Abstract

This paper studies the relationship between income distribution and international trade in the canonical trade setting with one change. Under the standard approach prices are a function of (constant) marginal costs and (constant) elasticities alone, implying that information on consumer income is of no value to a typical firm. To address this limitation the strategy space is expanded to include non-linear prices (i.e. potential to offer product lines). In equilibrium firms use information on the distribution of income to design a product for each income class, with associated prices that induce each group to optimally select their intended product. To achieve this outcome, some of these products are degraded relative to the first best while others exceed it. When countries with differing income distributions integrate, this has implications for the size of these distortions, influencing the gains from trade both within and across countries. The structure of trade and prices which emerge match a range of empirical patterns. The model also has novel implications for the speed of trade liberalization, industrial structure and factor prices. All these results are driven by firm strategy based on income difference alone as preferences are assumed to be identical and homothetic across countries, placing the distribution of income at the center of the analysis.

Key Words: Intra-industry trade, monopolistic competition
JEL Classifications: F12
1 Introduction

Models of international trade have traditionally used richness/heterogeneity on the supply side to gain insight into why countries trade and the likely implications. Any heterogeneity on the demand side is usually suppressed by positing that consumers have identical and homothetic preferences. While analytically convenient, this assumption implies that models of international trade ignore one of the most prominent differences across individuals, regions and countries: income and expenditure patterns. How to incorporate this important aspect of variation and analyze its implications represents a persistent challenge to the literature.

To confront this issue the typical approach relaxes the assumption of homotheticity, freeing up expenditure shares to depend not just on relative prices but also income levels.\(^1\) In essence this assumes that individuals with different incomes are hardwired to make different choices. The problem is then reduced to choosing an appropriate preference specification. In focusing on the preference structure the literature has overlooked an alternative possibility – firms may also be interested in income variation across consumers and try to exploit this variation to increase profits. The set of techniques a firm can employ is relatively rich but can be broadly summarized as a form of discrimination – charging different prices, offering different qualities and/or selling different sizes. Whether or not preferences are homothetic, the key feature under this approach is that individuals with different incomes are induced to make different choices. It is then entirely possible for low and high income consumers to face the same offerings from a firm but end up choosing differently. This choice can lead to natural variation in expenditure patterns, even among consumers with homothetic preferences. Moreover, discrimination generally does have implications for welfare outcomes. The open question is whether international trade tends to enhance the positive aspects of discrimination or magnify the negative ones.

The objective of this paper is to answer this question and explore the implications of income differences both within and across countries for international trade. In contrast to the non-homothetic literature, preferences will have the standard features of being identical

\(^1\)Caron et al. (2014), Markusen (2013), Fajgelbaum et al. (2011), Hallak (2010), Mitra and Trindade (2005), Simonovska (2010), Choi et al. (2009) and Fieler (2011) to list only some of the recent contributions.
and homothetic for all consumers. This is done solely to distinguish the analysis from the previous literature which starts by altering preferences.\textsuperscript{2} To further highlight the differences and simplify the analysis, the single sector structure of Krugman (1980) is adopted.\textsuperscript{3} The key insight that differentiates this paper from the previous literature is a focus on how a firm views and evaluates information relating to the distribution of income in the population. In the standard analysis the strategy space of a firm is restricted to linear prices which implies they are only interested in the curvature of the residual demand function when formulating their optimal strategies. Moreover, with Spence-Dixit-Stiglitz preferences the elasticity of residual demand is constant and the same for all consumers. This has relatively extreme implications for how firms are likely to react to changes in their information set. For example, if a firm was suddenly able to observe the income levels of each consumer, the best they could do under linear pricing is implement third degree price discrimination. However, with the elasticity of demand independent of income and the same for all consumers, a firm would not change their behaviour, continuing to charge the same price per unit for all types. Somewhat implausibly, this additional information would be essentially of no value to a firm.

To enable firms to exploit this information requires expanding a firm’s strategy space from exclusively linear prices to more general non-linear prices. This change places the nature of information possessed by the firm at the center of the analysis. In this context a plausible assumption is that the firm knows the aggregate distribution of income but not an individual consumer’s income. More formally this is a setting where a firm implements indirect discrimination (aka second degree price discrimination). If a firm optimally chooses to exploit this information, it does so through the design of a menu of options offered to a consumer (or more broadly a product line).\textsuperscript{4}

\textsuperscript{2}It should be pointed out that non-homothetic preferences may amplify the motivation to manipulate/discriminate by the firm.
\textsuperscript{3}Models based on non-homotheticity need at least two sectors for differences in expenditure patterns to influence equilibrium outcomes.
\textsuperscript{4}This product line is associated with goods of different characteristics/quality and prices that induce consumers with different income to select different options from the product line. In this sense a firm offers multiple products. However, this view of multi-product firms is much narrower than usual perspective employed in the international trade literature. See for example Bernard et al. (2011).
The iPad range offers a particularly neat illustration of a product line.\(^5\) The initial offerings only had one dimension of variation, the size of the memory: 16GB, 32GB and 64GB. The corresponding prices are $499 and $599 for the first two sizes. Using these price/GB relationships to predict the price of a 64GB machines gives $399 + $6.25(64) = $799, which is $100 more than the actual $699. The standard candidates to explain this deviation are cost or elasticity. Industry sources confirm that the marginal cost of a GB is constant, so this can’t explain the variation. Similarly the prices imply that the elasticity of demand is increasing in memory size, contrary to the typical assumption.\(^6\) Using the implied elasticity from the 16GB machine suggests that the 64GB iPad would be priced over $1100. The importance of understanding what is driving these price differences follows from the fact that many goods and services can be characterized in terms of product lines.

Under indirect discrimination the optimal strategy of the firm and the resulting monopolistically competitive equilibrium is now not just a function of the curvature of the demand functions but also their position. As a consequence the distribution of income is a fundamental determinant of the design of the equilibrium product line. A feature of this equilibrium is that product design is distorted relative to the first best. In general, products designed for low income types are below the first best, while the products targeted to the high types are above the first best.\(^7\) It then follows that welfare differences are more exaggerated than income differences. The critical role of the distribution of income in this outcome immediately implies that the integration of two countries with different income distributions alters product line design and consequently welfare. The model produces especially clear predictions if countries can be ranked in terms of income distribution. In particular countries which have a “good” distribution of income receive larger gains from trade, with these gains disproportionately concentrated at the bottom of the income distribution. In this case, trade reduces the distortions from indirect discrimination and the benefits are felt across the entire distribution of income. The opposite occurs in a country with a “bad” distribution of

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\(^5\)Since Apple launched the iPad in 2010 there has been a proliferation of firms supplying tablet computers, all of them using product lines.

\(^6\)See Fajgelbaum et al. (2011) or Hummels and Lugovskyy (2009).

\(^7\)Monopoly models of indirect discrimination predict the first result but not the second. See for example Maskin and Riley (1984).
income, as trade adds to the distortions from indirect discrimination. Since these distortions are not present in the standard model of international trade they represent a new dimension of the welfare analysis.

The model also allows the gains from trade to be decomposed into those derived from additional varieties and those associated with the design of the menu of choices. Critically, these two components respond differentially to the level of trade costs with important implications for trade liberalization. In particular, when trade barriers are relatively high, marginal liberalization is primarily about reducing the costs of serving a market and has little impact on menu design. Thus, for high trade barriers the gains from gradual liberalization follow a pattern familiar from the standard model. However, once trade barriers become sufficiently low, the potential for international arbitrage triggers a convergence in product design across countries. Since not all types in all countries gain from the convergence, there is potential for a gradual process of trade liberalization to stall – at the margin the negative effects for product design in one country can outweigh the saving from lower trade costs.

The model also has a number of predictions for observable outcomes that allow it to be evaluated relative to empirical findings. The first relates to the equilibrium price distribution, which is shown to be increasing and concave in income. This is consistent with previous findings in the literature – outcomes which have been interpreted as inconsistent with existing trade models (see Manova and Zhang (2012)). The second prediction relates to the specification of the gravity equation. In particular, the model predicts higher trade between countries with similar per capita income (holding dispersion constant) and higher trade between countries with similar income dispersion (holding per capita income constant). These “Linder” type predictions contrast with the existing non-homothetic literature which does not provide an aggregate gravity prediction. After appropriately controlling for endogeneity, these predictions are confirmed in a sample based on the World Input Output Database.

Extending the model to allow for additional sectors and factors has implications for the

\[ \text{Also see Choi et al. (2009), Bekkers et al. (2012) and Hummels and Lugovskyy (2009).} \]
location of production due to differences in income dispersion and also the behavior of factor prices. The richness of the factor price response to integration provides a perspective that can help explain why countries with large differences in income per capita can have changes in relative factor prices which are positively correlated. Such a finding is relatively common in the empirical literature but is contrary to a factor proportions view of the world. Nevertheless, it is consistent with a model where firms use indirect discrimination, and the implied changes in distortions then feedback through factor markets.

To develop these results the paper is broken into 3 sections. Section 2 constructs a general equilibrium monopolistically competitive model of indirect discrimination with three income types. This framework facilitates comparisons with both the previous trade literature based on general equilibrium models with linear pricing and also the partial equilibrium monopoly literature that analyzes indirect discrimination. Section 3 considers integration between countries with different income distributions, and examines the consequences of gradual liberalization while also providing empirical evidence on the observable predictions of the model. Section 4 expands the model by progressively adding an additional sector and then adding an additional factor to examine the implications of indirect discrimination for industrial structure and factor prices.

## 2 Model

The main elements of the model are familiar from Krugman (1980): one factor (inelastically supplied), a (common) cost function that exhibits constant marginal cost with a fixed overhead cost and one sector where consumers have Spence-Dixit-Stiglitz (SDS) preferences over products. To add within country income variation, these basic features are augmented by including three types of workers who differ in terms of labor endowment. The middle type, $M$, has an endowment normalized to unity, and the low type, $L$, has an endowment of $1 - \alpha$ while the high type, $H$, possesses $1 + \alpha$, with $\alpha \in (0, 1)$. Letting $\beta_i^I$ denote the fraction of population of country $i$ that is type $I \in \{L, M, H\}$, then country $i$ has an aggregate
endowment of:

\[ L_i = 1 + \alpha (\beta_i^H - \beta_i^L) \]

This three type set-up has the advantage of being sufficiently rich to allow for the first and second moments of the income distribution to be varied independently, yet still keeps the model relatively tractable. Note that simply adding within country income variation to Krugman (1980) does not alter any of that models results, the key departure involves allowing firms to utilize information on income distribution when setting non-linear prices.

2.1 Budget Constraint

Expanding firm strategy space to include non-linear pricing naturally has implications for firm behavior, which in turn alters the set of feasible options available to consumers as reflected in the budget constraint. A convenient conceptual approach is to have firms choose a general non-linear price schedule, \( \{T(q), q\} \) where \( T(q) \) is the payment required for a product with attribute \( q \), and have consumers view the menu as a set of two-part tariffs (price involves a fixed/access fee and a usage fee). From a modeling perspective this has the advantage that the fixed/access charge acts like a lump sum tax, allowing the budget constraint to be expressed in a relatively familiar form.

To see this assume that consumer type \( I \) purchases a product from firm \( i \) for total payment \( T_i^l \), and assume that type \( I \) actually buys the option on the menu specifically designed for their income group. Let the two components of price be given by an access fee, \( A_i^l \), and a usage based payment, \( p_i^l q_i^l \), where \( T_i^l = A_i^l + p_i^l q_i^l \). Note that the marginal price, \( p_i^l \), can be read off the inverse demand function (derived in the following section) given the firm’s choice of \( q_i^l \), which implies \( A_i^l = T_i^l - p_i^l q_i^l \). Therefore, if a consumer purchases the relevant version of each variety, then a consumer with gross income \( m^l \) has net income:

\[ m^l = m^l - \sum_i A_i^l = \sum_i p_i^l q_i^l \]

Hence, the main modification is in relation to net income. In the standard model (i.e. linear
prices) there is no difference between net and gross income \( (\bar{m}^I = m^I) \). However, under non-linear prices net income can diverge from gross income.

### 2.2 Preferences

Following Krugman (1980), consumers have SDS preferences.

\[
U^I = \left[ \sum_i q_i^I \right]^{1/\rho} \quad \text{and} \quad 0 < \rho < 1
\]

Note that since \( \rho \) is not indexed by \( I \), all consumers have the same preferences. Apart from using net income rather than gross income, the utility maximization program results in familiar expressions with the inverse demand for a variety targeted at consumer \( I \) by firm \( i \):

\[
p_i^I = \theta q_i^I (\rho - 1) \quad \text{with} \quad \theta^I = \frac{\bar{m}^I}{Q_i^I \rho} \quad \text{and} \quad Q^I = U^I
\]

Facing these residual demand curves a typical firm evaluates the surplus from serving consumer \( I \) in the following way:

\[
S_i^I (q) = \theta^I \int_0^{q_i^I} z^{\rho - 1} dz = \frac{\theta^I q_i^I \rho}{\rho}
\]

A feature of this framework is that it offers either a quantity or a quality interpretation. Since the quantity interpretation is familiar from other international trade applications of the SDS model I'll focus on the quality perspective which also appears in the indirect discrimination literature that analyzes monopoly behavior (see for instance Tirole (1988)). The implicit assumption is that each product is purchased in a single unit and the consumer has preferences over the quality level as measured by \( q^I \). Consider for example the choice of a tablet computer. Here the quantity purchased is typically a single unit, and the margin of choice for the consumer is memory size (giga-bytes).\(^9\) Other examples include printers, with the dimension of choice pages per minute, or televisions and screen size. Note that this

\(^9\)For example the screen size of the iPad is fixed and the dimension of choice is restricted to the memory size - 16GB, 32GB or 64GB.
interpretation has no direct implications for the specification of the utility function. However, it does have implications for how “price” is calculated. Under linear pricing the firm selects a price per unit of quality (eg giga-bytes, pages per minute, inches) and the consumer is free to select any positive quality level. As a consequence the price paid for a particular tablet computer is \( p^I q^I \). In this case higher quality products have a higher price, but it is proportionately higher. We now consider whether a firm can do better than linear pricing.

2.3 Profit Maximizing product lines

We make the typical assumption in the literature that the \( Q \) sector is populated by monopolistically competitive firms with a constant marginal cost (and unit labor requirement), \( w \), and a firm level fixed cost, \( w^F \).\(^{10}\) Since monopolistic competition implies an absence of strategic interactions, the profit maximizing program resembles that of an indirectly discriminating monopolist (see for example Maskin and Riley (1984)). In particular, using the surplus functions from above and the information on the distribution of types in the population, the typical firm chooses a menu of \( \{q^I, T^I\} \), \( I \in \{L, M, H\} \) to maximize

\[
\pi = \sum_I \beta^I (T^I - wq^I) - w^F
\]

subject to

\[
\frac{\theta^I q^I \rho}{\rho} - T^I \geq \frac{\theta^K q^K \rho}{\rho} - T^K, \forall I \neq K
\]

\[
\frac{\theta^I q^I \rho}{\rho} - T^I \geq 0, \forall I
\]

where (3) are the incentive compatibility constraints while (4) are the participation constraints. In a standard non-linear pricing problem the ordering of the \( \theta' \)s is enough to ensure that the single crossing property holds – implying that only three of these constraints bind, the incentive constraint for the high and middle types and the participation constraint for the low type.\(^{11}\) However, since the \( \theta' \)s are determined as part of an equilibrium outcome we cannot simply take for granted that \( \theta^H > \theta^M > \theta^L \). Nevertheless, we conjecture that this

\(^{10}\)Returning to the iPad example, the constant marginal cost of a GB does appear to be constant. According to iSuppli estimates, the cost of the 16 GB flash memory is $16.80, $33.60 (32 GB) and $67.20 (64 GB).

\(^{11}\)See Maskin and Riley (1984).
ordering holds (it is in fact satisfied in equilibrium) allowing the relevant constraints to be rewritten as:

\[
T^L = \theta^L \frac{q^{L\rho}}{\rho} \tag{5}
\]

\[
T^M = \left( \theta^M q^{M\rho}_M - \theta^L \frac{q^{L\rho}}{\rho} \right) + T^L = \theta^M \frac{q^{M\rho}}{\rho} - (\theta^M - \theta^L) \frac{q^{L\rho}}{\rho} \tag{6}
\]

\[
T^H = \frac{\theta^H q^{H\rho}}{\rho} - \theta^H \frac{q^{M\rho}_M}{\rho} + T^M = \theta^H \frac{q^{H\rho}}{\rho} - (\theta^H - \theta^M) \frac{q^{M\rho}}{\rho} - (\theta^M - \theta^L) \frac{q^{L\rho}}{\rho} \tag{7}
\]

These prices imply total revenues, along with total costs, of:

\[
TR = \beta^L T^L + \beta^M T^M + \beta^H T^H \tag{8}
\]

\[
TC = \beta^L wq^L + \beta^M wq^M + \beta^H wq^H + wF \tag{9}
\]

Taking first order conditions with respect to \( q^I \) defines optimal behavior of a firm:

\[
\theta^H q^{H\rho-1} = w \tag{10}
\]

\[
((\beta^M + \beta^H) \theta^M - \beta^H \theta^H) q^{M\rho-1} = \beta^M w \tag{11}
\]

\[
(\theta^L - (1 - \beta^L) \theta^M) q^{L\rho-1} = \beta^L w \tag{12}
\]

The value function is derived by observing that (8) is homogeneous of degree \( \rho \) in the vector of production designs, \( q^I \), which implies \( \sum_I \frac{\partial TR}{\partial q^I} = \rho TR \). Since marginal revenue of any design equals (constant) marginal cost it follows from (10)–(12) that the value function can be written as \( \frac{1-\rho}{\rho} \sum_I \beta^I q^I - wF \). Setting this equal to zero confirms that free entry output/characteristics must satisfy: \(^{12}\)

\[
\sum_I \beta^I q^I = F(\sigma - 1) \tag{13}
\]

\(^{12}\)This has a straightforward interpretation when \( q^I \) is a measure of quality. For example the tablet targeted to the high type might have 64 GB while the middle and low types might have 32 GB and 16 GB, but zero profits involve the average product sold to be \( \beta^H 64GB + \beta^M 32GB + \beta^L 16GB \).
where \( \sigma = \frac{1}{1-\rho} \) is the elasticity of demand. This says that the average attributes of a firm’s product line is the same as chosen by a social planner and also coincides with what arises in the standard model with linear prices. Given the aggregate endowment of labor is fixed, this implies the equilibrium number of firms, \( n_i \), is the same across all three scenarios in this single sector setting.\(^{13}\)

### 2.4 Equilibrium

Having derived the equilibrium attributes of each firm, the second issue is the allocation across income groups. To determine this, start by combining (10) and (12):

\[
(\theta^L - (1 - \beta^L)\theta^M)q^{L(\rho-1)} = \beta^L\theta^H q^{H(\rho-1)} \Rightarrow \left(\frac{\theta^L}{\theta^H} - (1 - \beta^L)\frac{\theta^M}{\theta^H}\right)\phi^{LH(\rho-1)} = \beta^L
\]

Now combine (10) and (11):

\[
\left( (\beta^M + \beta^H)\frac{\theta^M}{\theta^H} - \beta^H \right)\phi^{MH(\rho-1)} = \beta^M
\]

where \( \phi^{IK} \equiv \frac{q^I}{q^K} \) and \( \bar{\pi}^{IK} \equiv \frac{m^I}{m^K} \) which implies \( \frac{\theta^L}{\theta^K} = \frac{\bar{\pi}^{IK}}{\bar{\pi}^{IK}} \).

We will focus specifically on the relative design of products, \( \phi^{IK} \). Using these expressions, the equilibrium conditions for relative design can be written as:

\[
\beta^H \phi^{MH\rho} + \beta^M \phi^{MH} = (\beta^M + \beta^H)\bar{\pi}^{MH}
\]

(14)

\[
(1 - \beta^L)\phi^{LM\rho} + \beta^L \phi^{LM} \frac{\phi^{MH}}{\bar{\pi}^{MH\rho}} = \bar{\pi}^{LM}
\]

(15)

(or equivalently)

\[
(1 - \beta^L)\phi^{LH\rho} \frac{\phi^{MH}}{\phi^{MH\rho}} + \beta^L \phi^{LH} = \bar{\pi}^{LH}
\]

(16)

To complete the analysis of the equilibrium we need to derive the net incomes. For the

\(^{13}\)A multi-sector model is considered below.
low type net income follows from (5):

\[ \rho T^L = \theta^L q^L \rho = \bar{m} L n \Rightarrow \rho n T^L = \rho m L = \bar{m}^L. \] (17)

For the middle income group (6) implies:

\[ \rho T^M = \theta^M q^M(1 - \phi^L M \rho) + \rho T^L \]
\[ \Rightarrow \bar{m}^M = \frac{\rho (m^M - m^L)}{1 - \phi^L M \rho} = \frac{\rho (m^M - m^L)}{\phi^M H \rho - \phi^L H \rho} \phi^M H \rho \] (18)

While (7) gives:

\[ \bar{m}^H = \frac{\rho (m^H - m^M)}{1 - \phi^M H \rho} \] (19)

So the equilibrium product designs, \( \{\phi^L H, \phi^M H\} \), must satisfy:

\[ \beta^H \phi^M H \rho + \beta^M \phi^M = (\beta^M + \beta^H) \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{1 - \phi^M H \rho}{\phi^M H \rho - \phi^L H \rho} \right) \phi^M H \rho \] (20)

\[ (1 - \beta^L) \phi^L H \rho \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{1 - \phi^M H \rho}{\phi^M H \rho - \phi^L H \rho} \right) + \beta^L \phi^L H = \left( \frac{\alpha}{1 - \alpha} \right) (1 - \phi^M H \rho) \] (21)

Inspecting this system it is immediately apparent that any solution is determined solely by the distribution of income, \( \{\beta^I\} \), holding \( \alpha \) and \( \rho \) constant. The following proposition characterizes the nature of the equilibrium.

**PROPOSITION 1.** An autarky equilibrium exists and is unique.

(See appendix for proof.)

A distinctive aspect of this equilibrium is the distribution of output/quality across the income groups. As identified above, the aggregate feature of each firm is first best, but this property doesn’t carry over to the products offered to each income group.

**PROPOSITION 2.** For any non-degenerate income distribution, each firm always designs a menu that induces the low income group to purchase a product below the first best while offering the high
product that is above the first best. The product designed for the middle income group can be above, equal or below the first best depending on the characteristics of the income distribution.

This proposition is the basis of the difference between the current model and the previous trade literature and also helps to distinguish between partial and general equilibrium models of indirect discrimination. With this in mind there are three issues to highlight.

First, in contrast to a model with linear pricing, distortions exist in equilibrium. These distortions are a result of a firm’s differential ability to extract rents from the various income groups. Second, market power distorts output decisions both above and below the first best. While the usual downward monopoly distortion is evident for the low income group, it is always the case that the high income group receives a product above the first best i.e. distortions are bi-directional. Figure 1 depicts the pattern of distortions as a function of income. Outcomes above the social optimum provide a stark contrast to the linear price model. Note also that this result doesn’t arise in the canonical partial equilibrium monopoly model of indirect discrimination where the high type always receives the first best outcome (see Maskin and Riley (1984)). In that setting the positions of the residual demand curves are exogenous (i.e. \( \theta^I \) is given) and there is a single firm. Relaxation of either of these aspects can play a role in the result that a high income type is offered a product above the first best. In this single sector setting, the position of a residual demand curve is influenced by the net income of a consumer type.\(^{14}\) For the high income types, the capture of information rents raises their net income and consequently shifts their residual demand function out relative to the first best. Conditional on the position of this residual demand curve a firm has no incentive to distort a high type’s design since this product doesn’t concede information rents to any other type. Instead the problem is the residual demand curve of the high type is in the “wrong” position. The final point to emphasize is that welfare differences are more exaggerated than income differences. To see this, note from the symmetry of menus a consumer’s welfare is linear in product design i.e. \( U^I = n^1 q^I \). Since product design is above the first best for high types but below the first best for low types, it follows that differences in welfare outcomes must be more pronounced than income differences.

\(^{14}\)The role of entry is considered below.
To underscore this last point and to help facilitate the analysis to come, consider the indirect utility function. A key step in deriving this function relates to marginal price for group $I$:

$$p^I = \theta^I q^{I(\rho-1)} = \frac{\alpha^{IH}}{\phi^{IH}} w$$  \hspace{1cm} (22)$$

Indirect utility is then given by:

$$U^I = n^{1/\rho} \left( \frac{m^I}{np^I} \right) = n^{1-\rho} \left( \frac{m^H}{w} \right) \phi^{IH}$$
$$= \phi^{IH} U^H$$  \hspace{1cm} (23)$$

$$U^H = \rho n^{1-\rho} \left( \frac{\alpha}{1 - \phi^{MH\rho}} \right)$$  \hspace{1cm} (24)$$

We are now in a position to reflect on the implications of firms using non-linear prices when the only source of variation across consumers is the income they possess. Apart from
expanding a firm’s strategy space in a plausible way, all of the other assumptions of the standard general equilibrium model of monopolistic competition are retained - especially the assumption of homothetic preferences and a constant elasticity of residual demand. Nevertheless the differences in product design and welfare outcomes are striking. A key take-away is that the distribution of income is the primary determinant of the size of distortions and consequently welfare outcomes. Given countries differ substantially in their income distributions, this suggests if we start from an autarky situation, the size and relevance of the distortions will also vary considerably across countries. How does trade affect these distortions? How do these distortions affect the gains from trade? It is to these questions we now turn our attention.

3 Implications of International Trade

3.1 Free Trade in the Standard Model

As a benchmark consider the standard model where technology and preferences are as described above but firms are constrained to use linear prices. Since welfare of an income group is proportional to their share of income we only need to consider aggregate demand for a variety and the number of varieties (which are a function of endowments). Consequently, for a country with an endowment of $L_i$ that has access to $n_j$ varieties we have:

$$q_i = \frac{p^{-\sigma}wL_i}{n_jp^{1-\sigma}} = \frac{\rho L_i}{n_j} \quad \& \quad n_j = \frac{L_j}{\sigma F}$$

These generate welfare in country $i$ of

$$U_i = \rho n_j^{\frac{1}{\sigma}} L_i$$

Autarky is then a situation where $n_j = n_i$ and free trade involves $n_j > n_i$. Using $F$ to denote free trade and $A$ for autarky it follows that the gains from trade in the standard model for
country \( i \) have the form:

\[
\frac{U_{i,F}}{U_{i,A}} = \left( \frac{L_w}{L_i} \right)^{\frac{1}{\sigma-1}} = GFT_i
\]  

(25)

where \( L_w \) is the size of the labor endowment of the integrated countries. Whenever this country engages in free trade with another country the sole mechanism for welfare change is through the number of varieties. This makes relative size the only determinant of the gains from free trade: the more varieties accessed under free trade, the larger the gains from free trade.

3.2 Free Trade with Indirect Discrimination

Against this benchmark consider the integration of two countries with potentially different income distributions. While the nature of a country’s income distribution is critical for the design of the product line in autarky, it is the characteristics of the global income distribution that shape design under free trade. If countries have very different income distributions, then there will be very pronounced differences in product design across countries in autarky. To understand the implications of eliminating this variation through integration we’ll focus on two dimensions that are commonly emphasized when comparing income distributions (i) mean income, and (ii) income dispersion.

To trace through the consequences of integrating with another country use (23), (24) and (25) to derive:

\[
GFT_{i}^{H} = \left( \frac{1 - \phi_{iA}^{MH\rho}}{1 - \phi_{F}^{MH\rho}} \right) GFT_i
\]  

(26)

\[
GFT_{i}^{I} = \left( \frac{\phi_{F}^{IH}}{\phi_{iA}^{IH}} \right) GFT_{i}^{H}
\]  

(27)

Naming the two countries, Home, \( h \), and Foreign, \( f \), and assuming that Home has a higher per capita income than Foreign, leads to the following claim.

**Proposition 3.** If the likelihood ratio of Home’s income distribution dominates that of Foreign,
then Home’s gains from free trade are greater than the standard model while the opposite holds in the Foreign country. Furthermore, within the Home country, the proportional gain follows a rank that is inversely related to income. Consequently, the lowest income group in the Home country gains the most from trade. The converse holds in the Foreign country.

![Figure 2: The distribution of the gains from free trade](image)

Proposition 3 can be understood with the aid of Figure 2 which represents the set of income distributions in our three type setting. The vertical distance measures the share of high income types in the population while the horizontal distance measures the fraction of low income types. The fraction of middle income types is then implicitly defined as $\beta^M = 1 - \beta^L - \beta^H$ or either the vertical or horizontal distance from any point in the triangle to the $\beta^L + \beta^M = 1$ line. If we consider a Home country with an income distribution given by $\{\beta^L, \beta^H\}$ then all distributions with the same average income are given by the dotted line starting at the origin. Any distribution with lower mean income lies south-east of this line (all iso-mean-income lines are parallel). Indifference curves for the low and middle income types are plotted, with higher welfare for a type below a given indifference curve.\(^{15}\)

\(^{15}\)The indifference curve for the high type has the same slope as the middle income type. This is evident from comparing (28) and (29).
The relative slopes of the indifference curves at \( \{ \beta^L, \beta^H \} \) can be derived from (23) and (24). Expressing these relationships in terms of proportional changes we have:

\[
\hat{U}^H = \left( \frac{\rho \phi^{MH\rho}}{1 - \phi^{MH\rho}} \right) \hat{\phi}^{MH} \quad (28)
\]

\[
\hat{U}^M = \left( \frac{1 - (1 - \rho) \phi^{MH\rho}}{1 - \phi^{MH\rho}} \right) \hat{\phi}^{MH} \quad (29)
\]

\[
\hat{U}^L = \hat{\phi}^{LM} + \left( \frac{1 - (1 - \rho) \phi^{MH\rho}}{1 - \phi^{MH\rho}} \right) \hat{\phi}^{MH} \quad (30)
\]

Along an indifference curve these changes equal zero. This implies that for a given \( \Delta \beta^L > 0 \), the \( \Delta \beta^H \) required for \( \hat{U}^L = 0 \) (i.e. \( \hat{\phi}^{MH} < 0 \)) is greater than required for \( \hat{U}^M = 0 \) (i.e. \( \hat{\phi}^{MH} = 0 \)).\(^{16}\) Hence, at any common point the slope of the low types indifference curve is greater than that of the middle or high types indifference curve.

Using Figure 2 we can now see why a straight ranking of mean incomes is not generally sufficient to predict the distribution of the gains from trade across countries. If the lower mean income country also has a relatively small fraction of low income types, then the low types in the higher mean income country are served a product under free trade that is degraded relative to autarky. This occurs for income distributions below the dotted income line but above \( U^L \). A similar conclusion follows for distributions below the dotted income line but above \( U^M \), although product design for the middle type is now relatively degraded because there are too few middle types. Restricting the comparison to income distributions that can be ranked according likelihood ratios allows more definitive predictions to be made. The set of distributions dominated by \( \{ \beta^L, \beta^H \} \) is given by the triangle bounded by the dashed lines and the horizontal axis. Integration by a Home country with \( \{ \beta^L, \beta^H \} \) and a country within this set will deliver amplified gains from trade for the Home country, with the largest gains for the lowest income groups.

Proposition 3 provides a contrast to the standard model where homothetic preferences and linear pricing ensure that firms only focus on aggregate demand and not its composition: all consumers receive the same proportional gains from trade within a country. The

\(^{16}\)To see this suppose that the change in \( \beta^M < 0 \) and \( \beta^H > 0 \) is such that (20) remains satisfied for the same \( \phi^{MH} \). It then follows from (21) that \( \phi^{LM} > 0 \).
above proposition reveals that once firms are able to utilize information on income distribution, the distribution of the gains from free trade can vary significantly across income groups within a country. The ordering imposed by likelihood ratio dominance provides a sufficient condition for magnification of the gains from free trade relative to the standard model for the country with the higher GDP per capita. However, this ordering is also typically associated with a change in the dispersion of income, with the dominated Foreign country potentially having a lower variance of income. To consider these components separately, hold the GDP per capita constant across countries but vary income dispersion. A particularly neat parametrization is achieved by setting $\beta^L_i = \beta^H_i = \beta_i$ where $i \in \{h, f\}$. This implies mean income in both countries is unity and variance of income is given by $2\alpha^2\beta_i$.

PROPOSITION 4. If Home’s income distribution is a mean preserving spread of Foreign’s income distribution (i.e. $\beta_h > \beta_f$), then the low income group in the Home country receives gains from free trade that are lower than the standard model, while the gains for the middle and high income groups are higher. The converse holds in the Foreign country.

Once again the intuition for this proposition is captured in Figure 2. Moving along the dotted income line from the origin increases the dispersion of income while holding mean income constant. For a Home country with income distribution $\{\beta^L, \beta^H\}$, integrating with a low dispersion country offers better product design for the middle income group resulting in higher welfare for both the middle and high income groups. However, the smaller fraction of low types in the global economy leads to an inferior product for the low type and reduced gains from trade for the low type in Home. This logic is reversed when integration occurs with a country that has higher income dispersion. In this case there are now relatively more of both low and high income types in the global economy but fewer middle income types. This facilitates an improved design for the low income group, and amplified gains from free trade, but a less attractive product for the middle income type. The poorer design of the middle income product reduces the outside option for the high income group, which diminishes their gains from integration.

Together these propositions reveal that the gains from trade are fundamentally changed by indirect discrimination. In the standard model, relative size is the sole determinant of
the gains from trade: the smaller the country, the larger the gains from trade. In our three type model, this implies the country with the lower average income would gain the most from trade. With indirect discrimination, relative size is no longer enough to completely characterize the gains from trade. In fact a smaller country may have their variety gains from trade dramatically diminished by inferior product design. The main mechanism operates through the desire of firms to customize products to extract rents – better products generate more surplus but also concede information rents to higher income groups. This trade-off is resolved with reference to the distribution of income. The critical factor shaping the gains from trade is then the extent and nature of the difference between the national and global income distributions. Pronounced differences give rise to big differences not just between the number of varieties available but also between the menu of choices offered in autarky and free trade.

Under free trade the menu of choices is common to all countries and this implies that prices paid for a specific product will also be the same. However, since the distribution of income varies across countries the distribution of transaction prices will also vary. There is now an growing literature documenting the association between country characteristics and import prices. What does the indirect discrimination model imply about the distribution of import prices and to what extent is this consistent with the patterns observed in the trade data?

3.3 Comparing Prices Across Destination Markets

The prices implied by the free trade menu, $T^I$, have the property that they are increasing and concave in income: $T^L < T^M < T^H$ and that $\frac{dT^I}{dq} = p^I$. The concavity of the price schedule follows from the marginal price declining in income:

$$p^L = \frac{\bar{\alpha}^{LH} w}{\phi^{LH} w} > p^M = \frac{\bar{\alpha}^{MH} w}{\phi^{MH} w} > p^H = w$$

A typical finding in the empirical literature is that conditional on exporter-product pairs,
import unit values are increasing in destination per-capita income.\textsuperscript{17} Moreover the per-capita income elasticity is less than unity - suggesting a concave pricing function. Studies that consider within country income dispersion are less common, with Choi et al. (2009) and Bekkers et al. (2012) among the few that examine this dimension. Choi et al. (2009) find that differences in the dispersion of income across countries is associated with a less than proportional increase in the dispersion of import prices. In the context of their model they find this result puzzling but it is consistent with a concave price schedule of the type implied by indirect discrimination. While Choi et al. (2009) consider all HS 6 import prices for 26 importers, Bekkers et al. (2012) narrow their focus to 1260 HS 6 categories of final goods but expand the sample to over 100 countries. The motivation for this narrower set of products is a tighter mapping to consumer income. They find a negative correlation between within country inequality and per unit import prices.\textsuperscript{18} Consequently, the evidence tends to suggest that the distribution of import prices is concave with respect to per-capita income – as predicted by the model of indirect price discrimination.

3.4 Gradual Trade Liberalization

While the autarky/free-trade dichotomy offers a useful benchmark, it is typically not the case that trade costs are characterized by either of these extremes. Nevertheless, under the standard iceberg interpretation, a lowering of the trade costs monotonically increases welfare for all countries.\textsuperscript{19} While it is tempting to assume that a similar monotonicity applies in the indirect discrimination model, the following proposition reveals that all of the differences from the standard model arise only once trade barriers are sufficiently low.

**PROPOSITION 5.** Let $\tau \geq 1$ represent the iceberg transport cost between the Home and the Foreign country. Then there exists a transport cost $\bar{\tau}$ such that the gains from trade for a high income type

\textsuperscript{17}This finding is documented across a range of countries and also appears in data disaggregated to the firm level, see Manova and Zhang (2012).

\textsuperscript{18}Bekkers et al. (2012) conclude that is consistent with a model of non-homothetic hierarchic demand but inconsistent with non-homothetic models of quality or ideal variety. Each of these models employ the assumption of linear pricing.

\textsuperscript{19}The absence of tariff revenue implies the optimal trade cost is zero for all countries.
are:

\[
GFT_i^H(\tau) = \begin{cases} 
GFT_i(\tau) & \text{for } \tau \geq \tau \\
\frac{1-\phi_{iH}^{MH}}{1-\phi_{ir}^{MH}} GFT_i(\tau) & \text{for } \tau < \tau
\end{cases}
\]

and for income type \( I \neq H \)

\[
GFT_i^I(\tau) = \begin{cases} 
GFT_i(\tau) & \text{for } \tau \geq \tau \\
\left(\frac{\phi_{iH}^{IH}}{\phi_{ir}^{IH}}\right) GFT_i^H(\tau) & \text{for } \tau < \tau
\end{cases}
\]

where \( GFT_i(\tau) = \left(1 + \frac{L_i}{L_r} \left(\frac{\tau w_j}{w_i}\right)^{1-\sigma}\right)^{1-\tau} \) denotes the gains from trade in the standard model and \( \phi_{ir}^{IH} = \left(\frac{n_i}{n_i^{\phi_{ii}}} \phi_{ii}^{IH} + \frac{d_i}{n_i^{\phi_{ij}}} \phi_{ij}^{IH}\right) \) is a market share weighted average product design.

This says that when trade barriers are relatively high, the indirect discrimination model delivers the same proportional gains from trade as the standard model. Thus, all the changes described in Propositions 3 and 4 occur only after trade barriers are below \( \bar{\tau} \).

This demarcation can have important implications for a process of gradual trade liberalization. To see this consider the scenario described in Proposition 3 where the low average income country gains from trade due to an increase in the number of varieties but has these gains diminished by an inferior product design relative to autarky (lower \( \phi_{iH}^{IH} \) for all \( I \in \{L,M\} \)). According to Proposition 3 the reduction in quality is most pronounced for the low income group. Since all of the change in quality occurs for trade barriers below \( \bar{\tau} \) this suggests the possibility that the decline in quality may completely offset the standard gains from trade associated with further liberalization. If so, the gains from trade for a low income type may reach a maximum before free trade. An essential ingredient for such a result is a relatively large share of low income types in the population, since the decline in quality is more prominent the larger the share of low types. Taking this to an extreme, Figure 3 depicts a scenario where the gains from trade are in fact exhausted before free trade (where \( \beta_H = 1, \beta_L = 1, \rho = \frac{1}{2} \) and \( \alpha = \frac{3}{10} \)).

This intriguing interaction between trade barriers and welfare highlights a potential
downside to gradual trade liberalization: beyond a point one country simply may not benefit from further trade liberalization. Once again the root cause is differences across countries in the distribution of income and the associated design of products. When markets are segmented, access to additional varieties is the only source of gains from trade. As trade barriers fall, markets become more deeply integrated and product design becomes more universal. As we have seen, this standardizing of products doesn’t bring unambiguous gains to all consumers in all countries.

3.5 Gravity Equation

The potential for welfare outcomes to vary dramatically from the standard model raises the question of whether there is a similarly pronounced analogue for an observable outcome like trade flows. To explore this issue, let $d_{ij} = \left( \frac{\tau_{ij}w_j}{w_i} \right)^{1-\sigma}$ and $\bar{n}_i = \sum_j n_j d_{ij}$, and note that:

$$T_{ij}^H = d_{ij} \frac{\bar{m}_i^H}{\rho \bar{n}_i} (1 - \phi_{ij}^{MH\rho}) + T_{ij}^M, \quad T_{ij}^M = d_{ij} \frac{\bar{m}_i^M (\phi_{ij}^{MH\rho} - \phi_{ij}^{LH\rho})}{\phi_{ij}^{MH\rho} \phi_{ij}^{LH\rho}} + T_{ij}^L, \quad T_{ij}^L = d_{ij} \frac{\bar{m}_i^L \phi_{ij}^{LH\rho}}{\rho \bar{n}_i \phi_{ij}^{LH\rho}}$$
Then bilateral trade between importer $i$ and exporter $j$ is given by:

$$X_{ij} = n_j \left( \beta_i^H T_{ij}^H + \beta_i^M T_{ij}^M + \beta_i^L T_{ij}^L \right)$$

$$= \frac{n_j d_{ij} m_i^H}{n_i} \left( \beta_i^H (1 - \phi_{ij}^{MH}) + (\beta_i^H + \beta_i^M) \phi_{ij}^{MH} \frac{\phi_{ij}^{MH}}{\phi_i^{MH}} (1 - \phi_{ij}^{LM}) + \phi_{ij}^{LM} \frac{\phi_{ij}^{LM}}{\phi_i^{LM}} \right)$$

$$= \frac{n_j d_{ij} m_i^H}{n_i} \left( \beta_i^H + (\beta_i^H + \beta_i^M) \phi_{ij}^{MH} - \beta_i^H \phi_i^{MH} \right) \frac{\phi_{ij}^{MH}}{\phi_i^{MH}} + \left( \alpha_i^{LM} - (1 - \beta_i^L) \phi_{ij}^{LM} \phi_i^{LM} \right) \frac{\phi_{ij}^{LM}}{\phi_i^{LM}}$$

Note that

$$X_i = \sum_j X_{ij} = \frac{m_i^H}{\rho} \sum_j \left( s_{ij} \beta_i^H (1 - \phi_{ij}^{MH}) + s_{ij} (\beta_i^H + \beta_i^M) \phi_{ij}^{MH} \frac{\phi_{ij}^{MH}}{\phi_i^{MH}} (1 - \phi_{ij}^{LM}) + s_{ij} \phi_{ij}^{LM} \phi_i^{LM} \right)$$

$$= \frac{m_i^H}{\rho} \left( \beta_i^H (1 - \phi_{ij}^{MH}) + (\beta_i^H + \beta_i^M) \phi_{ij}^{MH} \phi_i^{MH} \frac{\phi_{ij}^{LM}}{\phi_i^{LM}} - \beta_i^H \phi_i^{MH} \right) + \left( \alpha_i^{LM} - (1 - \beta_i^L) \phi_{ij}^{LM} \phi_i^{LM} \right) \phi_i^{LM}$$

$$= \beta_i^H m_i^H + \beta_i^M m_i^M + \beta_i^L m_i^L$$

This implies the following expenditure shares for country $i$:

$$\frac{X_{ij}}{X_i} = s_{ij} \left( \frac{\beta_i^H + (1 - \beta_i^L) \phi_{ij}^{MH} - \beta_i^H \phi_i^{MH} \phi_i^{MH}}{\phi_i^{MH}} \right) + \left( \frac{\phi_{ij}^{LM}}{\phi_i^{LM}} \phi_i^{LM} \phi_i^{LM} \phi_i^{LM} \right)$$

$$s_{ij} = \frac{n_j d_{ij}}{\sum n_k d_{ik}}$$

Since $s_{ij}$ captures the standard gravity equation it is apparent that trade flows will deviate from the standard model to the extent that products from country $j$ diverge from the typical designs consumed in country $i$, $\phi_{ij}^{LM}$, and on the signs of $\phi_{ij}^{LM} \phi_i^{LM}$ and $\phi_{ij}^{LM} \phi_i^{LM} \phi_i^{LM}$. The interaction of these components will determine whether trade is above or below that predicted by the standard model. Note that the first component depends on the design choices of the exporter, $j$, while the second component depends on the income distribution in the importer, $i$. Before getting to the implications of the interaction, we’ll characterize behavior of each of these terms.
Starting with relative product design, there are two situations in which $\phi_{ij}^{IH} = \phi_{i}^{IH}$ for all $j$. First when $\tau \geq \bar{\tau}$ – markets are segmented. As we established above this coincides with the gains from trade in the standard model. This should not be too surprising since trade barriers in this set satisfy all of the primitive assumptions as well as the macro-level restrictions R1-R3 of Arkolakis et al. (2012). The second environment where $\phi_{ij}^{IH} = \phi_{i}^{IH}$ is under free trade. This is more interesting since it also satisfies all the restrictions of Arkolakis et al. (2012). In fact, $s_{ij}$, coincides with the form of R3' when $\tau_{ij} = 1$. However, as outlined in Propositions 3 and 4, knowledge of the domestic expenditure share and the trade elasticity, $\sigma - 1$, aren’t sufficient to calculate the gains from trade. So even though all the assumptions are met, the results from Arkolakis et al. (2012) no longer hold under indirect discrimination. This is all the more intriguing since the volume of trade will be exactly as predicted by the standard model, but the mapping to welfare will be fundamentally different.

What happens to trade when $\tau \in (0, \bar{\tau})$? To characterize the rank of $\phi_{ij}^{IH}$ for markets that are partially integrated consider the scenario of Proposition 3 where autarky relative designs in Foreign are uniformly superior to Home. In this case consumers in the Home country will prefer the designs offered in the Foreign market if trade barriers are low enough. In this sense, $f$, serves as the reference market. Since cross-hauling isn’t part of an equilibrium, product design in the $L$ and $M$ segments is shaped by the no arbitrage constraint. That is, the design of $q_{fj}^{L}$ will pin-down the design of $q_{hj}^{L}$ for $I \in \{L, M\}$.

To illustrate the implications of partial integration consider product design in the low income segment when trade barriers are sufficiently low for only this market segment to be constrained by the potential for arbitrage. This implies that a middle income type in the Home country must be indifferent between purchasing the local variant from a firm’s product line and cross-hauling the product designed for the Foreign low income type by the same firm. That is:

$$\theta_{h}^{M} \frac{q_{hj}^{L \rho}}{\rho} - T_{hj}^{L} = \theta_{h}^{M} \left(\frac{q_{fj}^{L \rho}}{\tau}\right) - T_{fj}^{L}$$

Since neither of the low types have an outside option $T_{hj}^{L} = \theta_{h}^{L \rho} q_{hj}^{L \rho}$ and $T_{fj}^{L} = \theta_{f}^{L \rho} q_{fj}^{L \rho}$, this
implies:

\[ q_{fh}^L = \frac{\tau^\rho (\theta^L_h - \beta^L_f \theta^L_f)}{\rho} \]  

\[ q_{ff}^L = \frac{\tau^\rho q_{fh}^L}{\rho} \Rightarrow q_{fh}^L = \tau^{-1} \gamma_{hf}^{L1} q_{ff}^L \]

This constraint implies that design for the low income market must satisfy the following first order conditions:

\[
\left( (\theta^L_f - (1 - \beta^L_f)\theta^M_f) + \tau^\rho (\theta^L_h - (1 - \beta^L_h)\theta^M_h) \right) q_{fh}^{L0-1} = \left( \beta^L_f + \gamma^L_{hf} \beta^L_h \tau^{-2} \right) \tau w_h
\]

\[
\left( (\theta^L_f - (1 - \beta^L_f)\theta^M_f) + \tau^\rho (\theta^L_h - (1 - \beta^L_h)\theta^M_h) \right) q_{ff}^{L0-1} = \left( \beta^L_f + \gamma^L_{hf} \beta^L_h \right) w_f
\]

The ratio of these conditions imply \( \frac{q_{fh}^L}{q_{ff}^L} = \left( \frac{\beta^L_f + \gamma^L_{hf} \beta^L_h \tau^{-2}}{\beta^L_f + \gamma^L_{hf} \beta^L_h} \right) \frac{\tau w_h}{w_f} \leq \left( \frac{\tau w_h}{w_f} \right)^{-\sigma} \) – which says that countering within product line arbitrage requires less of an adjustment for the Home firms. The intuition is relatively straightforward: under segmentation transport costs already ensure \( q_{fh}^L < q_{ff}^L \), so a Home firm’s product concedes fewer information rents, hence they have less to lose from international arbitrage and make less of an adjustment to counter cross-hauling. In addition, since the high end of the product line is never susceptible to cross-hauling, its design is never subject to integration which implies designs in the reference market have the following rank:

\[
\frac{\phi_{fh}^{LH}}{\phi_{ff}^{LH}} = \left( \frac{\beta^L_f + \gamma^L_{hf} \beta^L_h}{\beta^L_f + \gamma^L_{hf} \beta^L_h \tau^{-2}} \right)^\sigma \geq 1
\]

To recover the rank in the non-reference market note that:

\[
\frac{\phi_{fh}^{LH}}{\phi_{hh}^{LH}} = \frac{\phi_{fh}^{LH} q_{fh}^H q_{ff}^H}{\phi_{ff}^{LH} q_{ff}^H q_{hh}^H} = \frac{\phi_{fh}^{LH}}{\phi_{ff}^{LH} \tau^{2\sigma}} \leq 1
\]

Similar arguments can be constructed for the middle income market segment that show \( \phi_{fh}^{MH} \geq \phi_{ff}^{MH} \) and \( \phi_{hh}^{MH} \geq \phi_{hh}^{MH} \). These results imply that the local design is always weakly inferior to the overseas design when the likelihood ratio of Home’s income distribution dominates Foreign’s. Whether this translates into higher or lower trade flows than the
standard model depends on the sign of the second component of the interaction terms.

To see that these terms are capable of being either positive or negative, consider what happens as we approach free trade (i.e. \( \tau \to 1 \)). In this case variation in design across sources becomes relatively compressed so that \( \phi_{ij}^{IH} \to \phi^{IH} \). The signs of interest then depend on the following comparisons:

\[
\bar{\kappa}^{MH} = \frac{\beta^H_w}{\beta^H_w + \beta^M_w} \phi^{MH\rho} + \frac{\beta^M_w}{\beta^H_w + \beta^M_w} \phi^{MH} \leq \frac{\beta^H_i}{\beta^H_i + \beta^M_i} \phi^{MH\rho} \tag{32}
\]

\[
\bar{\kappa}^{LH} = (1 - \beta^L_w)\phi^{LH\rho} \bar{\kappa}^{MH\rho} + \beta^L_w \phi^{LH} \leq (1 - \beta^L_i)\phi^{LH\rho} \bar{\kappa}^{MH\rho} \tag{33}
\]

where \( \beta^l_w \) denotes the fraction of the world population with income \( l \). It is clear that when \( \beta^L_i = \beta^l_w \) then the LHS will be greater than the RHS and all the interaction terms will be positive. In this case the volume of trade under partial integration will be greater than predicted by the standard model.

In contrast, when (32) and (33) are both negative trade is below the standard gravity prediction. When is this most likely to occur? When the difference in per capita income is greatest. To see this consider \( \beta^L_j = 1 \) and ask how the value of trade changes as we vary \( \beta^H_h \) when we start from \( \beta^L_h = 1 \). The omission of the middle income type implies \( \phi^{MH\rho} = \bar{\kappa}^{MH} \) and reduces the equilibrium condition for low type design to \((1 - \beta^L_w)\phi^{LH\rho} + \beta^L_w \phi^{LH} = \bar{\kappa}^{LH} \). In a world with two countries we can construct the global income distribution as \( \beta^l_w = \frac{\beta^l_h + \beta^l_f}{2} \).

As a result the sign of the interaction term now depends on \( \phi^{LH} \leq \frac{\phi^{LH}(\phi^{LH\rho} + \phi^{IH})}{\beta^l_h} \). If both countries have the same income distribution (i.e. \( \beta^H_h = 0 \)) then the LHS is greater than the RHS and the interaction term is positive. However, as we increase \( \beta^H_h \) the RHS increases faster than the LHS and at \( \beta^H_h = 1 \) the interaction term is negative.\(^{20}\) This provides us with the following proposition

**PROPOSITION 6.** When countries income distributions can be ordered by likelihood ratio dominance and trade barriers are low enough for markets to be partially integrated, the indirect discrimination model predicts that the deviation from the standard gravity model can be either positive or

\(^{20}\)At \( \beta^H_h = 0 \) we have \( \frac{d\phi^{LH}}{d\beta^H_h} < 0 \) while the derivative of the RHS is \( \phi^{LH\rho} + \phi^{IH} > 0 \).
negative. More trade is predicted if the difference in GDP per capita is not too large. However, if the difference is relatively large, then the indirect discrimination model predicts lower trade volumes than the standard gravity model.

This result resembles the “Linder Hypothesis” in that it relates the volume of trade to differences in per-capita income: relatively small deviations in per capita income gives rise to augmented trade flows but relatively large differences reduce the volume of trade. To date it has been asserted that such a trade pattern can only be explained by non-homothetic preferences. What is interesting about the above result is that preferences are not only identical and homothetic, but they also impose the additional restriction that the elasticity of demand is constant. Nevertheless, simply allowing firms to maximize profits by exploiting information on income distribution in a relatively plausible way results in a positive correlation between similarity in income per-capita and trade. While there is variation across consumers on the demand-side, it is purely in terms of income rather than hardwired into preferences. The intuition is also relatively direct. When markets are partially integrated, a location that delivers a product design better than the average in an importing country faces two competing forces that shape trade flows. First, an above average design allows more rents to be extracted from the low types simply because a better product generates more rents. Second, a better product design allows the higher types to capture more information rents, which tends to suppress the volume of trade by lowering prices for the higher types. When $\beta_f^L$ is relatively high (given $\beta_f^H = 1$), the first effect dominates and trade flows are higher than predicted by the gravity equation. However, when $\beta_h^L$ is relatively high, the second effect dominates and trade flows tend to be smaller than the standard model would predict.

While a focus on the correlation between trade and per capita income differences is natural in this setting, the model also has implications for the volume of trade as income dispersion varies across countries. As in Proposition 4 consider a setting where Home’s income distribution is parameterized through $\beta_h$ to be a mean preserving spread of Foreign’s income distribution (i.e. $\beta_h > \beta_f$). To isolate the central mechanism let $\beta_f = 0$ – the Foreign country has no income heterogeneity. This last characteristic means that under segmentation
foreign consumers receive no information rents, while under partial integration they capture information rents by having the product offered to the low income consumer in Home as an outside option. Having an outside option implies that Home’s exports to Foreign can be expressed as:

$$n_h \tau_{fh}^M = \rho \left( m_f^M - m_h^L \right) \left[ \frac{\left( \gamma_M \phi_{hh}^{MH} + \gamma_L \phi_{hh}^{LH} \right)}{\gamma_M \phi_{h}^{MH} + \gamma_L \phi_{h}^{LH}} \left( \frac{1 - \phi_{hh}^{LM} / \tau^\theta}{1 - \phi_{h}^{LM} / \tau^\theta} \right) \right] + \rho m_h^L \left[ \phi_{hh}^{LH} \right]$$

(34)

To characterize trade flows as $\beta_h$ is varied, start by considering $\beta_h \approx \beta_f$. For partial integration to occur in this setting trade barriers must be relatively small, i.e. $\tau \approx 1$. This combination implies that $\phi_{hj}^{IK} \approx \phi_{h}^{IK}$, so there is little variation in design across locations and the terms in square brackets are both approximately unity. Consequently, trade flows are similar to the standard gravity prediction.

If we examine the other extreme, $\beta_h \to \frac{1}{2}$ a different result emerges. Now the Home country houses only high and low income types. Once again the foreign middle income type only captures information rents if trade barriers are sufficiently small to make the Home low’s product a viable outside option. This only occurs when $\theta_f^M / \tau^\theta \geq \theta_h^L$ – a condition that is met simultaneously for firms located in both countries, ensuring $\phi_{hj}^{LH} = \phi_{h}^{LH}$. In contrast, the trade cost that implies the home High income type prefers the foreign middle’s product as an outside option within a foreign firm’s product line is higher than the threshold trade cost for a similar incentive to arise within a home firm’s product line: $\phi_{hf}^{MH} > \phi_{h}^{MH}$. Combining these two features implies $\phi_{hh}^{LM} > \phi_{hf}^{LM}$ which gives:

$$\left( \frac{\gamma_M \phi_{hh}^{MH} + \gamma_L \phi_{h}^{LH}}{\gamma_M \phi_{h}^{MH} + \gamma_L \phi_{h}^{LH}} \right) \left( \frac{1 - \phi_{hh}^{LM} / \tau^\theta}{1 - \phi_{h}^{LM} / \tau^\theta} \right) < 1$$

Hence, trade flows are lower than predicted by the standard gravity model. We can summarize these results as follows.

**PROPOSITION 7.** If Home’s income distribution is a mean-preserving spread of Foreign’s ($\beta_h \geq \beta_f$) and trade costs are sufficiently small, then trade volume declines as the difference in income  

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21See appendix for derivation.
dispersion in the Home country increases (i.e. increase $\beta_h$).

This proposition augments the “Linder Hypothesis” by identifying differences in income dispersion as a characteristic that diminishes trade flows. The intuition derives from the enhanced ability of consumers to look abroad for outside options as trade costs fall, constraining the ability of firms to extract rents. For a given trade barrier, this mechanism is stronger the greater the difference in income dispersion across countries.

### 3.6 Augmented Gravity

To connect the analysis with the empirical literature on gravity, note that equation (31) has a multiplicative form consistent with the Head and Mayer (2013) definition of general gravity:

$$X_{ij} = D_i S_j v_{ij}$$  \hspace{1cm} (35)

where $D_i$ is an importer fixed effect, $S_j$ is an exporter fixed effect and $v_{ij}$ is a bilateral measure of accessibility. Due to balanced trade

$$M_i = \sum_k X_{ik} = \sum_k D_i S_k v_{ik} \quad \Rightarrow \quad D_i = \frac{M_i}{\sum_k S_k v_{ik}}$$

Substituting this result into (35) gives:

$$X_{ij} = \frac{M_i}{\sum_k S_k v_{ik}} S_j v_{ij}$$

Impose market clearing:

$$M_j = \sum_h X_{hj} = \sum_h \frac{M_h}{\sum_k S_k v_{hk}} S_j v_{hj} \quad \Rightarrow \quad S_j = \frac{M_j}{\sum_h \frac{M_h}{\sum_k S_k v_{hk}} v_{hj}}$$
Using this result gravity can be expressed in a structural form:

\[ X_{ij} = \frac{M_i \sum_k S_k v_{ik}}{\sum_i M_j \sum_h M_h \sum_k S_k v_{hk}} v_{ij} \]

\[ = \frac{M_i}{\Pi_i} \left( \frac{M_j}{\sum_i M_i \Pi_i v_{ij}} \right) v_{ij} \]  

(36)

where \( \Pi_i = \sum_k S_k v_{ik} \). This implies the indirect discrimination framework fits squarely within the structural gravity literature. Consequently, as emphasized by Egger and Nigai (2013), the validity of inference based on (36) depends fundamentally on the specification of \( v_{ij} \) since any unobserved trade costs will inevitably bias the estimates of observed trade costs and the importer and exporter fixed effects.

To account for this issue we follow the approach of Egger and Nigai (2013) who build on the work of Silva and Tenreyro (2006) and Fally (2012). In particular, Egger and Nigai (2013) propose a two step process to estimate gravity models, with the first stage employing a dummy variable model to provide an unbiased decomposition of trade costs into an exporter effect, an importer effect and a bilateral effect. This is achieved by using a short panel (two consecutive years) where the exporter and import effects are time dependent but constrains the bilateral effect to be constant over that short period – constrained analysis of variance (CANOVA). Bilateral trade costs can then be further decomposed in a second stage. They show that the standard one-step methodology is associated with pronounced bias in parameter estimates which can be minimized by the CANOVA procedure.

Table 1 provides a set of results to evaluate the predictions of Propositions 6 and 7. The estimates are based on the 40 countries included in the World Input Output Database.\(^{22}\) The approach is motivated by equation (31) which suggests a standard gravity formulation augmented by terms to reflect deviations from the typical specification due to both differences across trade partners in per capita income and income dispersion. Note that this direct link between the model and the aggregate gravity specification contrasts with the previous literature which has typically adopted ad hoc formulations at the aggregate level or relied on non-homothetic preferences that generate predictions about sectoral rather than aggregate

\(^{22}\)The appendix documents the data and sources used.
trade volumes (see Hallak (2010), Fieler (2011) and Caron et al. (2014)).

Table 1 decomposes the bilateral trade costs estimated in the first stage. The specification includes the typical list of candidate measures of trade costs and follows Eaton and Kortum (2002) with a flexible formulation of distance effects. It adds to the standard list of bilateral trade costs by including the absolute value of the differences in log per capita income between trading partners (“Linder Income”) and also differences in income dispersion as measured by absolute value of differences in the gini coefficient (“Linder Gini”). Consistent with Proposition 6 the coefficient on the Linder Income variable is negative and significant. Similarly the negative and significant coefficient on Linder Gini matches the prediction of Proposition 7.\footnote{Note that it is important to correct for bias using the CANOVA methodology. The typical one step procedure generates a positive and significant coefficient on “Linder Income”, matching the finding of Hallak (2010).}

It is worth emphasizing that Proposition 7 is solely due to variation in per capita income within a country. Without this variation, the model reduces to the standard gravity prediction – a result that arises because the first best, linear price and indirect discrimination (without income heterogeneity) models all generate the same equilibrium outcomes. The key reason is that neither of the formulations of imperfect competition distorts production or consumption decisions in a single sector setting.

Adding a second sector provides scope for market power in the monopolistically competitive sector to generate a production distortion. With linear pricing, this distortion reduces the sector’s size since price is above marginal cost. Nevertheless, the distortion is invariant to the distribution of income. This characteristic doesn’t hold for the indirect discrimination model. Exploring the implications for industrial composition and other features of the equilibrium is the topic of the next section.
Table 1: Decomposition of CANOVA Trade Costs for 2005

<table>
<thead>
<tr>
<th></th>
<th>Trade Costs (1)</th>
<th>Trade Costs (2)</th>
<th>Trade Costs (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linder Income</strong></td>
<td>-0.47***</td>
<td>-0.44***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.15)</td>
<td></td>
</tr>
<tr>
<td><strong>Linder Gini</strong></td>
<td>-4.95***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.06)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Miles: 0 - 374</strong></td>
<td>-3.42***</td>
<td>-2.89***</td>
<td>-2.70***</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td>(0.35)</td>
<td>(0.36)</td>
</tr>
<tr>
<td>375 - 749</td>
<td>-4.40***</td>
<td>-3.91***</td>
<td>-3.70***</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
<td>(0.28)</td>
<td>(0.30)</td>
</tr>
<tr>
<td>750 - 1499</td>
<td>-5.26***</td>
<td>-4.67***</td>
<td>-4.42***</td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(0.30)</td>
<td>(0.32)</td>
</tr>
<tr>
<td>1500 - 2999</td>
<td>-5.70***</td>
<td>-5.07***</td>
<td>-4.84***</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.28)</td>
<td>(0.29)</td>
</tr>
<tr>
<td>3000 - 5999</td>
<td>-6.35***</td>
<td>-5.71***</td>
<td>-5.39***</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.26)</td>
<td>(0.27)</td>
</tr>
<tr>
<td>Over 6000</td>
<td>-6.89***</td>
<td>-6.26***</td>
<td>-5.92***</td>
</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td>(0.25)</td>
<td>(0.27)</td>
</tr>
<tr>
<td><strong>Contiguity</strong></td>
<td>0.56***</td>
<td>0.46***</td>
<td>0.40***</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.15)</td>
<td>(0.15)</td>
</tr>
<tr>
<td><strong>Language</strong></td>
<td>1.62***</td>
<td>1.62***</td>
<td>1.61***</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.16)</td>
<td>(0.15)</td>
</tr>
<tr>
<td><strong>Colonial</strong></td>
<td>-0.66**</td>
<td>-0.68***</td>
<td>-0.68***</td>
</tr>
<tr>
<td></td>
<td>(0.26)</td>
<td>(0.25)</td>
<td>(0.25)</td>
</tr>
<tr>
<td><strong>PTA</strong></td>
<td>-0.12</td>
<td>-0.25</td>
<td>-0.21</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.17)</td>
<td>(0.20)</td>
</tr>
<tr>
<td><strong>Legal</strong></td>
<td>0.60***</td>
<td>0.48**</td>
<td>0.48**</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.19)</td>
<td>(0.19)</td>
</tr>
<tr>
<td><strong>Currency</strong></td>
<td>1.72***</td>
<td>1.55***</td>
<td>1.36***</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.17)</td>
<td>(0.16)</td>
</tr>
</tbody>
</table>

| Observations | 1,600 | 1,600 | 1,600 |
| R-squared    | 0.98  | 0.98  | 0.98  |
| Importer fe  | y     | y     | y     |
| Exporter fe  | y     | y     | y     |
| Specification| PPML  | PPML  | PPML  |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
4 Two Sector Model

Once again we follow convention in the literature and assume the second sector, $Y$, is a homogeneous good and the upper tier utility is Cobb-Douglas:

$$U = \left( \frac{Q}{\gamma} \right)^{\gamma} \left( \frac{Y}{1-\gamma} \right)^{1-\gamma}, \quad \gamma \in (0,1)$$

Assuming that $Y$ is supplied by a perfectly competitive industry and choosing this sectors output as the numeraire, the demand for each sector is then given by:

$$Y^I = (1-\gamma)\bar{m}^I, \quad Q^I = \frac{\gamma\bar{m}^I}{p} \quad \text{where} \quad p^I = \left[ \sum_i p_i \frac{\rho}{\rho-1} \right]^{\frac{\rho-1}{\rho}}$$

These expressions are familiar from the usual analysis. However, there is an additional feature to highlight. The Cobb-Douglas specification normally results in fixed budget shares, ruling out cross price effects. However, since demand in each sector is a function of $\bar{m}^I$, a model with indirect discrimination potentially has cross price effects and non-constant budget shares from gross income. In particular, this framework can mimic the non-linear Engel curves that motivate the use of non-homothetic preferences, as depicted in figure 4.

In this expanded model demand for variety, $i$, targeted to consumers with income level $I$, is given by:

$$p_i^I = \theta^I q_i^{\rho-1} \quad \text{with} \quad \theta^I = \frac{\gamma\bar{m}^I}{Q^I}$$

The two sector model can be applied to a number of trade issues since it maps directly into the conventional synthesis framework of Krugman and Helpman 1985. When there is only a single factor of production a positive trade cost is required for the location of production to be determined – generating a home market effect: the country with the larger national income will be home to a disproportionate share of firms in the $Q$ sector. This implies that the distribution of income within a country does not effect the location decisions of a firm – absolute market size is the only dimension that matters. The indirect discrimi-
nation model augments this analysis since, as we will now show, a location preference can arise even if both countries have the same per capita income and population size (and hence same aggregate income).

4.1 One Factor Model: Home Market Effect

As a starting point consider a situation where trade barriers are sufficiently high that both locations have a diversified industrial structure. To derive the value function of a firm located in country \(i\) we can once again use the property that the total revenue function is homogeneous of degree \(\rho\) in the vector of production designs \(\{q_{ji}\}\), \(\sum_j \sum_I \frac{\partial TR_i}{\partial q_{ji}} q_{ji} = \rho TR_i\). For designs serving local consumers, \(\frac{\partial TR_i}{\partial q_{ii}} = \beta_I w\) holds at an optimum while exports must

\[24\text{As usual, assume that } Y\text{ is freely traded.}\]
satisfy \( \frac{\partial TR_i}{\partial q_{ji}} = \tau \beta_j^l w \). This allows the value function to be written as:

\[
\pi_i^* = \frac{\sum \beta_i^l wq_{ii}^l + \sum \beta_j^l w\tau q_{ji}^l}{\sigma - 1} - wF \\
= \frac{\left(\sum \beta_i^l w\phi_i^lH\right) q_{ii}^H + \left(\sum \beta_j^l w\tau \phi_j^lH\right) q_{ji}^H}{\sigma - 1} - wF \\
= \frac{\left(\sum \beta_i^l w\phi_i^lH\right) q_{ii}^H + \left(\sum \beta_j^l w\tau \phi_j^lH\right) \tau^{1-\sigma} q_{ji}^H}{\sigma - 1} - wF
\]

Setting this equal to zero and using \( q_{ii}^H = \frac{\gamma \bar{m}_i^H}{n_i + \tau^{1-\sigma} n_j} \) we have

\[
\left(\sum \beta_i^l \phi_i^lH\right) \frac{\gamma \bar{m}_i^H}{(n_i + \tau^{1-\sigma} n_j)} + \left(\sum \beta_j^l \phi_j^lH\right) \frac{\gamma \bar{m}_j^H \tau^{1-\sigma}}{(\tau^{1-\sigma} n_i + n_j)} - F(\sigma - 1) = 0
\]

If we define local market size as \( \Omega_i = \left(\sum \beta_i^l \phi_i^lH\right) \gamma \bar{m}_i^H \) then we can write the system in the more familiar:

\[
\frac{\Omega_i}{(n_i + \tau^{1-\sigma} n_j)} + \frac{\Omega_j \tau^{1-\sigma}}{(\tau^{1-\sigma} n_i + n_j)} - F(\sigma - 1) = 0 \tag{38}
\]

From which it follows the country with the larger \( \Omega_i \) is also home to disproportionately more firms in the differentiated goods sector – the home market effect. This tells us that settings like those in Proposition 3, where the rank of a country’s \( \Omega_i \) follows its rank of income per capita, will exhibit a home market effect. However, a home effect is also evident when the two countries have the same average income, and therefore aggregate income.

**PROPOSITION 8.** If Home’s income distribution is a mean preserving spread of Foreign’s income distribution (i.e. \( \beta_h > \beta_f \)), then \( n_f > n_h \) for all \( \tau \). Consequently, the country with the lower dispersion of income will be the net exporter of the differentiated good.

(See appendix for proof.)

This proposition follows from the observation that adding income dispersion while holding mean income constant undermines the rent extracting abilities of a firm. When a population is homogeneous, there are no outside options for the highest income consumer. As consumer heterogeneity is increased, outside options emerge and progressively become
more important. Nevertheless, national income is identical in both locations, but firms will be disproportionately located in the country with lower income dispersion.

This outcome does not necessarily arise in a model with non-homothetic preferences. Consider the Stone-Geary utility function, a specification frequently adopted to allow for income elasticities that diverge from unity. It can be shown in this setting that when no consumer is constrained by the subsistence quantity in either sector, then reallocating income in a mean preserving manner does not alter demand for any product (see for example Markusen (2013)). Consequently, firms have no location preference based on income dispersion. A preference can arise for certain types of firms in the model of Fajgelbaum et al. (2011) when countries income dispersion can be ordered in a mean preserving manner. They consider a model with two sectors, where firms endogenously choose to be high or low quality in a free entry equilibrium. A key distinction between low and high quality goods is that the degree of substitution within the low quality segment is greater than within the high quality segment. Under these assumptions high quality firms will have a preference to locate in the high dispersion country if the expenditure share of the high quality product is relatively small for all income groups. The mechanism relies on the decrease in demand for the high quality goods by the lower income groups being less than the increase in the demand for the now even higher income consumers. This occurs if the expenditure on the high quality good is already relatively low (little scope for reductions at the lower end of the income distribution but scope for an increase on the high end). Hence, a non-homothetic specification does not generate a location preference based solely on differences in income dispersion across locations, since they require either a subsistence constraint to bind or differences in the elasticity of substitution.

The key to proposition 8 is that the greater income dispersion, the larger the incentive for a typical firm to distort consumption decisions. In a single sector setting these distortions only affected the allocation of output across consumer types. In a multi-sector setting, the distortions now influence the relative size of the sectors which generates a location bias with trade costs even when countries have exactly the same aggregate income. However, the implications of the variation in the distortion go beyond locational incentives and can
also have important implications for factor markets. To explore these issues we enrich the supply side of the economy by introducing a second factor of production and ask how factor returns are effected by the integration of countries that have different income distributions.

### 4.2 Two factors

To explore this issue consider a two factor model with relative factor prices of \( \omega \). We make the typical assumptions relating to technology in the two sectors, with the additional simplifying assumption about the cost functions in the \( Q \) sector: the marginal and fixed costs have the same factor intensity, \( c_q(\omega) \) and \( Fc_q(\omega) \). To help fix ideas consider the following parametrized marginal cost functions:

\[
c_y(\omega) = \omega^\eta \\
c_q(\omega) = \omega^{1-\eta} \text{ where } \eta \neq \frac{1}{2}
\]

The standard formulation of this model couches discussion of factor price relationships in terms of the Stolper-Samuelson Theorem. To isolate the new mechanism consider a situation where both countries have the same relative endowment. In addition assume that all types hold the same ratio of each factor, with the difference across types just reflected in the level of holdings. So once again countries are distinguished by the relative frequency of each type in the local population. This completely suppresses the standard Stolper-Samuelson channels since earnings within a country are perfectly correlated and factor proportions are the same across countries.

Insight into the behavior of factor prices in this setting can be gained by considering the outcomes when an economy is populated by a single type, \( \beta^I = 1 \). When the population is homogeneous, a typical firm has no incentive to distort any products sold. As a result, when \( \beta^I = 1 \), the first best is achieved and relative factor prices are also first best. Now add a small amount of heterogeneity when \( \beta^H = 1 \) or \( \beta^L = 1 \). The optimal response of the firm is to offer a product line that involves a degraded product being offered to the low type. However, the overall distortion will be relatively small - though the reason is different in each instance. If \( \beta^L = 1 \), then adding some high types only causes a small distortion
to the design of the product since the market is dominated by the low types. In contrast, when starting at $\beta^H = 1$ the distortion imposed on the low type is relatively large, but the aggregate impact is relatively small since there are so few of the low type in the population. In both cases the deviation from the first best is small but in the same direction – too few resources are devoted to the differentiated goods sector. This occurs not because the scale of any individual firm is too small, but because too few firms enter in equilibrium. That is, consumer heterogeneity undermines the ability of a firm to extract rents relative to the first best. With the differentiated goods sector smaller than the first best, then the demand for the factor used intensively in that sector will also be lower than the first best, which will cause the relative return to that factor to be lower as well.

This discussion implies that the relationship between the aggregate distortion and $\beta^I$ will be non-monotonic. That is, the aggregate distortion is composed of two elements: the size of the distortions and the frequency of types in the population. These forces are negatively related, the higher is $\beta^I$ the smaller the deviation from first best for type $I$, i.e. $\beta^I(\alpha^H - \phi^H)$. Since the distortion is minimized when $\beta^I = 1$, it follows that there must exist a $\beta^{L^*} < 1$, $\beta^{M^*} \geq 0$ and $\beta^{H^*} < 1$ where the aggregate distortion is maximized. Call this distribution $\beta^* = \{\beta^{L^*}, \beta^{M^*}, \beta^{H^*}\}$. Since the distortion is non-monotonic so is the deviation of relative factor prices from the first best. The relationship between factor prices and income distribution is depicted in Figure 5.

It then follows that the largest impact on relative factor prices is associated with integration of two countries with very different per capita incomes. Moreover, they will have a change in relative factor prices that are positively correlated. As Figure 5 makes clear, differences in income distribution can give rise to a rich set of possibilities. These are summarized in the following proposition.

**PROPOSITION 9.** If two countries with the same relative endowments integrate, then the response of factor prices are determined by the ranking of the $\beta^I$'s across countries and in relationship to $\beta^{I^*}$. If $\beta^H_i < \beta^H_j < \beta^{H^*}$, $\beta^L_i < \beta^L_j < \beta^{L^*}$, or $\beta^H_i > \beta^H_j > \beta^{H^*}$, $\beta^L_i > \beta^L_j > \beta^{L^*}$ and countries have the same mean income then $\text{corr}(\Delta \omega_f, \Delta \omega_h) < 0$. If $\beta^H_i > \beta^{H^*}$, $\beta^L_i < \beta^{L^*}$ while $\beta^H_j < \beta^{H^*}$, $\beta^L_j > \beta^{L^*}$ then $\text{corr}(\Delta \omega_f, \Delta \omega_h) > 0$. 38
Broadly this proposition says that when countries differ in terms of per capita income, then the change in relative factor prices is likely to be positively correlated. In contrast, countries with similar per capita incomes but differences in income dispersion are likely to have a negative correlation in factor price changes. The richness predicted by this proposition provides an additional perspective on the debate surrounding the globalization and income inequality. By construction income inequality does not change as countries integrate but relative factor prices do. Changes in the factor price distribution has received much attention due to the relative availability of data (as opposed to the issues associated with measuring income distribution) and a clear theoretical benchmark in the Stolper-Samuelson theorem. Concentrating on the behaviour of production/non-production wages (skill premium) in manufacturing a typical finding is that contrary to Stolper-Samuelson, the skill premium has not unambiguously decreased in labor abundant countries (developing countries). As emphasized by Zhu and Trefler (2005) in their sample of 20 developing and new industrialized countries, increased globalization has generated a roughly even split between rising and falling skill premium.
To date the literature has focused on identifying a mechanism to explain a positive correlation between relative factor price changes across the north and the south (FDI in Feenstra and Hanson and southern catch-up in Zhu and Trefler (2005)) but these models tend to provide a monotonic prediction. Hence, the choice appears to be between models that emphasize augmented Stolper-Samuelson mechanism (Mitra-Trindade in a model of non-homotheticities) or industrial migration due to technological change/transfer. Proposition 9 complements these frameworks by suggesting the difference or similarity in income distributions can alter underlying inefficiencies in a way that can have a positive or negative impact on the correlation of relative factor price change across locations. So while FDI, catch-up and product price changes are all no doubt relevant it might also be likely that the income distribution of the trading partners that integrate may also play are role in influencing relative factor prices.

5 Conclusion

This paper expands the strategy space of firms to include the possibility of designing product lines, introducing a role for income distribution in the standard model of international trade. This enriches firm behavior in a way that is present in the closed economy literature on firm strategy. However, in contrast to much of the previous closed economy literature, the current setting involves both within sector competition and a general equilibrium perspective. The resulting model is both tractable and provides a link between the distribution of income and the gains from trade. The link is related to firm strategy that indirectly discriminates between the various income classes, which in equilibrium results in a product line that differs from the first best allocation. Since the distortions have the largest negative welfare impact at the lower end of the income distribution, this is where the consequences of international integration are also most pronounced. However, trade can mitigate these welfare losses in countries with a “good” distribution of income, while amplifying them in countries with a “bad” distribution of income. These findings imply even more magnified changes under a process of gradual liberalization since the variety and design dimensions of welfare change
respond differentially to the level of trade barriers. In particular, design changes occur disproportionately at lower trade barriers, with the potential to derail the process of trade liberalization.

Aside from the welfare impact, the model offers a perspective on a range of empirical patterns that have previously been deemed inconsistent with the standard model of international trade based on identical and homothetic preferences. In particular, a number of findings on import prices and trade flows are consistent with the model. Additionally, the product line approach adopted also predicts a rich set of responses for factor prices from international integration that reflects the richness documented in studies of trade and income inequality. All these results follow from enriching firm strategy, rather than altering the structure of consumer preferences.
6 Appendix

6.1 Proof of Proposition 1

It is slightly simpler to work with \( \{\phi^{LM}, \phi^{MH}\} \), which must satisfy:

\[
\beta^H \phi^{MH\rho} + \beta^M \phi^{MH} = (\beta^M + \beta^H) \left( \frac{1 - \alpha}{\alpha} \right) \left( 1 - \phi^{MH\rho} \right) \tag{40}
\]

\[
(1 - \beta^L) \phi^{LM\rho} + \beta^L \phi^{LM} \phi^{MH} \left( 1 - \phi^{LM\rho} \right) = \left( \frac{1 - \alpha}{\alpha} \right) \left( 1 - \phi^{LM\rho} \right) \tag{41}
\]

Equation (40) implicitly defines a function \( M(\phi^{LM}, \phi^{MH}) = 0 \) and similarly for equation (41), \( L(\phi^{LM}, \phi^{MH}) = 0 \). Begin by noting that the slope of the first condition is positive while the slope of the second is negative. That is,

\[
\frac{d\phi^{MH}}{d\phi^{LM}} \bigg|_{M(\phi^{LM}, \phi^{MH})=0} = -\frac{\partial M}{\partial \phi^{LM}}/\partial \phi^{MH} > 0
\]

where

\[
\frac{\partial M}{\partial \phi^{LM}} = -\rho \phi^{LM\rho-1}(1 - \beta^L)\pi_{MH} < 0
\]

\[
\frac{\partial M}{\partial \phi^{MH}} = (\beta^M + \beta^H)\rho \phi^{MH\rho-1}(1 - \phi^{LM\rho}) + \rho \phi^{MH\rho-1}(1 - \beta^L) > 0
\]

\[
\frac{d\phi^{MH}}{d\phi^{LM}} \bigg|_{L(\phi^{LM}, \phi^{MH})=0} = -\frac{\partial L}{\partial \phi^{LM}}/\partial \phi^{MH} < 0
\]

where

\[
\frac{\partial L}{\partial \phi^{LM}} = (1 - \beta^L)\rho \phi^{LM\rho-1} + \frac{(1 - \beta^L)\rho \phi^{LM\rho-1}}{1 - \phi^{LM\rho}} + \frac{(1 - \beta^L)\phi^{MH}(1 - \phi^{LM\rho})}{1 - \phi^{MH\rho}} > 0
\]

\[
\frac{\partial L}{\partial \phi^{MH}} = \rho \phi^{MH\rho-1} \beta^L \phi^{LM} \phi^{MH}(1 - \phi^{LM\rho}) > 0
\]

Furthermore, \( L(\phi^{LM}, \phi^{MH}) = 0 \) implies that when \( \phi^{MH} = 0 \Rightarrow \phi^{LM\rho} = \frac{(1 - \alpha)}{(1 - \alpha) + \alpha(1 - \beta^L)} \leq 1 \), while \( \phi^{LM} = 0 \Rightarrow \phi^{MH} = 1 \). Using, \( M(\phi^{LM}, \phi^{MH}) = 0 \) implies that when \( \phi^{LM} = 0 \Rightarrow \phi^{MH\rho} + \phi^{MH} = \frac{(1 - \beta^L)}{(1 - \beta^L - \beta^H)} \leq 1 \). This allows the conditions to be plotted as:

6.2 Proof of Proposition 2

The first best product for income group \( I \) is \( q_1^I = \frac{\rho n l}{\pi \bar{w}} \). The product offered under indirect discrimination to type \( I \) is \( q^I = \frac{\bar{w} l}{\rho n \pi} \). From (17) and (22) we have \( q^L = \frac{\rho(1 - \alpha)}{n} \left( \frac{\phi^{LMH}}{\pi \pi} \right) \). From (16) it follows that \( \bar{\pi}^{LH} \in [\phi^{LH}, \phi^{LHM}] \) where the lower bound only arises if \( \beta^L = 1 \). Hence, \( q_1^L > q^L \) for \( \beta^L < 1 \).

Since \( q_1^L > q^L \), it must be the case that \( q^H > q_1^H \) and/or \( q^M > q_1^M \) (since firm scale is first best under indirect discrimination). Note that \( q^H > q_1^H \) requires \( \frac{\rho \alpha}{n(1 - \phi^{MH\rho})} > \frac{\rho(1 + \alpha)}{n} \) which
implies $1 - \phi^{MH}\rho - \alpha \phi^{MH}\rho < 0$. In contrast, $q^M > q^M_1$ requires $\frac{\beta\alpha}{n(1 - \phi^{LM}\rho)} (\frac{\phi^{LM}}{\rho}) > \frac{\rho}{n}$ which implies $1 - \phi^{MH}\rho - \alpha \phi^{MH} < 0$. So whenever $q^M > q^M_1$ then it must be the case that $q^H > q^H_1$ - which occurs if $\beta^H$ is sufficiently small. Finally, if $q^M \leq q^M_1$ then $q^H > q^H_1 \forall \beta^H \in [0, 1)$.

### 6.3 Proof of Proposition 3

Start by defining the set of distributions $\{b^L, b^M, b^H\}$ that are likelihood ratio dominated by $\beta \equiv \{\beta^L, \beta^M, \beta^H\}$. LRD requires $\frac{\beta^L}{b^L} \leq \frac{\beta^M}{b^M} \leq \frac{\beta^H}{b^H}$. The first part of this equality implies:

$$\frac{\beta^L}{b^L} \leq \frac{\beta^M}{b^M} \Rightarrow b^H \geq 1 - \left(1 - \frac{\beta^H}{\beta^L}\right) b^L$$

While the second part implies:

$$\frac{\beta^M}{b^M} \leq \frac{\beta^H}{b^H} \Rightarrow b^H \leq \frac{\beta^H}{1 - \beta^L} (1 - b^L)$$

This set is illustrated by the dashed lines in Figure 2.

Note that (28)-(30) implies the following differences:

$$\hat{U}^L - \hat{U}^M = \hat{\phi}^{LM}$$

$$\hat{U}^M - \hat{U}^H = \hat{\phi}^{MH}$$

Consequently the proposition requires that $\hat{U}^L > 0$ and $\hat{U}^H > 0$ when a country with an income distribution $\beta$ integrates with an country in the LRD set. To characterize the behavior of $\phi^{MH}$ consider the level sets $\phi^{MH}(\beta^L, \beta^H)$. The slope of a contour is given by:

$$\frac{d\beta^H}{d\beta^L} = -\frac{d\phi^{MH}/d\beta^L}{d\phi^{MH}/d\beta^H}$$

To evaluate the RHS use the system:

$$M(\beta, \phi) \equiv \beta^H \phi^{MH}\rho + (1 - \beta^L - \beta^H)\phi^{MH} - (1 - \beta^L) \left(\frac{1 - \phi^{MH}\rho}{1 - \phi^{LM}\rho}\right) = 0$$

$$L(\beta, \phi) \equiv (1 - \beta^L)\phi^{LM}\rho + \beta^L \phi^{LM}\phi^{MH} \left(\frac{1 - \phi^{LM}\rho}{1 - \phi^{MH}\rho}\right) - \left(\frac{1 - \phi^{LM}\rho}{\alpha}\right) (1 - \alpha) = 0$$
Therefore
\[
\frac{d\beta^H}{d\beta^L} = -\left(\frac{-M_{\beta^H}L_{\phi^LM} + L_{\beta^L}M_{\phi^LM}}{M_{\beta^H}L_{\phi^LM}}\right) = -\frac{\beta^H}{1 - \beta^L} + \frac{L_{\beta^L}M_{\phi^LM}}{M_{\beta^H}L_{\phi^LM}}
\]

Since \(L_{\beta^L} < 0\), \(M_{\phi^LM} < 0\) while \(M_{\beta^H} > 0\), \(L_{\phi^LM} > 0\), the second term on the RHS is positive, which implies the slope of the contour is decreasing in \(\beta^L\) but always greater than \(-\frac{\beta^H}{1 - \beta^L}\).

The figure below depicts the \(\phi^{MH}\) contours and the associated LRD set. Consequently, integration with any country which is LRD will result in a global income distribution that is also LRD and consequently lead to an increase in \(\phi^{MH}\) relative to autarky.

Since LRD implies \(\phi^{MH} > 0\), confirming that the gains for the middle type are greater than the gains for the high type. The ranking of the low and middle types requires integration to yield \(\hat{\phi}^{LM} > 0\). To see that this is also implied by LRD, consider that the slope of the \(\phi^{LM}\) contour is given by:
\[
\frac{d\beta^H}{d\beta^L} = -\left(\frac{-L_{\beta^L}M_{\phi^{LM}} + M_{\beta^L}L_{\phi^{MH}}}{M_{\beta^H}L_{\phi^{MH}}}\right) = -\frac{\beta^H}{1 - \beta^L} + \frac{L_{\beta^L}M_{\phi^{MH}}}{M_{\beta^H}L_{\phi^{MH}}}
\]

Since \(L_{\beta^L} < 0\), this implies that the second term is negative and less than \(-\frac{\beta^H}{1 - \beta^L}\). This implies that moving along the constraint \(b^H = \frac{\beta^H}{1 - \beta^L}(1 - b^L)\) from \(\beta\) results in higher \(\phi^{LM}\). However, if the slope of the contour is greater than \(-\frac{1 - \beta^L}{\beta^H}\), moving along \(b^H = 1 - \left(\frac{1 - \beta^H}{\beta^L}\right)b^L\) results in lower \(\phi^{LM}\). To see that this doesn’t occur, consider the slope of the contour when \(b^H = 0\). This implies \(\phi^{MH} = \hat{\alpha}^{MH}\) and the slope of the contour line when \(b^H = 0\) can be expressed as:
\[
-\left(\frac{1 - \beta^H}{\beta^L}\right) \left(\frac{1 - \beta^L - \beta^H}{1 - \beta^H}\right) \left(\frac{\phi^{LM\rho}(b^L) - \phi^{LM}(b^L)}{\phi^{LM\rho}(b^L)\phi^{LM}(b^L)}\right)
\]

To ensure that this is less than \(-\frac{1 - \beta^H}{\beta^L}\) requires \(\left(\frac{1 - \beta^L - \beta^H}{1 - \beta^H}\right) \left(\frac{\phi^{LM\rho}(b^L) - \phi^{LM}(b^L)}{\phi^{LM\rho}(b^L)\phi^{LM}(b^L)}\right) \geq 1\).

Writing this condition as:
\[
(1 - \beta^H)(\phi^{LM\rho} - \phi^{LM} - \phi^{LM\rho}\phi^{LM}) \geq \beta^L(\phi^{LM\rho} - \phi^{LM})
\]

This holds when \(\beta^L = 0\). Holding \(\beta^H\) constant, as \(\beta^L\) increases the LHS decreases while the RHS increases. This means if this condition fails for any \(\beta^L\) it will also fail for \(\beta^L = (1 - \beta^H)\). To evaluate this limit note that as \(\beta^L \to 1 - \beta^H\) we know from \(M(\beta, \phi) \equiv 0\) that \(\phi^{LM} \to 1\). This implies that there are really only two types in this setting, \(L\) and \(H\), and it is straightforward to show that the low types welfare increases proportionally more than the high types as \(\beta^L\) increases.

Hence LRD implies that the dominant country gains more than the standard model and the gains are proportionately higher the lower is the income.
6.4 Proof of Proposition 5

The central claim is that there exists a trade cost, $\tau$, such that above this trade cost the gains from trade are equivalent to the standard model and below that level the gains are manifestly different. Begin by considering trade costs that are sufficiently high that markets are segmented. In order for the gains from trade to be the same as the standard model we require that relative product design is not altered by trade barriers. From Proposition 1 we know that each isolated market has a unique equilibrium with the relative design in each market given by $\{\phi_{i,x}\}$. If firms from $j$ ship to $i$ then the first order conditions under segmentation are:

$$\frac{\partial \pi_j}{\partial q_{ij}^H} = \theta_i^H q_{ij}^{H-1} - \tau w_j = 0$$ (42)

$$\frac{\partial \pi_j}{\partial q_{ij}^M} = ((\beta_i^M + \beta_i^H) \theta_i^M - \beta_i^H \theta_i^H) q_{ij}^{M-1} - \beta_i^M \tau w_j = 0$$ (43)

$$\frac{\partial \pi_j}{\partial q_{ij}^L} = (\theta_i^L - (1 - \beta_i^L) \theta_i^M) q_{ij}^{L-1} - \beta_i^L \tau w_j = 0$$ (44)

However, it is immediately apparent that combining (42) and (43) and along with (42) and (44) reproduces the equilibrium conditions (14) and (15) which is solely a function of the distribution of income in country $i$ and $\rho$. Consequently, under segmentation firms from both locations offer the product line $\{\phi_{i,x}\}$ in country $i$.

To show the existence of $\bar{\tau}$, note that it is the location of the outside option which is relevant – i.e. for the income class immediately below type $I$, is the next best option within a product line local or not (this also implies that we should focus on the incentive constraints). Under segmentation the next best option is always strictly the local option. To illustrate the existence of $\bar{\tau}$ consider a setting where $\phi_{fI}^{LM} > \phi_{hI}^{LM}$ (as would arise under the conditions of Proposition 3) which implies a that within a Foreign firm’s product line the product designed for the low income consumer in the Foreign country is superior to the product designed for the low income consumer in the Home country. Since the information rents of the middle income consumer are determined by the product offered to low type, the relevant no arbitrage condition is:

$$\theta_{h}^M \frac{q_{hf}^{L}}{\rho} - T_{hf}^L > \theta_{h}^M \frac{q_{ff}^{L}}{\rho \tau^L} - T_{ff}^L$$

$$\frac{(\theta_{h}^M - \theta_{h}^L) q_{hf}^L}{\rho} > \frac{(\theta_{h}^M \theta_{h}^H - \theta_{f}^L) q_{ff}^{L}}{\rho}$$

$$\frac{(\theta_{h}^M - \theta_{h}^L) \phi_{h}^{LHP} \theta_{h}^H q_{hf}^{H}}{\rho} > \left( \frac{(\theta_{h}^M \theta_{h}^H 1 - \theta_{f}^L) \phi_{f}^{LHP} \theta_{f}^H q_{ff}^{L}}{\rho} \right)$$

$$\frac{(\theta_{h}^M - \theta_{h}^L) \phi_{h}^{LHP} \theta_{h}^H \tau}{\rho} > \left( \frac{(\theta_{h}^M \theta_{h}^H 1 - \theta_{f}^L) \phi_{f}^{LHP} \theta_{f}^H \tau}{\rho} \right)$$ (45)

Under segmentation the relative positions of the residual demand curves within a market are invariant to the trade cost, which implies the LHS of (45) is increasing in $\tau$. Using
the balanced trade condition it can be shown that \( \frac{\partial\theta^H}{\partial\tau^H} = \frac{1}{1 - \phi^M_{ij}} \left( 1 - \phi^M_{ij} \right)^{-\frac{1}{\rho}} \) which is decreasing in \( \tau \). Given the LHS is monotonically increasing in \( \tau \) while the RHS is monotonically decreasing, this implies that there must exist a \( \bar{\tau} \) such that a middle income Home consumer will find it attractive to arbitrage within a product line and purchase the product designed for the low income Foreign consumer. However, this implies that below \( \bar{\tau} \) the low income markets in the Foreign firm’s product line are no longer segmented. The optimal design for low income groups are now linked for a Foreign firm as (45) binds for \( \tau < \bar{\tau} \), resulting in \( q^H_{lf} = \left( \frac{\theta^M_{lf} - \theta^H_{lf}}{\theta^M_{hf} - \theta^H_{hf}} \right) q^L_{hf} \).

Note that it is not necessarily the case that at \( \bar{\tau} \) the low income markets are integrated within a Home firm’s product line – it is possible that they are still segmented. This implies that product design below \( \bar{\tau} \) will vary by income group, location and firm nationality – so the relative design for income group \( I \), in country \( i \), by firm \( j \) is \( \phi^H_{ij} \).

The gains from trade for a member of a high income group are:

\[
U^H_{iT} = \frac{1}{n_i^H} q^H_{ii} + \frac{1}{n_i^H} q^H_{ij} \quad \& \quad \rho T^H_{ij} = (1 - \phi^M_{ij})d_i n_i^H + \rho T^M_{ij}
\]

\[
\rho(n_i T^H_{ij} + n_j T^H_{ij}) = \bar{m}_i^H \left( \frac{n_i}{\bar{n}_i} + \frac{d_i n_i}{\bar{n}_i} \right) - \bar{m}_i^H \left( \frac{n_i}{\bar{n}_i} \phi^M_{ij} + \frac{d_i n_i}{\bar{n}_i} \phi^M_{ij} \right) + \rho \bar{m}_i^M
\]

\[
\Rightarrow \bar{m}_i^H = \frac{\rho(m_i^H - \bar{m}_i^M)}{1 - \phi^M_{ij}}
\]

Hence

\[
\frac{U^H_{iT}}{U^H_{iA}} = \left( \frac{1 - \phi^M_{iA}}{1 - \phi^M_{ij}} \right) \left( \frac{n_i}{\bar{n}_i} \right)^{1 - \rho}
\]

While the gains for someone with income \( I \) are:

\[
\frac{U^I_{iT}}{U^I_{iA}} = \left( \frac{n_i q^I_{ii} + n_j q^I_{ij}}{n_i^H q^H_{ii} + n_j^H q^H_{ij}} \right)^{\frac{1}{\rho}} = \left( \frac{n_i \phi^I_{ii} q^H_{ii} + n_j \phi^I_{ij} q^H_{ij}}{n_i^H \phi^I_{iA} q^H_{iA}} \right)^{\frac{1}{\rho}}
\]

\[
= \frac{\phi^I_{iA} \bar{n}_i^H}{\phi^H_{iA} n_i^H} \cdot \frac{\phi^I_{ij}}{q^I_{ij}}
\]
6.5 Derivation of equation (34)

Assume that trade costs are sufficiently small that all markets are partially integrated – home high views foreign middle as an outside option and foreign middle views home lows product as an outside option. This implies that the foreign middle income consumer captures information rents:

$$\theta_f^M q_{fj}^M - \rho T_{fj}^M = \theta_f^M \left( \frac{q_{hj}^L}{\tau} \right)^\rho - \rho T_{hj}^L$$

Which must also be reflected in the information rents captured by the home high income group:

$$\theta_h^H q_{hj}^H - \rho T_{hj}^H = \theta_h^H \left( \frac{q_{hj}^M}{\tau} \right)^\rho - \rho T_{hj}^M$$

$$(\theta_h^H - \theta_h^M) q_{hj}^M + (\theta_h^M - \theta_h^L) q_{hj}^L = \left( \frac{\theta_h^H}{\tau} - \theta_f^M \right) q_{fj}^M + \left( \frac{\theta_f^M}{\tau} - \theta_h^H \right) q_{hj}^L$$

$$\Rightarrow q_{fj}^M = \frac{(\theta_h^H - \theta_h^M)}{(\theta_h^H/\tau - \theta_f^M)} q_{hj}^M + \frac{(\theta_f^M - \theta_h^H / \tau)}{(\theta_h^H/\tau - \theta_f^M)} q_{hj}^L$$

$$= \gamma_M q_{hj}^M + \gamma_L q_{hj}^L$$

$$Q_{fj}^M = n(q_{fj}^M + q_{fj}^H)$$

$$= n(\gamma_M q_{hj}^M + \gamma_L q_{hj}^L + \gamma_M q_{hj}^M + \gamma_L q_{hj}^L)$$

$$= \gamma_M Q_{hj}^M + \gamma_L Q_{hj}^L$$

$$= Q_{hj}^H \left( \gamma_M \phi_{hj}^{MH} + \gamma_L \phi_{hj}^{LH} \right)$$

$$m_f^M = \frac{\rho(m_f^M - m_f^L)}{1 - \phi_{hj}^{LM} / \tau}$$

where

$$\phi_{hj}^{LM} = \frac{\phi_{hj}^{LM}}{\gamma_M + \gamma_L \phi_{hj}^{LM}}$$

$$\phi_{hj}^{LM} = s_f\phi_{hj}^{LM} + (1 - s_f)\phi_{hj}^{LM}$$
\[ T_{fh}^{M} = \theta_f^M q_{fh}^M - \theta_f^M \left( \frac{q_{hh}^L}{\tau} \right)^{\rho} + T_{hh}^L \]
\[ = \frac{\bar{m}_f^M}{Q_f^M} \left( q_{fh}^M - \left( \frac{q_{hh}^L}{\tau} \right)^{\rho} \right) + T_{hh}^L \]
\[ = \frac{\bar{m}_f^M}{\bar{n}_h q_{hh}^H \rho} \left( \frac{\bar{m}_h^{MP} - \bar{m}_h^{LP}}{\bar{m}_h^{MP}} \right) \left( 1 - \left( \frac{q_{hh}^L}{\bar{n}_f^M / \tau^{\rho}} \right) \right) + T_{hh}^L \]

\[ \rho (m_f^M - m_h^L) \left( \frac{\gamma M \phi_{hh}^{MP} + \gamma L \phi_{hh}^{LP}}{\gamma M \phi_{hh}^{MP} + \gamma L \phi_{hh}^{LP}} \right) \left( 1 - \phi_{hh}^{LM} / \tau^{\rho} \right) + T_{hh}^L \]

6.6 Proof of Proposition 8

The mean preserving spread is achieved by setting \( \beta_i^H = \beta_i^L = \beta_i \) where \( i \in \{h, f\} \), which implies relative size is \( \frac{\Omega_f}{\Omega_h} = \frac{(\beta_f \phi_{f}^{LM} + (1-\beta_f) \phi_{f}^{MP} + \beta_f) m_f^H}{(\beta_h \phi_{h}^{LM} + (1-\beta_h) \phi_{h}^{MP} + \beta_h) m_h^H} \). This is a function of the equilibrium set of designs and net incomes. Note that the set of designs solves:
\begin{align*}
\beta_i \phi_i^{MP} + (1 - 2 \beta_i) \phi_i^{MH} &= \bar{\alpha}_i^{MH} (1 - \beta_i) \\
(1 - \beta_i) \phi_i^{LP} / \phi_i^{MP} + \beta_i \phi_i^{LH} &= \bar{\alpha}_i^{LH}
\end{align*}

While net income is given by \( \bar{m}_i^H = \frac{\rho m_i^{MP} - \gamma m_i^{LP}}{\rho + \gamma (1-\phi_i^{MP})} \), \( \bar{m}_i^M = \frac{\rho m_i^{MP} - \gamma \bar{m}_i^{MP}}{\rho - \gamma (1-\phi_i^{MP})} \) and \( \bar{m}_i^L = \frac{\rho m_i^{LP}}{\rho - \gamma (1-\phi_i^{LP})} \).

To simplify the analysis consider behavior as \( \gamma \to 0 \). This ensures \( \bar{m}_f^H = \bar{m}_h^H, \bar{\alpha}_i^{MH} = \frac{1}{1+\alpha} \) and \( \bar{\alpha}_i^{LH} = \frac{1-\alpha}{1+\alpha} \). Note that \( (\beta_i \phi_i^{LM} + (1-\beta_i) \phi_i^{MP} + \beta_i) \) at \( \beta_i = 0 \) equals \( \frac{1}{1+\alpha} \). While at \( \beta_i = \frac{1}{2} \) \( \Omega_i = \frac{1+\phi_i^{LM}}{2} \times \frac{1-\alpha}{1+\alpha} \) from (47). Hence, \( \Omega_i \bigg|_{\beta=0} \geq \Omega_i \bigg|_{\beta=\frac{1}{2}} \), with \( \Omega_i \) monotonically declining over the interval \( \beta_i \in [0, \frac{1}{2}] \).

To see that the low dispersion country is the net exporter of the differentiated good use (38) to get:
\[ \frac{\Omega_f}{(n_f + \tau^{1-\sigma} n_h)} + \frac{\Omega_h \tau^{1-\sigma}}{(1-\sigma) n_f + n_h} = \frac{\tau^{1-\sigma} \Omega_f}{(n_f + \tau^{1-\sigma} n_h)} + \frac{\Omega_h}{(1-\sigma) n_f + n_h} \]
\[ \Rightarrow \frac{\Omega_f}{(n_f + \tau^{1-\sigma} n_h)} = \frac{\Omega_f}{(1-\sigma) n_f + n_h} \]

This implies that all firms that produce the differentiated good have the same sales to local consumers, \( \frac{\Omega_f}{(n_f + \tau^{1-\sigma} n_h)} = \frac{\Omega_h}{(1-\sigma) n_f + n_h} \), and also firms that export the differentiated good all have the same value of exports, \( \left( \frac{\tau^{1-\sigma} \Omega_f}{(n_f + \tau^{1-\sigma} n_h)} = \frac{\tau^{1-\sigma} \Omega_h}{(1-\sigma) n_f + n_h} \right) \). Therefore net exports of the differentiated good follow from the relative number of firms.
6.7 Equilibrium factor prices and firm behaviour

Shephard’s lemma gives:
\[
\frac{\partial c_y(\omega)}{\partial \omega} = \eta \frac{c_y}{\omega} = a_y L \\
\frac{\partial c_y(\omega)}{\partial (1/\omega)} = (1 - \eta) c_y = a_y K
\]

The \( q \) sector has the following cost function:
\[
C(\omega, q) = F c_q(\omega) + c_q(\omega) q = (F + q) c_q \\
(\text{using zero profit}) = (F + F(\sigma - 1)) c_q = \sigma F c_q
\]

This implies the following average cost function:
\[
\frac{C(\omega, q)}{q} = \frac{\sigma F c_q}{F(\sigma - 1)} = \frac{c_q}{\rho} = a_q L
\]

Shephard’s lemma gives:
\[
\frac{\partial AC_q(\omega)}{\partial \omega} = \frac{(1 - \eta) c_q}{\omega} = a_q L \\
\frac{\partial AC_q(\omega)}{\partial (1/\omega)} = \frac{\eta c_q}{\rho} = a_q K
\]

Which implies the following full employment conditions:
\[
K = a_y K Y + a_q K Q \\
L = a_q L Y + a_q L Q
\]

While it is possible to solve this system there is enough structure in the model to determine \( n \) and \( q \) from profit maximization, zero profits and market clearing for a typical differentiated good. Note that the equilibrium level of firm output is the same across pricing regimes. Using this constraint, the number of firms in the market must adjust to make sure that this is also the market clearing quantity.

Under 1\(^{st}\) degree price discrimination, the level of output is chosen to be efficient.
\[
\beta^L q^L + \beta^M q^M + \beta^H q^H = \frac{\beta^L \gamma \bar{m}_1}{n_1 c_q} + \frac{\beta^M \gamma \bar{m}_1}{n_1 c_q} + \frac{\beta^H \gamma \bar{m}_1}{n_1 c_q}
\]
\[
q = \frac{\gamma \bar{m}_1}{n_1 c_q} \\
\Rightarrow n_1 q = \frac{\gamma \bar{m}_1}{c_q} = Q_1
\]

Under 2\(^{nd}\) degree price discrimination, the level of output is chosen to be efficient for the high type, which then determines the outputs for the other types.
\[
\beta^L q^L + \beta^M q^M + \beta^H q^H = \frac{\beta^L \gamma \bar{m}_2}{n_2 c_q} \phi^{LH} + \frac{\beta^M \gamma \bar{m}_2}{n_2 c_q} \phi^{MH} + \frac{\beta^H \gamma \bar{m}_2}{n_2 c_q}
\]
\[
\Rightarrow n_2 q = \frac{\gamma \bar{m}_2}{c_q} = Q_2
\]
Since $nq$ is determined as a function of $\omega$ in both cases (via $\bar{m}$ and $c_q$), they also implicitly determine the size of the $y$ sector from the resource constraints. For example, the capital constraint implies:

$$Y_d = Y_s = \frac{K}{a_y} - \frac{a_q}{a_y} Q$$

Since the RHS is determined by other components of the system, market clearing in the $y$ sector requires:

$$\frac{(1-\gamma)\bar{m}}{c_y} = \frac{K}{(1-\eta)c_y} - \frac{\eta c_q}{\rho(1-\eta)c_y} nq$$

$$\frac{(1-\gamma)\bar{m}}{(1-\eta)c_y} = \frac{K}{(1-\eta)} - \frac{\eta c_q}{\rho(1-\eta)} \frac{\gamma m}{c_q}$$

$$(\rho(1-\eta)(1-\gamma) + \eta \gamma) \bar{m} = \rho K$$

Denoting the aggregate factor proportions as $\kappa$, we have under first degree price discrimination:

$$\omega = \left( \frac{(\gamma + (1-\gamma)\rho - \rho(1-\eta)(1-\gamma) - \eta \gamma)}{\rho(1-\gamma) - \rho \eta(1-\gamma) + \eta \gamma} \right) \kappa$$

Under indirect discrimination:

$$\omega = \left( \frac{\Delta(\gamma + (1-\gamma)\rho - \rho(1-\eta)(1-\gamma) - \eta \gamma)}{\rho(1-\gamma) - \rho \eta(1-\gamma) + \eta \gamma} \right) \kappa$$

where

$$\Delta = \frac{\bar{m}_2}{\bar{m}_1} = \frac{(\beta^L\alpha^L+\beta^M\alpha^M+\beta^H)\bar{m}_1^H}{(\beta^L\phi^L+\beta^M\phi^M+\beta^H)\bar{m}_1^H}$$

Note that $\Delta = 1$ whenever $\beta^I = 1$. However, for $\beta^I \in (0,1)$ then $\Delta > 1$, generating the required non-monotonicity between factor prices and $\beta^I$.

### 6.8 Data Appendix

All trade data are from the World Input-Output Database (WIOD), Timmer et al. (2012) and relate to the years 2005 and 2006. Bilateral trade flows are generated by aggregating across all sectors. Countries included: Australia, Austria, Belgium, Brazil, Bulgaria, Canada, China, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Malta, Mexico, Netherlands, Poland, Portugal, Romania, Russia, Slovakia, Slovenia, Spain, Sweden, Taiwan, Turkey, UK, USA.

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