

Based on these data, I believe that in the context of my problem, the production smoothing model with stockout avoidance is the appropriate model to use to model the firm’s inventory behaviour. My belief is based on the fact that the firm told me they aim for constant production levels in order to keep their costs low and prolong the lifetime of the plant. The firm is often unable to reach its production smoothing goal for a variety of reasons: I shall show through my model of how geological uncertainty, price fluctuations,

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1 By keeping production constant, adjustments do not have to be made to the plant on a monthly basis that might result in structural damage to the plant, and increased production costs.
and transportation costs influence their ability to meet their production targets.

### 3.2 Production Costs

I have extensive cost data for the firm. The cost data include mining and milling costs, ore transportation costs, processing costs, labour and administrative costs, as well as taxes and royalties. Of these costs, the processing plant is by far the costliest part of production, followed by the mine operating costs. It is the cost of kiln feed, and electricity that make the processing plant the most expensive part of the operation.

Figure 2: An Approximation of the Marginal Costs over the Sample Period for FeV

![Marginal Cost Approximation](image)

![FeV Production Volume](image)
In figure 2, I plot a crude approximation of the firm’s monthly marginal costs. For a large part of the sample, marginal cost is roughly constant, regardless of the amount produced each month, but there are distinct spikes throughout the sample. One of the main causes of these spikes is the disruptions to electricity supply from late 2008, and the associated sharp increases in electricity costs. The increases in marginal costs were coupled with decreases in the overall production level. Since the beginning of 2010, marginal costs and production volumes have stabilised.

3.3 Mining Data

3.3.1 Geology

In economics, researchers who have examined the decisions of mining firms typically have not included geological information, presumably because such data are unavailable, but also because of the complexity involved in modelling geological uncertainty. In papers concerned with mine production and productivity, the most common assumption is that the mine has a fixed quality of ore reserves for a pre-determined lifespan. In such a setting, there is no way to see what the influence of ore quality and accessibility has on the production process.

Six pits have been active at various times during the firm’s lifetime, with no more than two pits being active at once. I have exploration data for three of these pits. The three pits that I do not have data for where either mined by the previous owners, or were mined before the firm started to keep detailed records (approximately 2004).

Table 3.3.1 summarises the number of blast hole locations per pit for which I have co-ordinates. In total there are 3,574 locations of which most are in pit 6. For some holes there is missing data, so in total I have data for 2,916 locations. In some cases, the missing data relates to blast holes that were planned, but not executed, and in other cases, the data

2I calculate the marginal costs as ∆(Total Costs) / ∆(Production Volume).
was not fully recorded. It is not possible to differentiate between the two cases.

<table>
<thead>
<tr>
<th></th>
<th>No of Hole Locations</th>
<th>No of Locations with Assay Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pit 1</td>
<td>774</td>
<td>661</td>
</tr>
<tr>
<td>Pit 2</td>
<td>66</td>
<td>0</td>
</tr>
<tr>
<td>Pit 5</td>
<td>226</td>
<td>127</td>
</tr>
<tr>
<td>Pit 6</td>
<td>2508</td>
<td>2128</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>3574</strong></td>
<td><strong>2916</strong></td>
</tr>
</tbody>
</table>

Each blast hole sample is approximately ten centimetres in diameter and 100 – 120 metres long. The sample is divided in half: one half is stored as a reference, and the other half is crushed and sent to an onsite laboratory. At the laboratory, the sample is analysed and its mineral content is recorded. I have approximately 30,000 sample values across the property regarding the Vanadium content at each observation. For each observation I further have the x-, y-, and z-co-ordinate, as well as the drill hole identification. From this data, it is possible to get a sense of the distribution, and quality of ore in the sampled areas.

Before 2005, there are four distinct spikes in this ratio; the largest of these spikes is in mid-2002 with ore outstripping waste 150:1. During the first few years of the mine’s life, ore was easily accessible, but since 2005, it has been more costly to access, as it requires deeper pits, and more waste extraction.

### 3.3.2 Mining Production and Locations

In figure 3, I show the tons of ore mined each month at the Vanadium mine. The tons of ore mined have the same general pattern exhibited in production, but seems to have greater volatility. The mine only shutdown twice, both times during 2009. There is a small inventory of ore, as each month the tons of ore mined, and the tons milled differ by 1,000 – 30,000 tons. The firm is only able to keep a small inventory of ore, as it is too costly to
keep a larger inventory.

Figure 3: Tons of Ore Mined

In figure 4, I depict the historical ore-to-waste ratio for mining activities for the period November 2005 – March 2011. Ore is defined as any material extracted that can be used for the production of FeV. Any other material extracted that is unsuitable for production is defined as waste. The ore-to-waste ratio is a good indicator of ore accessibility. Despite the large fluctuations in this ratio in the first few years of the firm’s life, it is more common for the ore-to-waste ratio to be below a ratio of 1:1. The firm has a target ore-to-waste ratio of 1:2.2, which is represented in figure 4 by the dashed line. The ratio does not continuously remain under the solid line for longer than three months, except in the period

3The ratio is officially 1:2, but from analysing their monthly mining reports, I find that they are uncon-
2008 – mid-2009. This period corresponds to a time of severe cost shocks from electricity prices, and decreased demand due to the GFC. From the graph, it appears that there are times when it is optimal for the firm to continue mining sub-optimal areas in order to keep costs low.\footnote{This is re-enforced by statements made in their monthly mining reports, and trends observed throughout the industry.}

Figure 4: Changes in Mining Locations from November 2005 – March 2011

From the firm’s monthly mining reports, I have data regarding where the firm mined each month.\footnote{I do not have any information concerning the distance of the pits from the plant. I only have transportation costs for the last fifteen months of my sample.} In figure 4, I further show the monthly changes in mining location. The location movements in the figure are: 2 = move to a new pit; 1 = move within pit; 0 = no movement by a ratio of 1:2.2, and only comment on ratio’s below this level. I shall use this level to study their movements.
change in location. Most of their movements are within pits, with two big movements to
new pits in the sample period. I only have these data from the end of 2005 which limits
the number of new pits I observe.

The graph clearly shows that after a few periods of observing ore-to-waste ratios below
its target level, the firm either makes a move within the pit, or to another pit. There are
times when the firm moves within pits even though they observes a high ore-to-waste ratio,
and there are times when they stay in the same location despite observing low ratios. I shall
examine the factors that drive these decisions that seem to contradict “optimal” behaviour
by combining mining and production decisions.

4 Within-Pit Model of Mining and Production

In this paper, I develop a structural model of the firm’s mining activities which mimics
their monthly mining and production decisions. In order to make the model tractable, and
keep it interesting, I shall split the decisions regarding mining and production and allow
for feedback between the two models so as to fully capture the dynamics of the decisions
made at the plant. In the vein of [Seiler 2010], I split the firm’s monthly decision into two
consecutive decisions: the mining production decision, and the plant production decision.
In the model time, is discrete, \( t = 1, 2, \ldots \), and the firm discounts the future at \( \beta = 0.98 \).
Initially, I only consider the firm’s within-pit mining decisions.

4.1 Mining Production Stage

4.1.1 Mining Exploration

In the first stage of production, the firm chooses the mining location. Each pit is divided
into \( i = 1, 2, \ldots, n \) equally-sized locations. The left-hand side of figure [5] illustrates how
the pit is divided into \( n \) equally-sized locations. Each location is discretised in accordance
with the the firm’s exploration data as described previously. This implies that the depth of each location \(d_{i,k}\) is considered up to 100 metres, and is discretised at every ten metre interval so that \(k = 1, \ldots, 10\). This discretisation is illustrated on the right hand side of figure 5. Based on the analysis results, the firm knows the expected average ore-to-waste ratio for each section of the sample in each location, \(\varrho(d_{i,k})\). This information is known to the firm when it makes its monthly mine and plant production decisions.

Figure 5: Illustration of Within-Pit Exploration

At each point in time, the firm can choose whether to continue mining in its current location, or to move to the next best location. The decision to move to a new location is denoted by the switching function \(I = \{0, 1\}\), where 0 indicates no movement, and 1 indicates a movement to the next location. Once the firm has left a specific location, that location goes back into the pool of potential locations. Therefore, it is possible to return to that location at a later stage; this is consistent with what I observe in the data.

The detailed exploration performed in each pit prior to mining allows the firm to form expectations regarding the ore grade in different locations. The firm can therefore form an expectation regarding the ore-to-waste ratio that can be obtained from continuing mining
in its current location, and from moving to the next location. The firm bases this expectation on an ergodic distribution of the ore across the property. To determine this ergodic distribution I turn to the geological literature and a paper by Carle and Fogg [1997]. This paper builds on Besag [1974]'s seminal work the use of Markov Random Fields in spatial analysis. Carle and Fogg [1997] use one-dimensional Markov chain models of spatial variability to determine transition probabilities between states in geological processes.\(^6\) This model is based on the assumption that a local occurrence of a state depends entirely on the nearest occurrence of another (or the same) state. I use the methodology set out in Carle and Fogg [1997] for calculating the transition probabilities between states. I divide the ore-to-waste ratio into four states that reflect the quality of the ore: no ore; low grade ore; medium grade ore; and high grade ore.

One dimensional Markov chains are dependent on distance between observations, as well as on direction (i.e. vertical, horizontal, or diagonal). As in any geological study, I have abundant data in the vertical direction from blast hole samples. Spatial Markov chains based on this method will perform well, and I can accurately model the ergodic distribution of movements within the same location. My data is less abundant in the horizontal direction. Carle and Fogg [1997] provide a framework to combine two or three one-dimensional Markov chains into a two- or three-dimensional Markov chain. This is the framework that I shall use to determine the ergodic distribution across locations.\(^7\)

4.1.2 Value of Mining

In any period, the firm can only mine in one location. A period is defined as the length of time required to mine 10 metres in one location. Once the firm has chosen the mining location, the total volume of ore and waste extracted in time period \(t\) is \(v_t\). The ore-to-

\(^6\)There are several alternative methods in the geological literature that are commonly used in modelling spatial distributions. I have chosen this specific method as it is the only one that is based on Markovian processes and is therefore suitable for the model under consideration.

\(^7\)This section is still under development. Please contact me at cmarais@student.unimelb.edu.au with any questions based on this or any other sections.
waste ratio in each location at every depth depends on the mining location and the ergodic distribution. The volume of ore $\kappa_t$ extracted every period is given by

$$
\kappa_t = \begin{cases} 
\nu_t \vartheta_t(d_{i,k}) & \text{if } I = 0 \text{ and } k = k + 1 \\
\nu_t \vartheta_t(d_{i',k}) & \text{if } I = 1 \text{ and } k = 1 
\end{cases}
$$

where $i'$ is the next best location. The values of $\vartheta_t(d_{i,k})$ and $\vartheta_t(d_{i',k})$ are simulated from the ergodic distribution.

Increases in the total volume mined in a specific location, have cost implications. For every depth, there is an associated per unit cost $\delta_k$ where $\delta_10 > \delta_9 > \ldots > \delta_1$. Mining costs follow a step-function so that in order to extract $\nu_t$, the firm incurs the following costs,

$$
C_M(\nu_t) = \begin{cases} 
\delta_k \nu_t & \text{if } I = 0 \text{ and } k = k + 1 \\
\delta_1 \nu_t & \text{if } I = 1 \text{ and } k = 1 
\end{cases}
$$

Costs increase as the mining depth increases due to the greater complexity of mining at greater depths. This increase in extraction costs as $V_{i,t}$ increases, forms an incentive for the firm to move to a new mining location even if the total stock of ore has not been extracted from the current location. Further, there is a one-off fixed cost to moving within the the pit to the next location, $F_M$. This cost represents the movement of equipment, and any additional costs related to moving between locations. I assume the cost parameters are constant throughout the pit.

Once $\nu_t$ has been extracted and the average ore-to-waste ratio is observed, the ore is separated from the waste so that the amount of ore mined is given by equation (1). Waste is dumped next to the pit and the ore is transported back to the processing plant. I assume transportation costs are determined in proportion to the distance the ore travels as,

$$
\tau_t = \tau \kappa_t
$$

where $\tau_t$ is the total transportation cost at time $t$, and $\tau$ is the constant, pit-specific cost of transporting one ton of ore to the plant. $\tau$ represents the costs of operating vehicles.

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8 I have transportation cost data for the last 18 months, and this is not an unrealistic assumption. It is only large swings in the price of diesel that would really challenge this assumption.
to transport the ore back to the plant which includes costs related to diesel, labour, and maintenance.

I can now state the firm’s single-period payoff function from the mining decision $u_{M,t}$ for movements within a pit as

$$u_{M,t} = -\tau_t - C_M(\nu_t) - F_M(I = 1).$$

(4)

### 4.2 Plant Production Stage

In the second stage of the model, the firm decides on the quantity to produce in each period $y_t$ based on the realisations from the mining production stage, demand, and the firm’s expectations regarding future mining production volumes.

#### 4.2.1 FeV Demand

After the mining volumes and the ore-to-waste ratio have been realised, the firm observes the quantity of FeV demanded in the current period. I assume that the quantity of FeV demanded $D_t$ depends only on the selling price of FeV, which results in a stochastic form of downward-sloping demand. The demand function follows the basic form

$$D_t = \alpha - \beta p_t,$$

(5)

where the parameters $\alpha$ and $\beta$ are constant over time.

I further assume that the selling price of FeV $\{p_t\}$ evolves according to an exogenous AR(1) Markov process

$$\log p_t = \rho_0 + \rho_1 \log p_{t-1} + \sigma \varepsilon_t,$$

(6)

where $\varepsilon_t$ is an independent and identically distributed Gaussian error term with mean zero and variance one. I further impose a reflecting lower bound of 20 USD per kilogram and an upper bound of 160 USD per kilogram. These reflecting bounds are necessary
in order to prevent the AR(1) process from generating unrealistic values. The bounds were chosen to be twenty per cent above and below any the highest and lowest amounts observed, respectively. I believe the bounds represent a realistic reflection of future prices. As described previously, the firm has no control over selling prices even though they are a large player in the market for FeV they have no direct control over setting their prices which is done by the firm’s holding company.

4.2.2 Plant Production Process

I assume plant production follows a simple production process where the amount of FeV produced is fully determined by ore volumes. Production in period \( t \) is given by

\[ y_t = a \kappa_{i,t}, \quad (7) \]

where \( a \) describes production. This production process can be viewed as a Leontief production function, with \( a \) being constant over time. There is a cap \( \bar{y} \) on the amount of FeV that can be produced in any period.

Production costs are described by

\[ C_p(y_t) = c_{P,1}y_t, \quad (8) \]

where \( c_{P,1} \) is the constant marginal cost of production. This form is chosen for the cost function as opposed to the more traditional quadratic cost function so as to not impose a functional form on the data. FeV is stored in a secured warehouse, and its storage is captured by the marginal cost component. FeV does not naturally deteriorate over time, and theft is not common. There is a large cost to shutdown both the plant and the mine, but as I do not consider the firm’s decision whether to operate or not, these costs are not explicitly included in the model.

The firm’s single-period payoff function of the plant’s production \( u_{P,t} \) is given by

\[ u_{P,t} = p_t D_t - C_F(y_t). \quad (9) \]
Once the firm decides how much to produce \( y_t^* \), the required amount of ore is delivered to the plant based on the inverse of the production function,

\[
\kappa_t^* = \frac{y_t^*}{a}.
\] (10)

When \( \kappa_t^* \) arrives at the plant, the firm realises its current period production of FeV and costs. After FeV is produced, and the current period demand \( D_t \) is observed, the inventory level of FeV for the following period is determined by the transition equation

\[
X_{t+1} = X_t + y_t - D_t.
\] (11)

I assume the firm has a maximum storage capacity of \( \bar{X} < \infty \). In order to complete the picture of the firm’s demand process, I make two additional assumptions; first, I assume that the firm must meet the full demand in the current period, and cannot withhold goods in the expectation of future price increases; second, I assume that if demand in the current period is greater than available inventories, the firm will satisfy demand up until the total amount of inventories, i.e. sales is \( \min(X_t, D_t) \). The firm is not able to rollover this excess demand to the next period.

### 4.3 Value Functions

The firm chooses a finite sequence of decision rules \( \theta_t = \{(\nu_t), (y_t|\nu_t)\} \) to maximise

\[
\max_{\theta_t} \mathbb{E}_{\theta_t} \left\{ \sum_{t=0}^{\infty} \beta^t g_t(\theta_t)|V_t, \phi_t, p_t, X_t \right\},
\] (12)

\( g_t(\theta_t) \) represents the two consecutive decisions regarding mining and plant production that the firm has to make in each time period,

\[
g_t(\theta_t) = u_{M,t}(\nu_t) + u_{P,t}(y_t), \quad 0 < \nu_{t,t} \leq \bar{\nu} \text{ and } 0 < y_t \leq \bar{y}.
\] (13)

I use two choice-specific value functions to describe the firm’s consecutive decisions. The choice-specific value function of mining \( V_M(I; \varrho_t(d_{t,k}), p_t, X_t) \) is

\[
V_M(I; \varrho_t(d_{t,k}), p_t, X_t) = \max_{0 < \nu_{t,t} \leq \bar{\nu}} \left[ u_{M,t}(\nu_t) + \mathbb{E}_{\nu_t} V_P(y_t|\nu_t, p_{t-1}, X_{t-1}, \varrho_{t-1}(d_{t,k})) \right].
\] (14)
The firm chooses the mining location based on expectations regarding the plant’s production, and demand in the current period. Notice here that $\beta$ is not included in equation (14) as the plant production decision is made in the same time period as the mining decision. The expectation in equation (14) can be expanded as

$$
E_y V_P(y_t|p_{t-1}, X_{t-1}, V_{i,t}) = \int V_P M_P(p_t|p_{t-1}) M_X(X_t|p_{t-1}, X_{t-1}, \varrho_t(d_{i,k})).
$$

(15)

$M_p$ is the exogenous transition probability of prices determined by the AR(1) Markov process in equation (6). $M_X$ is the endogenous transition probability on inventories implied by the model.

When picking the mining location $I$, the firm chooses the location that minimises mining costs while maximising the expected value of production in the current period. The location decision tradeoff comes from the fact that as the ore-to-waste ratio deteriorates, a larger volume, i.e. a greater depth, must be mined in order to meet expected demand. Mining costs increase as the volume extracted in a particular location increases. The firm trades-off the increase in cost and the ore-to-waste ratios in the current location, against the cost of switching cost and the next location’s ore-to-waste ratios. The mine aims to meet the expected demand from the plant, so there is an optimal location which will maximise equation (14).

The choice-specific value function of production $V_P(y_t; p_t, X_t, \varrho_t(d_{i,k}))$ is

$$
V_P(y_t; p_t, X_t, \varrho_t(d_{i,k})) = \max_{0<y_t\leq \bar{y}} \left[ u_P(y_t) + \beta E_I V_M(I|p_t, X_t, \varrho_t(d_{i,k})) \right],
$$

(16)

where

$$
E_I V_M(I|p_t, X_t, \varrho_t(d_{i,k})) = \int M_\varrho(\varrho_{t+1}(d_{i,k})|\varrho_t(d_{i,k})) V_M(I|p_t, X_t).
$$

(17)

$M_\varrho$ is the transition probability implied by the ergodic distribution of the ore body. $M_I$ is the endogenous transition in location implied by the model. The firm’s plant production decision depends on its expectations regarding current period demand, as well as expectations regarding mining activities, and mining costs in the following period.

The firm considers three factors when making the production decision. First, the firm must ensure it can meet the current period demand either from inventories, or through
further production. Second, the firm wishes to maximise its current period profit by keeping production costs as low as possible. Production costs increase exponentially with the quantity produced so that it is in the firm’s interest to smooth production and avoid large, costly production decisions. This implies that there is an implicit optimal level at which the plant should be run. Third, the firm must consider its inventory levels. The firm holds inventories to avoid the costly production decisions just mentioned. Further, the firm holds inventories to account for the fact that the desired volume of ore is not necessarily available in each period. In every period, the mine has a decision between two locations, and is constrained by the ore-to-waste ratios in each of these locations.

5 Conclusion

In this paper I have considered the decisions of a mining firm regarding where to mine, how much to produce, and the volume of inventories to hold. I have developed a structural framework for integrating both the mining and production decisions made at resource companies without imposing restrictive assumptions on the dynamics of the model. This model is unique in the way it uses both geological and economic principles to address the firm’s decision making problem. Features of this model are still under development, so results are still pending. Once I am able to model the firm’s within-pit mining decision, I shall move on to their decision to move between pits. The ultimate goal of this research is to develop a complete model of mining which can be used in many applications. In my thesis I shall specifically look at how this model can predict the firm’s decision to upgrade its current plant by moving it to a different location on the property to make more areas of the property economically viable.
References


