Longevity, social security, and public health programs in a dynastic model of capital accumulation, health investment, and fertility

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Abstract
This paper investigates health investment for old-age longevity, capital accumulation, and fertility in a dynastic family model with moral hazard from survival-contingent annuities. We first establish the social optimum and then the general equilibrium allocations with social security, health subsidization, or public health provision financed by labor income taxation. Private annuities induce not only more health spending and more savings but also fewer children. Social security and health subsidization exacerbate the moral hazard. However, social security and public health with free access and level control can decentralize the social optimum. Calibration to the US economy yields quantitative implications.

Keywords: Social security; Public health policies; Annuities; Longevity; Investment; Fertility

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1. Introduction

Developed nations have instituted social security and public health programs for several decades (see, e.g., Aaron, 1985; Lee and Tuljapurkar, 1997). While social security programs are mainly pay-as-you-go (PAYG) programs, public health programs fall into several types. Some countries such as Canada and the UK provide all residents with nearly free access to publicly funded health services. In the US, however, the access to public health programs is limited to the underage or the elderly or low-income population through Medicare or Medicaid. Residents without access to public health programs in the US may receive health insurances from companies or institutions exempted from income taxation. This tax exemption is health subsidization in essence. In fact, the US is the only major developed country that offers no universal access to health care. The recent reform of the US health system, the Patient Protection and Affordable Care Act (ACA), was designed to enhance access to health care by expanding public health insurance and subsidizing private insurance coverage.

At the same time, these countries have observed dramatic increases in longevity and significant declines in fertility, leading to rapid population aging. In OECD countries, total fertility rates have declined dramatically from an average 2.7 in 1970 to 1.7 in 2008, while life expectancy at birth has increased by more than 10 years since 1960, reaching 79.1 in 2007 (OECD, 2010a, 2010b). Public pensions and public health expenditures were averaged over 7% and 6.4% of GDP respectively across the OECD countries (OECD, 2010c) in tandem with population aging. By contrast, private health spending accounted only for about 2.6% of GDP on average across the OECD countries in 2008 (OECD, 2010d). In the US, for example, there were upward trends in the ratio of public to private health expenditure and in longevity in the time series data for the period 1870-2000 as noted in Tang and Zhang (2007). The most recent data shows that health expenditure reached 17.7% of GDP in the US in 2011, much higher than the counterpart 11.2% in Canada and 9.4% in the UK (OECD, 2013).

According to conventional wisdom based on lifecycle models, public pension annuities, like private annuities, induce excessive health spending for longer life as individuals cannot internalize the impact of longevity on annuity returns. Nevertheless, social security and government health programs still receive majority support in most countries. Reform proposals mainly focus on the financial sustainability rather than removal of these public programs. The purpose of this paper is to build up a general equilibrium model of health investment for
longevity, fertility, and savings through actuarially fair annuity markets to shed light on some important questions concerning the moral hazard and government policies. What are the socially optimal capital accumulation, fertility, and health investment for longevity in a dynastic model concerning the welfare of all generations? What is the difference in health spending, longevity, fertility and capital intensity across the different health systems in the presence of longevity-dependent annuities of savings and social security? What is the interaction between the government health programs and social security? Is there any socially optimal combination of government health program and social security?

In this paper we investigate health investment for old-age longevity, capital accumulation, and fertility in a dynastic family model with moral hazard from survival-contingent annuity benefits. We first establish the social optimum and then analyze general equilibrium allocations with social security, health subsidization, or public health provision financed by labor income taxation. Private annuities induce not only more savings and more health spending for longer life as found in the literature but also more bequests and fewer children. Social security and health subsidization exacerbate the moral hazard, because social security annuities strengthen the incentive to live longer and because health subsidization reduces the user cost of health investment. Social security reduces fertility under a plausible condition that the taste for the welfare of children exceeds the fraction of time for childrearing. Social security increases health investment until the marginal contribution of health investment to longevity vanishes. Social security also makes up the shortfall of old-age resources to individuals who overestimate private annuity returns and overinvest in health for longevity.

However, social security and public health with free access can be used together to decentralize the social optimum under the same condition social security reduces fertility. If public health spending is low, its marginal contribution to longevity is high, and thus a rise in public health spending induces more savings but lower fertility. At higher levels of public health spending, its marginal contribution to longevity diminishes, whereas the labor income tax financing public health is more likely to increase fertility. Combining social security with public health helps to remove the tax effect on fertility. Calibrating the model to the US economy yields quantitative implications; in particular, the US health spending exceeds the socially optimal level by 52%.
Our results are consistent with some empirical evidence in the literature. There are empirical studies finding a negative effect of social security on fertility (e.g., Cigno and Rosati, 1992; Zhang and Zhang, 2004) and a negative effect of longevity on fertility (e.g., Ehrlich and Lui, 1991; Zhang and Zhang, 2005). Social security and health expenditure together therefore tend to raise the old-age dependency ratio and accelerate population aging. In addition, Barro (1989), Perotti (1996), Zhang and Zhang (2004), among others, find evidence of a positive effect of social security on growth using cross-country data. Barro and Sala-i-Martin (1995) and Barro (1997), among others, find empirical evidence that life expectancy has a positive effect on economic growth when income is low, and that the growth effect fades away when income is high; Zhang and Zhang (2005) also find a positive growth effect of life expectancy at a decreasing rate. Moreover, health capital is found to have a positive effect on growth as well in Bloom and Canning (2003), Gyimah-Brempong and Wilson (2004), and Canning and Mahal (2010), for instance.


The effects of social security are more controversial. Social security is harmful for capital accumulation and growth in lifecycle-saving models (e.g., Feldstein, 1974; Bruce and Turnovksy, 2013) but neutral in dynastic family models in which altruistic parents leave more
bequests to children to mitigate the tax burden of social security transfers from workers to the elderly (e.g., Barro, 1974). Social security may increase capital intensity and reduce fertility as shown in a dynastic model of Becker and Barro (1988) and in lifecycle models of Rosati (1996) and Wigger (1999), among others. Social security may reduce fertility and increase human capital investment and the growth rate as shown in Zhang (1995) and Zhang and Zhang (2007), and can further enhance welfare in the presence of human capital externalities in Yew and Zhang (2009, 2013). Using lifecycle models, Davies and Kuhn (1992) and Philipson and Becker (1998) show that social security, like a private longevity-contingent claim, induces more health spending for greater longevity and more lifecycle savings because agents fail to internalize the effect of longevity extension on annuity returns. Zhang, Zhang and Leung (2006) show that pension and health subsidies raise longevity even further but reduce lifecycle saving and future output. Tang and Zhang (2007) argue that health subsidies can enhance longevity and promote growth. Bethencourt and Galasso (2008) argue that health care and social security are political complements through majority voting in a lifecycle model as health care increases longevity and thus increases social security annuities.

Existing analyses of health investment for longevity extension are typically based on lifecycle models with fixed fertility. The current paper uses a dynastic family model with a lifecycle dimension to analyze capital accumulation, fertility, and health investment for longevity in a general equilibrium framework with annuity markets. This extension enriches incentives via intergenerational altruism and relaxes lifecycle constraints via intergenerational transfers. Intuitively, mutual material support between the elderly and offspring within families interacts with decisions on savings, longevity and fertility. Using a dynastic family model, we can determine the socially optimal allocation on a broader basis including the welfare of all generations for optimal social security and health programs. Through the comparison with the social optimum, we can capture the effects of private annuities, social security annuities and public health programs on fertility and intergenerational transfers as additional channels of moral hazard arising from longevity-dependent claims. We can also compare different public health programs and social security as practised in developed countries in terms of departures in health spending, capital intensity, and fertility from the social optimum. The comparison yields useful policy implications that are different from those in the literature based on lifecycle considerations alone.
The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 establishes the social planner’s allocation. Section 4 characterizes equilibrium allocations in a decentralized economy with social security. Section 5 considers health subsidization. Section 6 decentralizes the social optimum into an equilibrium allocation through social security and public health with free access. Section 7 provides examples at steady states with specific functional forms and explores quantitative implications through calibrating the model to the US economy. Section 8 concludes the paper with some policy suggestions.

2. The model

Time is discrete in this model, extending from period 0 to infinity ($t = 0, 1, ..., \infty$). The economy is inhabited by overlapping generations of a large number of identical agents who live for three periods. In the first period of life, agents live in childhood and make no decision. Agents work in the second period of life and retire in the third period. The length of life in the first and second periods are fixed at one but the length of life in the third period, $T(m_t) \in (0,1]$, is increasing and concave in health investment per worker, $m_t \in R_+$, in the second period. Thus, longevity equals $2 + T(m_t) \leq 3$ in the current model.

The preference of the coexisting old and middle-aged agents in a family is defined over the middle-aged agent’s consumption, $c_t \in R_+$, the old agent’s consumption, $d_t \in R_+$, weighted with the length of life in old age, $T(m_{t-1})$, and the number of children, $n_t \in R_+$, of all generations:

$$V(0) = \sum_{t=0}^{\infty} \alpha^t \{ \beta T(m_{t-1})U(d_t) + \alpha[U(c_t) + \rho G(n_t)] \}, \quad (1)$$

where $\alpha \in (0,1)$ and $\beta \in (0,1)$ are the weights (discount factors) on average utility from the middle-aged agent’s consumption and from the old parent’s consumption, respectively, and $\rho > 0$ is the weight on utility from the number of children relative to that from middle-age consumption. The utility function $U: R_+^2 \to R$ is twice differentiable, strictly increasing and strictly concave, and meets the Inada conditions. The utility specification in (1) is an extension of the Millian utility for a dynastic family (Mill, 1848) in Yew and Zhang (2009, 2013) to allow for the role of longevity and health investment. It also extends the lifecycle utility in Philipson and Becker (1998) and many others to consider intergenerational interactions motivated by altruism.
In period $t$, a middle-aged agent devotes $\nu n_t$ units of time endowment to rearing children, where $0 < \nu < 1$ is fixed and thus fertility has an upper bound $1/\nu$. The remaining $1-\nu n_t$ units of time are devoted to working that earns $(1-\nu n_t)w_t$ where $w_t \in R_+$ is the wage. A middle-aged agent receives (gives) a bequest (gift) $b_t$ from (to) his or her old parent in period $t$ if $b_t > 0$ ($< 0$), taking the decisions of siblings as given. He or she spends the earnings and the received bequest on own middle-age consumption, $c_t$, own health expenditure, $m_t$, and retirement savings via an actuarially fair annuity market, $s_t \in R_+$. An old agent receives annuity income from savings $(1+r_t)s_{t-1}T(m_{t-1})/\bar{T}(\bar{m}_{t-1})$, consumes $d_t T(m_{t-1})$, and leaves (receives) a bequest (gift), $b_t$, to (from) each child, where the bar above a variable indicates its average level in the economy. The budget constraints are:

$$c_t = b_t + (1-\nu n_t)w_t - s_t - m_t, \quad (2)$$

$$d_t = \frac{[(1+r_t)s_{t-1}T(m_{t-1})-b_t n_{t-1}]}{T(m_{t-1})}. \quad (3)$$

As in Philipson and Becker (1998), an agent cannot internalize the negative effect of his or her own longevity on the annuity returns via average longevity $\bar{T}(\bar{m}_{t-1})$. The intergenerational transfers relax resource constraints from an individual’s lifetime to a dynastic family horizon.

The production of a single final good has constant returns to scale:

$$Y_t = f(K_t, N_t) \quad (4)$$

where $Y_t \in R_+$, $K_t \in R_+$, and $N_t \in R_+$ are output per worker, physical capital per worker, and labor per worker, respectively. Capital depreciates fully within one period that corresponds to 40 years in the present model with lifecycle considerations. The function $f: R_+^2 \rightarrow R_+$ is assumed to be twice differentiable, strictly increasing and strictly concave, and meet the Inada conditions. Let the final good be a numeraire and assume competitive pricing:

$$w_t = f_N(K_t, N_t), \quad (5)$$

$$1 + r_t = f_K(K_t, N_t). \quad (6)$$

Markets clear when

$$K_{t+1} = s_t/n_t, \quad (7)$$
\[ N_t = 1 - vn_t. \]  
\( (8) \)

Denote the size of the working population in period \( t \) by \( L_t \in R_+ \), which evolves according to \( L_{t+1} = L_t n_t \). Feasibility in the economy is

\[ c_t = f(K_t, 1 - v n_t) - K_{t+1} n_t - m_t - d_t T(m_{t-1})/n_{t-1}. \]  
\( (9) \)

To measure the efficiency loss of moral hazard arising from longevity-dependent annuity returns, we investigate the socially optimal allocation next.

3. The social optimum

The social planner maximizes utility in (1) by choice of \( \{d_t, c_t, n_t, K_{t+1}, m_t\}_{t=0}^{\infty} \)

\[ V(0) = \max_{\{d_t,c_t,n_t,K_{t+1},m_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \alpha^t \{\beta T(m_{t-1}) U(d_t) + \alpha[U(c_t) + \rho G(n_t)]\}, \]  
\( (10) \)

subject to feasibility in (9), given an initial state \((m_{-1}, K_0, n_0)\).

Notice that the dynamic progralming in (10) may not be concave, partly because of the product of two concave functions \( T(m_t) U(d_{t+1}) \) in utility (1) and partly because of the product of two choice variables \( K_{t+1} n_t \) in feasibility (9). Thus, we assume:

**Assumption 1.** The period utility function together with feasibility

\[ \beta T(m_{t-1}) U(d_t) + \alpha[U(f(K_t, 1 - vn_t) - K_{t+1} n_t - m_t - d_t T(m_{t-1})/n_{t-1}) + \rho G(n_t)] \]

is strictly concave in \((m_{-1}, K_t, n_{-1}, d_t, K_{t+1}, m_t, n_t)\).

For \( t \geq 0 \), the first-order conditions of (10) for interior solutions are:

\[ d_t: \quad \beta U'(d_t) = \frac{\alpha U'(c_t) d_t}{n_{t-1}}, \]  
\( (11) \)

\[ n_t: \quad \rho G'(n_t) + \frac{\alpha U'(c_{t+1}) d_{t+1} T(m_t)}{d_t} = U'(c_t) [\nu f_N(K_t, 1 - vn_t) + K_{t+1}], \]  
\( (12) \)

\[ K_{t+1}: \quad \alpha U'(c_{t+1}) f_K(K_{t+1}, 1 - vn_{t+1}) = U'(c_t) n_t, \]  
\( (13) \)

\[ m_t: \quad \beta U(d_{t+1}) T'(m_t) = U'(c_t) + \frac{\alpha U'(c_{t+1}) d_{t+1} T'(m_t)}{d_t}. \]  
\( (14) \)
Equations (11) and (13) capture intergenerational links through altruistic bequests. Together, they also imply lifecycle savings behaviour \( \beta U'(d_{t+1}) f^*_K(K_{t+1}, 1 - vn_{t+1}) = U'(c_t) \). Equations (12) and (14) capture novel interactions between fertility and health investment. The second term on the left-hand side of (12) is the marginal benefit of having an additional child at time \( t \) through relaxing the feasibility constraint in the next period in (9). This marginal gain becomes larger when fertility becomes lower or when longevity becomes higher as in an aging population. However, the second term on the right-hand side of (14) is the marginal cost of health investment for longer life that intensifies the feasibility constraint in the next period in (9). This marginal loss becomes larger when fertility becomes lower or when the marginal gain in longevity from health investment becomes larger.

The transversality conditions are:

\[
\lim_{t \to \infty} \alpha^t \left[ \alpha U'(c_t) f^*_K(K_t, 1 - vn_t) \right] K_t = 0,
\]

\[
\lim_{t \to \infty} \alpha^t \left[ \frac{\alpha U'(c_t) d_t T(m_{t-1})}{n_{t-1}^2} \right] n_{t-1} = 0,
\]

\[
\lim_{t \to \infty} \alpha^t T'(m_{t-1}) \left[ \beta U(d_t) - \frac{\alpha U'(c_t) d_t}{n_{t-1}} \right] m_{t-1} = 0,
\]

where \( T'(m_{t-1})[\beta U(d_t) - \alpha U'(c_t)d_t/n_{t-1}] > 0 \) for an interior solution for health expenditure.

The conditions ensuring the social optimum are given below.

**Proposition 1.** If a sequence \( \{c_t^{SP}, d_t^{SP}, m_t^{SP}, n_t^{SP}, K_t^{SP}\}_{t=0}^{\infty} \) with an initial state \( (m_{-1}, n_{-1}, K_0) \) satisfies the technology in (4), the feasibility in (9), the first-order conditions in (11)-(14), and the transversality conditions, then it is socially optimal under Assumption 1.

**Proof.** The proof is relegated to Appendix A. ■

This social optimum is based on the wellbeing of all generations concerning consumption, capital accumulation, fertility, and health investment for longevity. In what follows, we compare the social optimum with equilibrium allocations with private annuities, social security annuities, health subsidies, or public health in turn.
4. Equilibrium with PAYG social security

Let the social security tax rate be $\tau_t^B$ and the social security benefit be $B_t$ conditional on old-age longevity. The household’s budget constraints become

$$\begin{align*}
c_t &= b_t + (1 - vn_t)w_t(1 - \tau_t^B) - s_t - m_t, \\
d_t &= \frac{[(1 + r_t)\bar{\tau}_{t-1}T(m_{t-1}) + B_tT(m_{t-1}) - b_t\bar{n}_{t-1}]}{T(m_{t-1})}.
\end{align*}$$

As in some countries such as France and Germany, the amount of social security benefits is paid in proportion to a retiree’s own contributions in working age according to a replacement rate $\psi$. Thus, a worker who has more children (hence more time for rearing children and less time for working) will not only earn less wage income today but also receive less social security benefits in old age. The government budget constraint for social security is given by

$$B_t = \psi_t(1 - vn_{t-1})w_{t-1} = \bar{n}_{t-1}\tau_t^B(1 - vn_t)w_t/\bar{T}(m_{t-1}).$$

In this decentralized economy, a dynastic family chooses $\{b_t, n_t, m_t, s_t\}_{t=0}^{\infty}$ to maximize utility in (1) subject to budget constraints (15) and (16), knowing the earnings dependent benefit formula and taking the prices, taxes and replacement rates as given. For $t \geq 0$, the first-order conditions for interior solutions are:

$$\begin{align*}
b_t: &\quad \alpha U'(c_t) = \beta U'(d_t)n_{t-1}, \\
n_t: &\quad \rho G'(n_t) = U'(c_t)(1 - \tau_t^B)vw_t + \beta U'(d_{t+1})[\psi_{t+1}vn_wT(m_t) + b_{t+1}], \\
s_t: &\quad \beta U'(d_{t+1})(1 + r_{t+1}) = U'(c_t), \\
m_t: &\quad \beta T'(m_t)\left[U(d_{t+1}) + \frac{U'(d_{t+1})b_{t+1}n_t}{T(m_t)}\right] = U'(c_t).
\end{align*}$$

Using the budget constraint (16), we rewrite these conditions in a way to enhance comparisons with those in the social planner’s problem:

$$\begin{align*}
n_t: &\quad \rho G'(n_t) + \frac{\alpha U'(c_{t+1})d_{t+1}T(m_t)}{n_t} = U'(c_t)\left[(1 - \tau_t^B)vw_t + K_{t+1} + \frac{\beta(1 - vn_{t+1})w_{t+1}}{(1 + r_{t+1})(1 - vn_t)}\right], \\
s_t: &\quad \alpha U'(c_{t+1})(1 + r_{t+1}) = U'(c_t)n_t.
\end{align*}$$
We define the competitive equilibrium below:

**Definition 1.** A competitive equilibrium with an initial state \((m_{-1}, K_0, n_{-1})\) and with PAYG social security financed by a labor income tax is a sequence of allocations \(\{b_t, c_t, d_t, m_t, n_t, s_t, K_t, Y_t\}_{t=0}^{\infty}\), prices \(\{r_t, w_t\}_{t=0}^{\infty}\) and government policies \(\{\tau^B_t, \psi_t\}_{t=0}^{\infty}\) such that: (i) taking the prices and the government policies as given, firms and households optimize and their solutions satisfy the budget constraints (15) and (16), the technology (4), and the optimal conditions (5), (6), and (18)-(24); (ii) the PAYG social security budget (17) is balanced, and (iii) markets clear: \(K_{t+1} = s_t/n_t\) and \(N_t = 1 - vn_t\); (iv) \(X = \bar{X}\) for \(X = m, n\) by symmetry.

From a comparison between (24) and (14), the net marginal benefit (NMB) of health expenditure in this competitive equilibrium, the left-hand side less the right-hand side of (24), is greater than the counterpart in the socially optimal allocation in (14):

\[
\frac{u'(c_t)T'(m_t)n_t}{T(m_t)} \left[ K_{t+1} + \frac{\tau^B_{t+1}(1-vn_{t+1})w_{t+1}}{(1+r_{t+1})} \right] > 0.
\]

The departure of a competitive economy from the socially optimal allocation concerning health expenditure occurs here because agents do not internalize the negative effect of extended longevity on the average returns to private annuities and to social security. As a consequence, they over-invest in health for longevity, compared to the socially optimal allocation, in line with the finding in Philipson and Becker (1998). These excessive incentives of health investment due to private annuities and social security are shown by the two terms appear in the square bracket on the left-hand side of (24). Note that when agents over-invest in health for longer life, they may also over-save for their old-age consumption.

With endogenous fertility, the fertility level in the competitive equilibrium may also depart from the socially optimal level. Using \(w_t = f_N(K_t, 1 - vn_t)\), the NMB of having a child in the competitive equilibrium, the left-hand side less the right-hand side of (22), minus the counterpart in the socially optimal allocation in (12), denoted as \(\text{DNMB}_n\), is equal to
From the first term on the right-hand side, the social security contribution tends to increase the NMB of having a child in equilibrium over the NMB in the social optimum because it reduces the time cost of rearing children. From the second term, the social security contribution tends to reduce the NMB of having a child in equilibrium because it increases the forgone social security benefits for rearing children. The DNMB\(_n\) could be negative when agents over-save for old-age consumption so that the rate of return to saving, \((1 + r_{t+1})\), is sufficiently low, and that the future wage rate, \(w_{t+1}\), is sufficiently high. Intuitively, the amount of this over-saving becomes larger when stronger altruism induces more savings for more bequests to children to mitigate the increased tax burden of social security on children. This can be inferred from (18) and (20): a larger \(\alpha\) motivates more bequests \(b_{t+1}\) (hence larger \(c_{t+1}\) and smaller \(d_{t+1}\)) to retain the balance \(\alpha U'(c_{t+1}) = \beta U'(d_{t+1})n_t\) in (18), and the decline in \(d_{t+1}\) is consistent with the over-saving (lower \(c_t\)) in (20).

Because of this over-saving induced by social security annuities, social security is more likely to reduce fertility in the present model than in the existing ones with fixed longevity such as Zhang and Zhang (2007), Yew and Zhang (2009), and Yew and Zhang (2013). This departure from the socially optimal fertility level increases with social security. In the steady state with \(c_{t+1} = c_t\), the first-order condition (23) with respect to savings boils down to \(\alpha(1 + r) = n\). As the equilibrium rental rate of capital falls with savings or investment but rises with labor via \(1 + r = f_k(K, 1 - \nu n)\), if social security annuities increase savings in the steady state, it must reduce fertility via \(\alpha f_k(K, 1 - \nu n) = n\), and vice versa.

From the above comparisons, the efficiency losses mentioned above are in fact magnified by social security in an economy with competitive annuities markets, as social security further distorts agents’ decisions on saving, health investment and fertility. Our result is thus in line with those in Davies and Kuhn (1992) and Philipson and Becker (1998) that social security never raises welfare in an environment with moral hazard. Our consideration of fertility choice enriches the contents of moral hazard and population aging and magnifies the magnitudes of the moral hazard concerning longevity and savings.

5. Equilibrium with health subsidization and PAYG social security
As practiced in countries such as the US, it is also relevant to ask how health subsidization affects allocation and welfare. Suppose that the government subsidizes health expenditure at a rate $\mu^m_t$ financed by a labor income tax rate $\tau^m_t$ under a separate and balanced budget:

$$\mu^m_t \bar{m}_t = \tau^m_t (1 - \nu \bar{n}_t) w_t.$$ 

Accordingly, the household budget constraint at working age becomes

$$c_t = b_t + (1 - \nu n_t) w_t (1 - \tau^B_t - \tau^m_t) - s_t - (1 - \mu^m_t) m_t.$$ 

For $t \geq 0$, the first-order conditions with respect to $b_t$ and $s_t$ are the same as (18) and (20), respectively, but those with respect to $n_t$ and $m_t$ become:

$$n_t: \rho G'(n_t) = U'(c_t)(1 - \tau^B_t - \tau^m_t) v w_t + \beta U'(d_{t+1}) \psi_{t+1} v w_t T(m_t) + b_{t+1}, \quad (26)$$

$$m_t: \beta T'(m_t) \left[ U(d_{t+1}) + \frac{U'(d_{t+1}) b_{t+1} n_t}{\tau(m_t)} \right] = U'(c_t)(1 - \mu^m_t). \quad (27)$$

Using (16), (18), and (20), we rewrite (26) and (27) as

$$n_t: \rho G'(n_t) + \frac{\alpha U'(c_{t+1}) d_{t+1} T(m_t)}{n_t^2} = U'(c_t) \left[ (1 - \tau^B_t - \tau^m_t) v w_t + K_{t+1} + \frac{\tau^B_{t+1}(1 - \nu n_{t+1}) w_{t+1}}{(1 + r_{t+1})(1 - \nu n_t)} \right], \quad (28)$$

$$m_t: T'(m_t) \left\{ \beta U(d_{t+1}) + \frac{U'(c_t) n_t}{\tau(m_t)} \left[ K_{t+1} + \frac{\tau^B_{t+1}(1 - \nu n_{t+1}) w_{t+1}}{(1 + r_{t+1})} \right] \right\} = U'(c_t)(1 - \mu^m_t) + \frac{\alpha U'(c_{t+1}) d_{t+1} T'(m_t)}{n_t}. \quad (29)$$

From (29) and (24), the NMB of health expenditure in equilibrium with health subsidies is higher than that in equilibrium without health subsidization. From the NMB of health expenditure with health subsidies, the left-hand side less the right-hand side of (29), minus the counterpart in (24) without health subsidies, we obtain a positive residual $U'(c_t) \mu^m_t > 0$. Intuitively, health subsidies increase health investment by reducing the cost of health investment, and thus further magnify the efficiency loss from over-investing in health induced by longevity-dependent annuities.

From (28) and (22), the NMB of having a child in equilibrium with health subsidies is higher than that in equilibrium without health subsidies: $U'(c_t) \tau^m_t v w_t > 0$. Intuitively, higher
labor income taxes for health subsidies reduce the time cost of rearing a child. Thus, health subsidization financed by labor income taxes may raise fertility.

However, when higher health subsidies increase health spending and fertility, it would reduce middle-age consumption and thus increase the marginal cost of saving in (23). In particular, in the steady state with $c_{t+1} = c_t$, equation (23) reduces to $\alpha(1 + r) = n$. Consequently, health subsidization can reduce savings when it raises fertility, and vice versa.

As both health subsidies and social security increase the NMB of health investment, they cannot achieve the socially optimal outcome in a model that already has overinvestment in health induced by longevity-dependent annuities. Can we find government policies to fully remove the efficiency losses from the moral hazard? We explore this possibility next.

6. Equilibrium with public health and PAYG social security

Public health is available in some countries like Canada with free access to residents financed by payroll taxes and with bargaining over bulk funding between hospitals and the government. Differing from health subsidization, the amount of public health is now chosen by the government, $\bar{m}_t = \tau_t^m(1 - v\bar{m}_t)w_t$, where $\tau^m < 1$ is the labor income tax rate for public health.

Having free access to public health, the household’s budget constraints become:

$$c_t = b_t + (1 - \tau_t^B - \tau_t^m)(1 - vn_t)w_t - s_t,$$

$$d_t = \frac{(1 + \psi_t)\zeta_{t-1}\bar{\zeta}(\bar{m}_{t-1}) + B_t\bar{T}(\bar{m}_{t-1}) - b_{t-1}n_{t-1}}{\bar{T}(\bar{m}_{t-1})}.$$  \hspace{1cm} (31)

Here, longevity depends on public health spending.

For $t \geq 0$, the first-order conditions for an interior solution are:

$$b_t: \alpha U'(c_t) = \beta U'(d_t)n_{t-1},$$

$$n_t: \rho G'(n_t) = U'(c_t)(1 - \tau_t^B - \tau_t^m)vw_t + \beta U'(d_{t+1})[\psi_{t+1}vw_t\bar{T}(\bar{m}_t) + b_{t+1}],$$

$$s_t: \beta U'(d_{t+1})(1 + r_{t+1}) = U'(c_t).$$  \hspace{1cm} (34)

Using (32), (34) and the budget constraint (31), we rewrite (33) as

$$n_t: \rho G'(n_t) + \frac{\alpha U'(c_{t+1})d_{t+1}\bar{T}(\bar{m}_t)}{n_t^2} =$$
Definition 2. A competitive equilibrium with PAYG social security and public health from an initial state \((\bar{m}_{-1}, K_0, n_{-1})\) is a sequence of allocation \(\{b_t, c_t, d_t, n_t, s_t, K_{t+1}, N_t, Y_{t}\}^\infty_{t=0}\), prices \(\{r_t, w_t\}^\infty_{t=0}\), and government policies \(\{\tau^B_t, \tau^m_t, \bar{m}_t, \psi_t\}^\infty_{t=0}\) such that: (i) taking prices and government policies as given, firms and households optimize and their solutions satisfy the budget constraints (30) and (31), the technology (4), the optimal conditions (5), (6), and (32), (34) and (35); (ii) government budgets for PAYG social security and public health are balanced, and (iii) markets clear: \(K_{t+1} = s_t/n_t\) and \(N_t = 1 - \nu n_t\); (iv) \(X = \bar{X}\) for \(X = m, n\) by symmetry.

The socially optimal government policies are given below.

Proposition 2. The socially optimal government policy \(\{\tau^B_t, \tau^m_t, m_t, \psi_t\}^\infty_{t=0}\) is characterized implicitly by the following difference equations:

\[
U'(c_t) \left[ (1 - \tau^B_t - \tau^m_t) \nu w_t + K_{t+1} + \frac{\tau^B_{t+1}(1-\nu n_{t+1})w_{t+1}}{1+\tau_{t+1}(1-\nu n_t)} \right].
\] (35)

\[
vw_t(\tau^B_t + \tau^m_t)(1 + r_{t+1})(1 - \nu n_t) = \tau^B_{t+1}(1 - \nu n_{t+1})w_{t+1},
\] (36)

\[
\beta U(d_{t+1})T'(m_t) = U'(c_t) + \frac{aU'(c_t+1)T'(m_t)}{n_t},
\] (37)

\[
\psi_t(1 - \nu n_{t-1})w_{t-1} = \frac{n_{t-1}\tau^B_t(1-\nu n_t)w_t}{T(m_{t-1})},
\] (38)

\[
m_t = \tau^m_t(1 - \nu n_t)w_t,
\] (39)

where \(m_t\) and the coefficients on \(\tau^B_t, \tau^m_t, \) and \(\psi_t, \forall t \geq 0\), are evaluated at the socially optimal allocation, \(\{d^S_t, c^S_t, m^S_t, K^S_{t+1}, n^S_t\}^\infty_{t=0}\), and when factors prices are equal to marginal products. In laissez faire, the net marginal benefit of fertility is lower and the net marginal benefit of health expenditure is higher than the socially optimum, other things being equal. Neither PAYG social security nor public health alone can be socially optimal.

Proof. The task is to find a sequence of government policy \(\{\tau^B_t, \tau^m_t, m_t, \psi_t\}^\infty_{t=0}\) to transform the system of equations characterizing the competitive equilibrium allocation in Definition 2 to the same system of equations characterizing the socially optimal allocation in Proposition 1, knowing the socially optimal allocation \(\{d^S_t, c^S_t, m^S_t, K^S_{t+1}, n^S_t\}^\infty_{t=0}\). The superscript SP is
omitted for ease of notation unless it is necessary. Note first that the technology in (4) is the
same in both the competitive equilibrium allocation with PAYG social security and public health
and the socially optimal allocation. Using the household budget constraints (30) and (31), the
government budget constraints, the market clearing condition for physical capital (7), and
substituting \( f(K_t, 1 - v n_t) = (1 + r_t)K_t + (1 - v n_t)w_t \) (due to the constant-return-to-scale
technology), we obtain feasibility in (9) for the economy:
\[
d_t T(m_{t-1}) = n_{t-1}[f(K_t, 1 - v n_t) - c_t - m_t - K_{t+1}n_t].
\]

The government chooses the level of health investment to maximize the agent’s utility
subject to feasibility as in the planner’s problem, when the government combines its budget
constraints with the agent’ constraints. The optimal condition with respect to health investment is
(37), which is the same as (14). From this socially optimal level of health investment, the
government budget constraint for public health, \( m_t = \tau_t^m (1 - v n_t)w_t \), can be used to determine
the tax rate \( \tau_t^m \) in (39) given \( w_t = f_N(K_t, 1 - v n_t) \) in (5).

The optimal condition with respect to the private intergenerational transfer in (32) is the
same as (11) in the socially optimal allocation. Substituting (6) and (32) into the optimal
condition (34) concerning life-cycle savings obtains the same optimal condition (13) in the
socially optimal allocation.

Comparing (35) with the socially optimal condition (12) concerning fertility, the NMB of
having a child in the competitive equilibrium with social security and public health (i.e. the left-
hand side less the right-hand side), evaluated at the socially optimal allocation, minus the
counterpart in the socially optimal allocation is equal to:
\[
DNMB_n = U'(c_t) \left[ (\tau_t^B + \tau_t^m)vw_t - \frac{\tau_t^{B+1}(1 - v n_{t+1})w_{t+1}}{(1 + r_{t+1})(1 - v n_t)} \right].
\]  
(40)

Without restricting these taxes to zero, the wedges created by \( \tau_t^B \), \( \tau_t^m \) and \( \tau_t^{B+1} \) in the square
bracket of the above expression should be equal to zero in order to obtain the socially optimal
condition concerning fertility. This justifies condition (36) in which the coefficients on
government policy variables are evaluated at the socially optimal allocation: \( vw_t = v f_N(K_t, 1 - v n_t) \), and \( (1 - v n_{t+1})w_{t+1}/[(1 + r_{t+1})(1 - v n_t)] = (1 - v n_{t+1})f_N(K_{t+1}, 1 - v n_{t+1})/ [f_K(K_{t+1}, 1 - v n_{t+1})(1 - v n_t)]. \)
Note that when only public health is present ($\tau_t^B = 0$, and $\tau_t^m > 0$, for $\forall t \geq 0$), $\text{DNMB}_n$ in (40) is positive, meaning that the NMB of having a child in the competitive equilibrium in (35) is always greater than that in the socially optimal allocation in (12). Intuitively, public health with free access financed by labor income taxes reduces the time cost of having a child and thus raises fertility. This requires a negative effect of social security ($\tau_t^B > 0$ for $\forall t \geq 0$) on fertility for the full cancelation of wedges caused by public health financed by labor income taxation. Comparing $\text{DNMB}_n$ in (40) with that in (25), public health financed by labor income taxes tends to induce higher fertility and thus mitigate the negative effects of longevity-dependent (private and social security) annuities on fertility.

The transversality conditions of the consumer’s problem are essentially the same as the transversality conditions of the social planner’s problem by using the capital market clearing condition $s_t = n_t K_{t+1}$ and the equilibrium pricing rules. In short, conditions (36)-(39) with the coefficients on $\{\tau_t^B, \tau_t^m, \bar{m}, \psi_t\}_{t=0}^{\omega}$ evaluated at $\{d_{t}^{SP}, c_{t}^{SP}, m_{t}^{SP}, K_{t+1}^{SP}, n_{t}^{SP}\}_{t=0}^{\omega}$ transform the equilibrium conditions into the same system of equations as (9), (11)-(14), and transversality conditions in the socially optimum. Thus, these conditions implicitly determine the socially optimal policy $\{\tau_t^B, \tau_t^m, m_t, \psi_t\}_{t=0}^{\omega}$ that eliminates all efficiency losses in a decentralized economy with a moral hazard problem from longevity-dependent annuities. ■

As an essential part of the optimal government policy, public health spending with free access needs the government to curb over-spending on health for longevity. As mentioned earlier, the ratios of health expenditure to GDP in Canada and the UK are indeed much lower than that in the US. As a side effect, however, the labor income tax financing public health may induce higher fertility. Thus, social security can be used together to mitigate the side effect of public health on fertility if social security can reduce the NMB of having a child.

**Lemma 1.** The optimal tax rate for public health is positive if and only if social security reduces fertility.

**Proof.** From (36) and (40), we have

$$
\tau_t^m = \frac{\tau_{t+1}^B(1-\nu n_{t+1})w_{t+1}}{(1+\tau_{t+1})(1-\nu n_t)\nu w_t} - \tau_t^B > 0.
$$
As mentioned below (25), the inequality above means that the negative effect of higher social security on fertility through raising forgone social security benefits for having a child dominates the positive effect through reducing the cost of time rearing a child (after tax wage).

This result highlights the interaction between public health and social security via endogenous fertility. It is relevant since, as mentioned earlier, there is empirical evidence on a negative effect of social security on fertility.

To derive more specific and transparent results, we provide an example with the popularly used CRRA utility and Cobb-Douglas production technology in the steady states of the various cases.

7. Examples at steady states: CRRA utility and Cobb-Douglas technology

The utility function is specified as a CRRA function:

\[ U(x) = \frac{x^{1-\sigma}-1}{1-\sigma}, x = c, d, \sigma > 0; \ G(n) = \frac{n^{1-\gamma}-1}{1-\gamma}, \gamma > 0. \] (41)

The function linking old-age longevity to health spending takes the following form:

\[ T(m) = p \left( \frac{m}{\delta+\epsilon m} \right)^{\varphi}, \ p \in (0,1], \delta > 0, \epsilon \geq 1, \varphi \in (0,1), \] (42)

which ensures \(0 \leq T(m) < 1 \) for \( m \geq 0 \) and gives rise to

\[ T'(m) = \frac{\delta \varphi T(m)}{m(\delta+\epsilon m)} > 0, \]

\[ T''(m) = - \frac{\delta^2 \varphi (1-\varphi)T(m)}{m^2(\delta+\epsilon m)^2} - 2 \frac{\delta \epsilon \varphi T(m)}{m(\delta+\epsilon m)^2} < 0. \]

Here, \( p \) is an autonomous factor; \( \delta, \varphi, \) and \( \epsilon \) affect longevity negatively. For a given level of health spending, the health-spending elasticity of old-age longevity

\[ \frac{mT'(m)}{T(m)} = \frac{\delta \varphi}{(\delta+\epsilon m)} \]

increases in \( \delta \) and \( \varphi \) but decreases in \( \epsilon \). This function is rich enough to capture the relationship between health spending and longevity and is less likely to get into corner solutions than an exponential function does according to our experiences from numerical experiments.

The production function takes the Cobb-Douglas form:

\[ Y_t = AK_t^\theta (1 - vn_t)^{1-\theta}, A > 0, \theta \in (0,1), \] (43)

where \( A \) is the total factor productivity parameter and \( \theta \) is the share parameter of capital. The competitive factor prices are: \( w_t = (1-\theta)Y_t/(1 - vn_t) \) and \( 1 + r_t = \theta Y_t/K_t. \)
Even with such functional specifications, the model is still too complex to be tractable for its full dynamic path. Thus, we will mainly focus on the analysis of the steady state equilibrium. In what follows, we first characterize the steady state of the social optimal and equilibrium paths, respectively, and then investigate the implications of private annuities, social security, health subsidization or public health for fertility, capital intensity, health investment, and longevity at the steady state. The gap in steady state between the social optimum and equilibrium allocations is meaningful in terms of variations in fertility, health investment, capital intensity, and output.

7.1. The steady state of social optimum

From Section 3 and the specific functions, the steady-state of the social optimum is derived as:

\[ c = \frac{nK(1-\alpha\theta)-ma\theta}{\alpha\theta(1+E_1)}, \quad (44) \]

\[ d = \left( \frac{\beta n}{\alpha} \right)^{\frac{1}{\sigma}} c, \quad (45) \]

\[ K = (\alpha\theta A/n)^{1/(1-\theta)}(1-vn), \quad (46) \]

\[ \rho n^{-\gamma} + c^{-\sigma} \left\{ \frac{\alpha E_1 c}{n} - K \left[ \frac{(1-\theta)vn}{(1-vn)a\theta} + 1 \right] \right\} = 0, \quad (47) \]

\[ \beta T'(m) \left[ \frac{(\beta n/\alpha)^{\frac{1}{\sigma}} \cdot e^{1-\sigma-1}}{1-\sigma} \right] - c^{-\sigma} \left[ 1 + \frac{T'(m)\alpha}{n} \left( \frac{\beta n}{\alpha} \right)^{\frac{1}{\sigma}} c \right] = 0, \quad (48) \]

where

\[ E_1 \equiv T(m)(\beta/\alpha)^{\frac{1}{\sigma}} n^{\sigma-1} > 0. \quad (49) \]

If \( \sigma > 1 \) in (48) as typically assumed in the literature on macroeconomic studies, then the existence of an interior solution for health spending requires \( (\beta n/\alpha)^{\frac{1}{\sigma}} e^{1-\sigma} - 1 < 0 \) or \( c > (\beta n/\alpha)^{\frac{1}{\sigma}} \) in (48). In (46), capital per worker is negatively related to fertility as in the neoclassical growth theory.

7.2. The steady state with PAYG social security

Using the functional forms in (18)-(21) and \( s = nK \), the steady-state solution with a time-invariant tax rate for social security \( \tau_i^B = \tau^B \) is

\[ c = \frac{nK(1-\alpha\theta)-ma\theta}{\alpha\theta(1+E_1)}, \quad (50) \]

\[ d = \left( \frac{\beta n}{\alpha} \right)^{\frac{1}{\sigma}} c, \quad (51) \]
\[ b = -\frac{T(m)}{n} \left( \frac{\beta n}{\alpha} \right)^{\frac{1}{\sigma}} c + \frac{nK[\theta + t^B(1-\theta)]}{\alpha \theta}, \]

\[ V'_{n}^{\tau} \equiv \rho n^{-\gamma} + c^{-\sigma} \left\{ \frac{\alpha E_{1}c}{n} - K \left[ \frac{1}{n} - \frac{1 - \theta}{1 - \nu n} \left( \frac{1 - \tau^{m} \nu n + t^B}{\alpha \theta} \right)^{1} + 1 \right] \right\} = 0, \tag{52} \]

\[ V'_{m}^{\tau} \equiv \beta T'(m) \left\{ \frac{\beta n}{\alpha} \left( \frac{1 - \sigma}{\sigma} c^{1 - \sigma - 1} \right) \right\} - c^{-\sigma} \left[ 1 + \frac{T'(m)\alpha}{n} \left( \frac{\beta n}{\alpha} \left( \frac{1 - \sigma}{\sigma} c^{1 - \sigma - 1} \right) \right) - \frac{nK T'(m)}{T(m)} \left( \frac{\theta + t^B(1-\theta)}{\theta} \right) \right] = 0, \tag{53} \]

where \( E_{1} \) is given in (49), and \( K \) is given in (46). Absent social security, a laissez faire steady state differs from the socially optimal steady state in that the NMB of health investment in (53), denoted as \( V'_{n}^{\tau} \), has an additional positive last term compared to the counterpart in (48) as noted earlier. If evaluated at the social optimum, the NMB of health investment in laissez faire would be positive and thus induce over-investment in health.

Note that bequests are increasing with the tax rate for social security as in a typical dynastic family model because altruistic parents can raise bequests to offset the increased tax burden on children. Also, a higher tax rate for social security annuities reduces the NMB of having a child, denoted as \( V'_{n}^{\tau} \), if \( \alpha > \nu n \) on the one hand, but always increases the NMB of private health spending further on the other hand. These two factors tend to induce extra private health spending and fewer children.

### 7.3. The steady state with health subsidization and PAYG social security

At the steady-state with time-invariant tax rates \( (\tau^{B}, \tau^{m}) \), substituting the functional forms in (28) and (29) yields

\[ V'_{n}^{s} \equiv \rho n^{-\gamma} + c^{-\sigma} \left\{ \frac{\alpha E_{1}c}{n} - K \left[ \frac{1}{n} - \frac{1 - \theta}{1 - \nu n} \left( \frac{1 - \tau^{m}\nu n + t^B}{\alpha \theta} \right)^{1} + 1 \right] \right\} = 0, \tag{54} \]

\[ V'_{m}^{s} \equiv \beta T'(m) \left\{ \frac{\beta n}{\alpha} \left( \frac{1 - \sigma}{\sigma} c^{1 - \sigma - 1} \right) \right\} - c^{-\sigma} \left[ 1 + \frac{T'(m)\alpha}{n} \left( \frac{\beta n}{\alpha} \left( \frac{1 - \sigma}{\sigma} c^{1 - \sigma - 1} \right) \right) - \frac{nK T'(m)}{T(m)} \left( \frac{\theta + t^B(1-\theta)}{\theta} \right) \right] = 0. \tag{55} \]

The NMB of private health spending in (55) exceeds that in (48) due to the moral hazard from longevity-dependent annuities exacerbated by social security and health subsidization. Also, a higher tax rate for social security has a negative direct effect on the NMB of having a child,
denoted as $V^s_n$, if $\alpha > \nu n$ in (54), but it has a positive direct effect on the NMB of private health spending in (55). These two forces tend to induce extra health spending and fewer children. Moreover, health subsidization also has positive direct effects on the NMBs of having a child and health spending.

7.4. The steady state with public health and PAYG social security

Using the functional forms in (32)-(34) and $s = nK$, the steady-state solution with PAYG social security and public health that offers free access to all agents is

$$c = \frac{nK(1-\alpha\theta)-\alpha\theta m}{\alpha\theta(1+E_1)},$$  

(56)

$$d = \left(\frac{\beta n}{\alpha}\right)^{\frac{1}{\sigma}}c,$$  

(57)

$$b = -\frac{\tau(m)}{n} \left(\frac{\beta n}{\alpha}\right)^{\frac{1}{\sigma}}c + \frac{nK[\theta+r^B(1-\theta)]}{\alpha\theta},$$

$$V^u_n \equiv \rho n^{-\gamma} + c^{-\sigma} \left\{ \frac{aE_1c}{n} - K \left[ \left(\frac{1-\theta}{1-\nu n}\right)\left(\frac{(1-r^B)\nu n+r^B\alpha}{\alpha\theta}\right) + 1 \right] + \frac{\nu m}{1-\nu n} \right\} = 0,$$  

(58)

$$m = \tau^m(1-\theta)nK/\alpha\theta,$$  

(59)

where $E_1 \equiv T(m)(\beta/\alpha)^{\frac{1}{\sigma}}n^{\frac{1}{\sigma}-1} > 0$ in (49) and $K = (\alpha\theta A/n)^{1/(1-\theta)}(1-\nu n)$ in (46).

Unlike health subsidization, the government now can directly control the amount of health spending and offer free access to health services, thereby avoiding the moral hazard via private health investment in the presence of longevity-dependent annuities. The system of equations, except (59), determines the steady state allocation as a function of public health spending. The constraint in (59) can then determine the tax rate for any targeted level of public health spending. Alternatively, one can use all the equations to determine the allocation by changing the tax rate for public health.

**Proposition 3.** For $\alpha > \nu n$, an increase in the tax rate for PAYG social security, $\tau^B$, reduces fertility, $n$, and increases capital per worker, $K$, and output per worker, $Y$, in the steady state, given the tax rate for health spending, $\tau^m$.

**Proof.** The proof is relegated to Appendix A. □
From Lemma 1, a negative net effect of social security on fertility is essential for optimal public health spending. Proposition 3 gives the exact condition $\alpha > \nu n$ for a negative fertility effect of social security. Intuitively, the stronger the taste for the welfare of children, the more likely the negative fertility effect of social security via the bequest cost of having a child dominates the positive fertility effect of social security via the time cost of rearing a child. Conversely, a rise in public health spending along with a rise the tax rate has a positive direct effect on the net marginal benefit of having a child, thereby tending to raise fertility and reduce capital intensity. The socially optimal combination of social security and public health with the specific functions is given below:

**Proposition 4.** Suppose that $\alpha > \nu n$, given the functional forms in (41)-(43). The socially optimal PAYG social security and public health financed by labor income taxation in the steady state are given by

\[
(\tau^B)^* = \frac{\nu n \theta}{\alpha - \nu n} > 0, \quad (60)
\]

\[
(\tau^m)^* = \frac{\nu \theta}{(1-\theta) n K} > 0, \quad (61)
\]

where $n, m,$ and $K$ are evaluated at the socially optimal allocation.

*Proof.* The proof is relegated to Appendix A. ■

The optimal tax rate for public health is proportionate to the socially optimal ratio of health spending to investment in capital per family and increasing with the taste for the welfare of children and the capital’s share in production. Also, the optimal tax rate for social security is increasing with the socially optimal level of fertility and the optimal tax rate for public health.

Among all cases in the present paper, the optimal level of public health spending overcoming the moral hazard problem is expected to be lower than those in cases with private health spending. The level of health spending is expected to be the highest in the case with both social security annuities and health subsidies as in the US. These speculations are consistent, for example, with the most recent data from the OECD (2013): the ratio of health expenditure to GDP is 17.7% in the US, 11.2% in Canada, and 9.4% in the UK in 2011, given that the US has a subsidized private health system and that Canada and UK have (almost) free access to a public funded health system. As the model is complicated for analytical comparisons, the task next is to
compare the cases and determine the socially optimal tax rates for social security and public health numerically. In doing so, we shall also compare these socially optimal tax rates with those in the US to shed light on policy issues.

7.5. Numerical examples

Due to the complexity of tracking down the full dynamic path in the present model, the numerical task only focuses on the steady states in different cases with different policies and compares them with the social optimum in terms of health spending, fertility, capital intensity, and output. The numerical experiments are based on the steady state solutions with the CRRA utility function in (41), the health technology in (42), and the Cobb-Douglas production function in (43).

The calibration starts with the health technology for longevity. Concerning lifecycle, imagine that agents start making decisions at age 20, work for 40 years, and have less than 40 years during old age. The length of a period is set as 40 years. To capture the nonlinear relationship between old-age longevity and health spending in (42), we use three observations for average life expectancy at 20, which equals 40(1 + T(m)), and average health spending (constant 2005 US dollars) in 1960-1970, 1980-1990, and 2000-2009 in the US in Table 1. These observations yield the calibration \( \varphi = 0.7752, \epsilon = 1, \delta = 910, \) and \( p = 0.5111 \) for a good fit as shown in the right panel of Table 1 and in Figure 1. This good fit provides not only plausible levels of \( T(m) \) at each level of health spending but also plausible values of the derivatives \( T'(m) \) and \( T''(m) \) that are important in determining the allocations of the economy.

[Table 1 goes here.]

[Figure 1 goes here.]

We calibrate the other parameters with social security and health subsidization using observed values of the variables in the US economy in Table 2 and using available estimates of some of the parameters in the literature.

[Table 2 goes here.]

For calibration with unisex in our model, we use fertility at 1.03 (half of the actual rate of 2.06 on average given in Table 2). The saving rate in our closed economy, \( \alpha \theta \), is calibrated to match the observed ratio of investment to GDP adjusted by fertility. We use a standard value of
the capital share parameter: \( \theta = 0.3267 \). To match the value of the ratio of investment to GDP of 0.1906 in Table 2, \( \alpha = 0.5837 \) is chosen accordingly.

Time for rearing children can be measured by subtracting the time spent by young parents on working 0.7165 in Table 2 from the one unit time endowment. This leads to \( \nu n = 0.2835 \) and thus \( \nu = 0.2752 \) for \( n = 1.03 \). From the level of output per capita and the ratio of investment to output in Table 2 and from \( \alpha = 0.5837, \theta = 0.3267, \nu = 0.2752, \) and \( n = 1.03 \), the productivity parameter, \( A = 2809 \) is chosen accordingly.\(^1\) Using the values for total health spending (% of GDP) and health subsidization (% of total health spending) in Table 2 and given \( \theta = 0.3267 \), we can obtain the tax rate for subsidizing health spending in the US economy, \( \tau^m = (\mu^m m/Y)/(1 - \theta) = (0.1618 \times 0.39)/(1 - 0.3267) = 0.0937 \).

Finally, the levels of preference parameters \((\beta, \gamma, \sigma, \rho)\) are chosen to generate values of fertility, output, and proportional allocations of output as closely as possible to the observed values of these variables in Table 2. In setting the value of \( \beta = 0.43 \), we consider the possibility that the discounting factor on utility from old-age consumption \( \beta \) is smaller than the discounting factor on utility from middle-age consumption and the welfare of future generation \( \alpha \). Also, fertility being less inter-temporally elastic than consumption is important to account for secular decline in fertility in the US according to Greenwood et al. (2005). Table 3 gives the list of the calibrated values of the parameters, or the baseline parameterization.

[Table 3 goes here.]

The calibration yields a plausible parameterization to quantify the effects of private annuities, social security and health subsidization in comparison with the social optimum in terms of health spending, capital intensity, fertility and output. We also compare the actual rates of social security and health subsidization in the U.S. economy with the numerical optimal public policies consisting of social security and public health in counterfactual experiments.

The numerical results at steady states are reported in Table 4 in five different cases: (i) the laissez faire economy, (ii) the economy with only social security, (iii) the economy with only health subsidization, (iv) the US economy with social security and health subsidization, and (v)

\(^1\) We do not scale down the actual output per capita in calibrating the productivity parameter, \( A \), here. This is because the scaled values for output per capita and the productivity parameter, \( A \), will generate different optimal tax rates compared with the non-scaled counterparts due to the non-linearity of the longevity function in our model.
the social optimum achieved via the optimal government policies consisting of social security and public health with free access.

[Table 4 goes here.]

Let us first look at the social optimum. In case (v) of Table 4, the socially optimal rate of social security contribution is \((\tau^B)^* = 0.19\) and the socially optimal level of public health spending is \(m^* = 4462\) or 11.48% of output which can be financed by a tax rate \(\tau^m = 0.1705.\)

The social optimum serves as the expanded Modified-Golden rules governing not only capital accumulation but also fertility and longevity through health spending. Any departure from this golden rule is regarded as dynamically inefficient in the present model.

In case (i) (laissez faire: \(\tau^B = \tau^m = 0\)), the moral hazard from private annuities engenders a 11.72% rise in health spending per worker and a 2.93% rise in capital per worker but a 1.48% decline in fertility in comparison with the social optimum. As a result, it increases output per worker by 1.39%. The rise in health spending per worker is substantial.

In case (ii) with the US social security contribution rate and without health subsidy \((\tau^B = 0.124, \tau^m = 0)\), the departure from the social optimum becomes larger because social security annuities worsen the moral hazard problem. Now, health spending increases by 18.35%, capital intensity increases by 16.37%, fertility decreases by 7.65%, and output increases by 7.47%. Such strong effects of social security differ significantly from those that focus on capital accumulation without the moral hazard via endogenous longevity or without endogenous fertility.

In the case with the US ratio of health subsidies to labor income and without social security \((\tau^B = 0, \tau^m = 0.0937)\), the departure from the social optimum becomes larger in health spending but smaller in other dimensions. Health spending increases dramatically by 42.78%, capital intensity increases by 2.31%, fertility decreases by 1.18%, and output increases by 1.1%. Since health spending increases much more than output does, the ratio of health spending to output is the highest in this case. Intuitively, health subsidization exacerbates the moral hazard problem through reducing the cost of longevity relative to the costs of consumption and children. In comparison with the laissez faire, health subsidies at the US level also increase

\(^2\) Note that, given the negative longevity externality, positive externalities from capital are no longer essential for optimal policies. This feature differs from Zhang and Zhang (2007) and Yew and Zhang (2009) where the optimal social security tax rate should be zero without the positive externality in production or education.
health spending substantially. However, health subsidies increase fertility and reduce capital per worker and output per worker in comparison with the laissez faire.

In case (iv) with the US social security contribution rate at $\tau^B = 0.124$ and the US ratio of health subsidies to labor income at $\tau^m = 0.0937$, the calibrated US economy has a larger departure from the social optimum in all dimensions than the above cases do: health spending increases by 51.75%, capital intensity increases by 16.88%, fertility decreases by 7.86%, and output increases by 7.69%. Although it is debatable whether the health system in a country is public or subsidized, in some OCED countries such as UK and Canada, the health system is closer to a public system to which the access is nearly free, and the total health spending is strongly influenced by the government budget (e.g. through bulk funding negotiation). By contrast, the health system in the US is largely private and is subsidized in various ways (e.g. subsidizing health insurance directly or indirectly via tax exemption on employer-provided health insurances). If our interpretation of the health system is correct, the numerical results suggest that the socially optimal ratio of health spending to output at 11.48% is closer to the 11.2% counterpart in Canada and 9.4% in the UK than to the 17.7% in the US (OECD, 2013).

### 8. Conclusion

The literature has been concerned with excessive health spending for longevity induced by longevity-dependent annuities. In the present paper, we broaden the concern for savings and fertility decisions. Such extended considerations justify the interaction between social security and public health in terms of the effects on allocation and welfare, compared to their separate effects in the literature. With endogenous fertility, government health programs financed by income taxes alone cannot achieve the social optimum. Moreover, as the existing analysis of health investment and longevity is typically based on lifecycle models, we analyze them along with social security and public health programs in a dynastic family model. The dynastic model relaxes lifecycle constraints and enriches incentives through intergenerational altruism and intergenerational transfers.

The use of the dynastic model allows us to determine the socially optimal allocation concerning the welfare of all generations. The social optimum allows us to compare the distortions of private annuities, social security annuities, and various public health programs, and to explore socially optimal government policies. We have shown analytically that social security
and health subsidization not only strengthen incentives from longevity-dependent annuities for over-investment in health expenditure as found in the literature but also distort fertility. In an example using CRRA utility and Cobb-Douglas technology, we have shown analytically that social security indeed increases private health expenditure and reduces fertility under plausible conditions. Further, we have found analytically that social security and public health together can achieve the social optimum. Public health with free access allows the government to set an optimal level of health spending, while the side effect on fertility via income taxes can be removed by social security under empirically plausible conditions.

Quantitatively, our numerical results based on plausible parameterization help to explain why health spending in the United States with a subsidized private health system is much higher than those in Canada and the United Kingdom with a universal public health system. Also, the numerical results suggest that the United States should increase social security and provide universal access to publically funded health care as in other industrial nations. On the contrary, major European economies should downsize social security.
Appendix A

Proof of Proposition 1. Define the state vector and the return function in each period as

$$q_t = (m_{t-1}, n_{t-1}, K_t),$$

$$W(t) \equiv W(q_t, d_t, q_{t+1}) = \beta T(m_{t-1})U(d_t) + \alpha [U(c_t) + \rho G(n_t)].$$

We then obtain:

$$W_{m_{t-1}}(t) = T'(m_{t-1}) \left[ \beta U(d_t) - \frac{\alpha U'(c_t)d_t}{n_{t-1}} \right] > 0 \text{ (for an interior solution),}$$

$$W_{m_t}(t) = -\alpha U'(c_t),$$

$$W_{K_t}(t) = \alpha U'(c_t)f_K(K_t, 1 - vn_t) > 0,$$

$$W_{K_{t+1}}(t) = -\alpha U'(c_t)n_t,$$

$$W_{d_t}(t) = T(m_{t-1}) \left[ \beta U'(d_t) - \frac{\alpha U'(c_t)}{n_{t-1}} \right],$$

$$W_{n_{t-1}}(t) = \frac{\alpha U'(c_t)d_tT(m_{t-1})}{n_{t-1}^2} > 0,$$

$$W_{n_t}(t) = \alpha [-U'(c_t)(vN_f(K_t, 1 - vn_t) + K_{t+1}) + \rho G'(n_t)].$$

where $W(t)$ is increasing in $m_{t-1}, K_t,$ and $n_{t-1}.$

From the variational approach, any optimal interior solution $(d^*_t, q^*_{t+1})_{t=0}^\infty$ to

$$\sup_{(d_t, q_{t+1})_{t=0}^\infty} \sum_{t=0}^\infty \alpha^t W(q_t, d_t, q_{t+1})$$

also solves

$$\sup_{(d_t, q_{t+1})_{t=0}^\infty} [W(q^*_t, d_t, q_{t+1}) + \alpha W(q_{t+1}, d^*_{t+1}, q^*_{t+2})].$$

The first-order (necessary) conditions for an interior solution for $t = 0, 1, 2 \ldots$ are:

$$W_{d_t}(t) = 0,$$

$$W_{n_t}(t) + \alpha W_{n_t}(t + 1) = 0,$$

$$W_{K_{t+1}}(t) + \alpha W_{K_{t+1}}(t + 1) = 0.$$
$W_{m_t}(t) + \alpha W_{m_t}(t + 1) = 0.$

These first-order conditions correspond to (11)-(14) and lead to

$$\alpha^t W_{n_t}(t)n_t = -\alpha^{t+1} W_{n_t}(t+1)n_t,$$

$$\alpha^t W_{K_t+1}(t)K_{t+1} = -\alpha^{t+1} W_{K_t+1}(t+1)K_{t+1},$$

$$\alpha^t W_{m_t}(t)m_t = -\alpha^{t+1} W_{m_t}(t+1)m_t.$$

The transversality conditions are therefore given by:

$$\lim_{t \to \infty} \alpha^t W_{n_{t-1}}(t)n_{t-1} = 0, \lim_{t \to \infty} \alpha^t W_{K_t}(t)K_t = 0, \lim_{t \to \infty} \alpha^t W_{m_{t-1}}(t)m_{t-1} = 0,$$

which correspond to the transversality conditions given earlier.

Let $W^*(t)$ be evaluated on the feasible path $(d_t^*, m_t^*, K_t^{*+1}, n_t^*)_{t=0}^\infty$ satisfying the first-order conditions, the transversality conditions and Assumption 1. And let $(d_t, m_t, n_t, K_t^{*+1})_{t=0}^\infty$ be any alternative feasible (nonnegative) sequence starting from the same initial state $(m_{-1}, K_0, n_{-1}).$ We want to show

$$\lim_{T \to \infty} \sum_{t=0}^T \alpha^t \left[ W^*(m_{t-1}^*, m_t^*, K_t^*, K_{t+1}^*, n_{t-1}^*, n_t^*, d_t^*) - W(m_{t-1}, m_t, K_t, K_{t+1}, n_{t-1}, n_t, d_t) \right] \geq 0.$$

Define

$$\Delta_T = \sum_{t=0}^T \alpha^t \left[ W^*(m_{t-1}^*, m_t^*, K_t^*, K_{t+1}^*, n_{t-1}^*, n_t^*, d_t^*) - W(m_{t-1}, m_t, K_t, K_{t+1}, n_{t-1}, n_t, d_t) \right].$$

By strict concavity of $W(m_{t-1}, m_t, K_t, K_{t+1}, n_{t-1}, n_t, d_t)$ in Assumption 1, we have

$$\Delta_T \succ \sum_{t=0}^T \alpha^t \left[ W^*_{m_{t-1}}(t)(m_{t-1}^* - m_{t-1}) + W^*_{m_t}(t)(m_t^* - m_t) + W^*_{K_t}(t)(K_t^* - K_t) + W^*_{K_{t+1}}(t)(K_{t+1}^* - K_{t+1}) + W^*_{n_{t-1}}(t)(n_{t-1}^* - n_{t-1}) + W^*_{n_t}(t)(n_t^* - n_t) + W^*_{d_t}(t)(d_t^* - d_t) \right] + \sum_{t=0}^T \alpha^t \left[ [W^*_{m_t}(t) + \alpha W^*_{m_t}(t + 1)](m_t^* - m_t) + [W^*_{K_t}(t) + \alpha W^*_{K_t}(t + 1)](K_t^* - K_t) + \alpha W^*_{K_{t+1}}(t)(K_{t+1}^* - K_{t+1}) + W^*_{n_t}(t)(n_t^* - n_t) + \alpha W^*_{n_t}(t)(n_t^* - n_t) \right].$$

Since $W^*_{d_t}(t) = 0$ for $t = 0, 1, 2, \ldots$, the terms containing this factor drop out. The initial terms drop out too because $m_{-1}^* = m_{-1}, K_0^* = K_0,$ and $n_{-1}^* = n_{-1}$. Using the intertemporal conditions
\[ W_n^*(t) + \alpha W_n^*(t + 1) = 0, \quad W_{n_t}^*(t + 1) = 0, \quad \text{and} \quad W_n^*(t) + \alpha W_n^*(t + 1) = 0, \]
the terms with \((m_t^n - m_t), (K_t^n - K_{t+1}^n), \) and \((n_t^n - n_t)\) for \(t = 0, 1, ..., T - 1\) also drop out. The remaining terms are

\[
\begin{align*}
\alpha^T & \left[W_{m_t}^*(T)(m_T - m_T) + W_{K_t}^*(T)(K_{T+1}^* - K_{T+1}^*) + W_{n_T}^*(T)(n_T^* - n_T) \right] = \\
& -\alpha^{T+1} \left[W_{m_T}^*(T + 1)(m_{T+1} - m_T) + W_{K_T}^*(T + 1)(K_{T+1}^* - K_{T+1}^*) + W_{n_T}^*(T + 1)(n_{T+1}^* - n_T) \right] \geq \\
& -\alpha^{T+1} \left[W_{m_T}^*(T + 1)m_{T+1}^* + W_{K_T}^*(T + 1)K_{T+1}^* + W_{n_T}^*(T + 1)n_{T+1}^* \right], \forall m_T, K_{T+1}, n_T \geq 0,
\end{align*}
\]

since \(W_{m_t}^*(T + 1) > 0, W_{K_t}^*(T + 1) > 0, \) and \(W_{n_t}^*(T + 1) > 0. \) Thus, \(\Delta_T > -\alpha^{T+1} [W_{m_T}^*(T + 1)m_{T+1}^* + W_{K_T}^*(T + 1)K_{T+1}^* + W_{n_T}^*(T + 1)n_{T+1}^*]. \) From the transversality conditions, we obtain

\[
\lim_{T \to \infty} \Delta_T > 0 \text{ for all alternative feasible allocations } m_t \geq 0, K_{t+1} \geq 0, n_t \geq 0 \text{ outside the socially optimal allocation.} \]

**Proof of Proposition 3.** By differentiating \(V_n^u = 0\) in (58) with respect to \(\tau^B\) via consumption, \(c,\) capital stock, \(K,\) and fertility, \(n,\) we obtain:

\[
\frac{dn}{d\tau^B} = \frac{1}{V_{nn}^u} \left[ - \frac{\partial V_n^u}{\partial \tau^B} \right],
\]

where \(V_{nn}^u < 0\) for a concave programing problem and

\[
\frac{\partial V_n^u}{\partial \tau^B} = -\frac{c^{\alpha(1-\theta)K}n^{(1-vn)}}{\alpha \theta (1-vn)} < 0, \text{ if } \alpha > vn.
\]

Thus, \(dn/d\tau^B < 0.\) Consequently, we obtain \(dK/d\tau^B > 0, \) \(dY/d\tau^B > 0, \) through \(K = (\alpha \theta A/n)^{1/(1-\theta)} (1 - vn), \) and \(Y = AK^{\theta} (1 - vn)^{1-\theta}. \)

**Proof of Proposition 4.** We use the first-order-condition approach in steady state. Note that equations (56) and (57) concerning consumption in young age and old age are the same as (44) and (45), and that (46) applies in both cases for capital. Using the optimal condition in (60) and (61) in (58) makes it the same as the counterpart (47) in the social planner’s allocation concerning fertility. Note that \((\tau^B)^*\) is positive when the taste for the welfare of children is sufficiently large, that is when \(\alpha > vn.\) When the government chooses \(m\) subject to its budget constraints and the household budget constraint, the first-order condition in steady state is the same as (48) in the social planner’s allocation. Equation (61) is essentially the government budget constraint for public health, \(m = \tau^m (1 - \theta)nK/\alpha \theta,\) used to solve for \(\tau^m. \)

30
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Table 1. Health spending and life expectancy at 20 in the US economy- data versus simulation

<table>
<thead>
<tr>
<th>Period</th>
<th>Data</th>
<th>Calibration:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \delta = 910, \epsilon = 1, \varphi = 0.7752, p = 0.5111 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Health</td>
<td>Life expectancy at 20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>spending</td>
<td>longevity</td>
</tr>
<tr>
<td>1960-70</td>
<td>1127.67</td>
<td>52.79</td>
<td>0.325</td>
<td>52.9244</td>
</tr>
<tr>
<td>1980-90</td>
<td>3134.81</td>
<td>56.045</td>
<td>0.4011</td>
<td>56.7801</td>
</tr>
<tr>
<td>2000-09</td>
<td>6770.1</td>
<td>58.541</td>
<td>0.4635</td>
<td>58.5413</td>
</tr>
</tbody>
</table>

Sources: data for life expectancy is from Centers for Disease Control and Prevention; data for health spending is from Centers for Medicare and Medicaid services.

Table 2. Observations in the US economy

<table>
<thead>
<tr>
<th>Variables</th>
<th>Average 2000-2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fertility</td>
<td>2.06</td>
</tr>
<tr>
<td>Time spent on working</td>
<td>0.7165</td>
</tr>
<tr>
<td>Investment (% of GDP)</td>
<td>0.1906</td>
</tr>
<tr>
<td>Total health spending (% of GDP)</td>
<td>0.1618</td>
</tr>
<tr>
<td>Health subsidization (% of total health spending)</td>
<td>0.39</td>
</tr>
<tr>
<td>Life expectancy at 20 (average 2000-2009)</td>
<td>58.541 years</td>
</tr>
<tr>
<td>Effective rate of pension contribution</td>
<td>0.124</td>
</tr>
<tr>
<td>GDP per capita (constant 2005 US dollar)</td>
<td>41,842.3</td>
</tr>
</tbody>
</table>

Note: Time spent on working refers to the fraction (%) of time parents with full employment status and children under 18 spent on working on an average day.

Source: data for fertility are from OECD Family Database; data for time spent on working are from the US Bureau of Labor Statistics; data for investment is from IMF (2011); data for health spending and GDP per capita (constant 2005 US dollar) are from Centers for Medicare and Medicaid Services; data for life expectancy are from Centers for Disease Control and Prevention; and data for the effective rate of pension contribution are from OECD (2011).
Table 3. List of baseline parameterization

<table>
<thead>
<tr>
<th>Population</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 40$</td>
<td>Number of years per period</td>
</tr>
<tr>
<td>$v = 0.2752$</td>
<td>Fixed time rearing a child</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Utility</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.5837$</td>
<td>Intergenerational discounting factor</td>
</tr>
<tr>
<td>$\beta = 0.43$</td>
<td>Taste parameter for parental old-age consumption</td>
</tr>
<tr>
<td>$\rho = 0.77$</td>
<td>Taste parameter for the number of children</td>
</tr>
<tr>
<td>$\sigma = 1.0009$</td>
<td>Reciprocal of inter-temporal elasticity of substitution (consumption)</td>
</tr>
<tr>
<td>$\gamma = 1.3762$</td>
<td>Reciprocal of inter-temporal elasticity of substitution (fertility)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Production of final output</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 0.3267$</td>
<td>Capital share</td>
</tr>
<tr>
<td>$A = 2809$</td>
<td>Total factor productivity</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Function of life extension in old age</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta = 910$</td>
<td>Positive factor on marginal efficiency of health spending</td>
</tr>
<tr>
<td>$\epsilon = 1.000$</td>
<td>Negative factor on marginal efficiency of health spending</td>
</tr>
<tr>
<td>$\varphi = 0.7752$</td>
<td>Return factor on health spending</td>
</tr>
<tr>
<td>$p = 0.5111$</td>
<td>Autonomous factor of longevity</td>
</tr>
</tbody>
</table>
Table 4. Numerical results at steady states

<table>
<thead>
<tr>
<th>Policies</th>
<th>fertility</th>
<th>old life</th>
<th>health</th>
<th>Capital</th>
<th>Output</th>
<th>( \frac{m}{Y} ) (%)</th>
<th>( \frac{\Delta m}{m} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i. Laissez faire</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau^B = \tau^m = 0 )</td>
<td>1.1013</td>
<td>0.4488</td>
<td>4985</td>
<td>6823</td>
<td>39402</td>
<td>12.65</td>
<td>11.72</td>
</tr>
<tr>
<td>ii. Social security</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau^B = 0.124, \tau^m = 0 )</td>
<td>1.0324</td>
<td>0.4518</td>
<td>5281</td>
<td>7714</td>
<td>41763</td>
<td>12.65</td>
<td>18.35</td>
</tr>
<tr>
<td>iii. Health subsidy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau^B = 0, \tau^m = 0.0937 )</td>
<td>1.1047</td>
<td>0.4608</td>
<td>6371</td>
<td>6782</td>
<td>39290</td>
<td>16.22</td>
<td>42.78</td>
</tr>
<tr>
<td>iv. The US calibration</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau^u = 0.124, \tau^m = 0.0937 )</td>
<td>1.03</td>
<td>0.4635</td>
<td>6771</td>
<td>7748</td>
<td>41849</td>
<td>16.18</td>
<td>51.75</td>
</tr>
<tr>
<td>v. Social optimum</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau^u = 0.19, \tau^m = 0.1705 )</td>
<td>1.1179</td>
<td>0.4426</td>
<td>4462</td>
<td>6629</td>
<td>38861</td>
<td>11.48</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: Numerical results are based on the calibrated parameterization in Table 3; the percentage change in health spending in the last column is compared with the social optimum; in the social optimum, public health spending equals health spending.
Figure 1. The calibrated old-age longevity, $T(m)$, in the US economy

Note: The three ♦-shaped points are observed old-age longevity divided by 40 at the corresponding levels of health spending. The curve is based on the calibration $\delta = 910, \epsilon = 1, \varphi = 0.7752, p = 0.5111$. 