Margin debt and portfolio margin requirements

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Abstract

This paper investigates the effects of a change in the margin rules of the U.S. stock market. These rules determine how much investors can borrow to leverage their investments. Since the 1929 stock market crash, margin loans have been tightly regulated by the Securities and Exchange Act Regulation T. Between 2005 and 2008, the Securities and Exchange Commission modified these margin rules because they were perceived as not adequately reflecting investment risk. The amended rules have made it more attractive for investors to borrow by opening new margin accounts and diversifying their investment positions. This paper tests the hypothesis that the change in the margin rules has accelerated growth in margin debt across the U.S. stock market. It provides statistical evidence that the beginning of this acceleration can be dated to the change in the rules. Since the 2008 financial crisis, margin debt has grown rapidly, reaching previously unseen levels. This worrying trend has been intensified by record low interest rates and rising stock values. These facts present new incentives for reassessing the efficacy of margin rules and margin requirements.

Keywords: Margin debt, margin requirements, portfolio margining, financial regulations, structural change, U.S. stock market.

JEL: G18, G28

1. Introduction

Investors who purchase securities can borrow part of the purchase price from financial institutions, such as brokerage firms, by opening margin accounts. They are required to deposit a portion of the purchase price, the margin, which represents the initial equity in the accounts. The portion that must be deposited is called the initial margin requirement and is calculated by following margin rules that are regulated by the U.S. Federal Reserve Board. The margin debt held by investors with their financial institutions is secured by the purchased securities. There is also a maintenance margin requirement that determines the amount necessary to be kept as collateral in the margin account for the duration of an investment. Whenever the margin is below the requirement, the broker issues a margin call. Investors typically use margin accounts to leverage their investments and increase their purchasing power.

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Since the 1929 stock market crash, the U.S. Federal Reserve Board has been given authority to regulate margin loans with the Securities and Exchange Act Regulation T of 1934. One of the objectives of this act is the “control of monetary aggregates” (Climan, 1978). The U.S. Federal Reserve Board has achieved this by establishing an initial margin requirement for margin loans. This requirement sets a minimum equity position on the date of a credit-financed security transaction. The initial margin requirement for listed stocks has changed 22 times between 1934 and 1974 and has been as high as 100 percent and as low as 40 percent. Since 1974, however, the initial margin requirement has been fixed at 50 percent of the current market value of the stock.

During the 1980s and 1990s, margin requirements were often perceived as too high and not accurately representing investment risks. In order to more accurately represent risk, the Option Clearing Corporation developed a new portfolio margining methodology where portfolio margin requirements were calculated by using the Theoretical Intermarket Margining System (TIMS). TIMS was first implemented in 1997 to calculate the net capital requirements for brokers’ proprietary portfolios of listed options (SEC, 1997; GAO, 1998). However, this margining methodology was not used to margin customer accounts prior to 2005. After two proposals from the New York Stock Exchange, the Securities and Exchange Commission (SEC) approved the use of the portfolio margining methodology under a temporary pilot programme (SEC, 2002, 2004, 2005).

The pilot programme was implemented in three phases. Phase I began on July 14, 2005 and permitted the use of the portfolio margining methodology only for margin accounts with listed broad-based indices and exchange-traded funds derivatives. Phase II began on July 11, 2006 and included listed stock options and securities futures (SEC, 2006a). Phase III began on April 2, 2007 and included equities, equity options, unlisted derivatives and narrow-based index futures. Unlike Phase I, the changes in margin requirements in Phases II and III were more significant because they targeted a much wider array of securities. Furthermore, Phase III was widely advertised in the media after its approval on December 12, 2006 (SEC, 2006b), more than 3 months before it would become effective. By 2008, the SEC ended the pilot programme, thus making portfolio margining permanent and leading to the amendment of the margin rules of the Financial Industry Regulatory Authority (FINRA), the New York Stock Exchange (NYSE) and the Chicago Board Options Exchange (CBOE).

The new portfolio margin requirements are substantially lower than the original margin requirements, which were termed strategy-based margin requirements by the SEC. The primary difference between these two methodologies lies in the way a portfolio with a single underlying instrument is margined, meaning that all derivatives in the portfolio are on the same underlying instrument. Consider, for example, the portfolio in Figure 1 with several investment positions listed A to I. Strategy-based margining initially requires determining an allowable decomposition of the portfolio into hedging strategies (or offsets) as illustrated in Figure 1(a). It then requires calculating the margin requirements for each offset in the decomposition as a current (or maximum, in some cases) loss associated with that offset. The sum of these losses constitutes the required margin. Portfolio margining, however, con-

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1It is now known as System for Theoretical Analysis and Numerical Simulations (STANS).
Figure 1: Examples of margining approaches. The terms “call butterfly”, “bull put” and “put condor” are standard terminology for investment positions in finance; refer, for example, to Cohen (2005) for formal definitions.

considers the entire portfolio to be a single fixed offset. The portfolio is subjected to numerical simulations in which the price of the underlying security is assumed to change according to a pre-specified system of 11 valuation points. All positions in the portfolio are evaluated with respect to their potential gains and losses for each valuation point. The potential portfolio loss (or gain) is then determined by adding the potential gains and subtracting the potential losses of all positions. The greatest value among the 11 net losses constitutes the required margin. It should be noted that, under the amended rules, margining can be performed with either methodology.

In a 2006 promotional communication to its clients, the CBOE provided several numerical examples that demonstrated the potential benefit of the new approach to margining. Two of these examples are reproduced in Figure 2. In all of the examples provided by the CBOE, including the two reproduced in this paper, portfolio margining produced lower margin requirements; in some cases, more than 40 times lower. The difference is particularly noticeable for portfolios containing options, i.e., the securities that were part of Phases II and III of the pilot programme.

This paper hypothesises that the amendments to the margin rules introduced between 2005 and 2008 stimulated investor borrowing due to the lower margin requirements; as a result, there was an accelerated growth in margin debt. The lower initial and maintenance margin requirements made it more attractive for new investors to open margin accounts and for existing investors to diversify their investment positions. This paper provides statistical evidence that the onset of this rapid growth in margin debt can be dated to the change in margin rules, which should be apparent during Phases II and III as discussed earlier. The proposed hypothesis is tested on models of the margin debt series for the U.S. stock market, after adjustments for stock market inflation. This is a monthly series and the only data on margin debt publicly available.

Figure 3 depicts the margin debt series for the U.S. stock market from July 2001 to
February 2013. It is worth noting that, prior to the change in margin rules, investors already displayed bullish borrowing behaviour, as margin debt grew by 72% over the three years between September 2002 and July 2005. Between July 2006 and its peak in July 2007, however, the upward trend in margin debt accelerated by 63% in a single year.

Section 2 initially discusses a theoretical model formalising the effect of margin requirements and stock price movements on margin loans. The main hypothesis of the paper is then stated and discussed. Section 3 describes the empirical model and methods used to test the proposed hypothesis. The methods used provide rigorous ways to test for structural change in a non-stationary process. Section 4 presents the data and the major results supporting this paper’s hypothesis. Finally, Section 5 provides a discussion on the implications of the change in margin rules for margin debt and margin requirements.

2. Hypothesis

The existing literature on margin requirements is surprisingly sparse given the long history of the regulation. Most of the research has been conducted prior to 2005. The earliest work on the topic is by Moore (1966) and studies the influence of margin requirements on investors’ equity ratios. Later research conducted on margin requirements has mostly focused on their impact on stock volatility; see, for example, Hardouvelis (1990), Hardouvelis and Peristiani (1992), Salinger (1990) and Hardouvelis and Theodossiou (2002). Hardouvelis (1990) found that margin requirements appeared to be an effective policy tool in curbing destabilising speculation by lowering stock price volatility, while several studies, including Schwert (1989), Kupiec (1989), Salinger (1990) and Hsieh and Miller (1990), found that margin requirements had no significant impact on stock price volatility. This debate is beyond the scope of this paper.

There are also differing views on the connection between margin debt and stock prices because the evidence is both weak and sparse. This connection is typically made by looking at the impact of margin requirements on stock prices. Grube et al. (1979) showed that lower
margin requirements were associated with higher prices but not the reverse. Luckett (1982) found that higher margin requirements tended to reduce equity holdings.

However, Greenspan (2004) argued that not only was margin debt a small fraction of stock market capitalisation, but it was also very unlikely to affect stock price. In fact, the total amount of NYSE margin debt varied between 2.3% and 2.9% of the Standard & Poor’s 500 index market capitalisation at its pre-2008 crisis peak and at its crisis low (Scaggs and Russolillo, 2013). Fortune (2000) observed that the ratio of margin debt to stock value (S&P 500) was high when stock prices were low and vice versa, contradicting the conventional view that increased margin debt leads to higher stock prices. It is in the light of this weak connection between margin debt and stock prices that investors are assumed to be the price takers in the model of margin requirements and margin loans shown below.

Early attempts to model margin requirements and margin loans include Ricke (2003) and Fortune (2000). Ricke (2003) found that, under certain conditions, the availability of margin credit can cause a bubble in the price of a risky asset. This assertion is beyond the scope of this paper. In contrast, the following model illustrates the effect of margin requirements and stock price movements on margin loans.
Assume that an investor’s entire wealth is invested into a single asset and that the investor takes the maximum margin loan possible. Let \( w \) be the initial wealth of the investor, \( q \) be the size of the margin loan and \( m \) be the margin requirement. Without any loss of generality, it is assumed that the initial and maintenance margin requirements are the same. The following relationship must then hold:

\[
\frac{w}{w + q} \geq m
\]  
(1)

where (1) is tight for the maximum value of \( q \). Also, let \( p \) be the asset price, \( n \) be the number of purchased stocks and \( \pi \) be the portfolio value. Then \( \pi = p \cdot n \) and the following relationship must be satisfied:

\[
\frac{\pi - q}{\pi} = m
\]  
(2)

for the maximum value of the margin loan. Suppose that the price of the asset changes from \( p \) to \( \tilde{p} = r \cdot p \), where \( r \) is the ratio of the new and old asset price. Then the value of the portfolio changes to \( \tilde{\pi} = \tilde{p} \cdot n = r \cdot \pi \).

If \( r \geq 1 \), then

\[
\tilde{\pi} \geq \pi \quad \Rightarrow \quad \frac{\tilde{\pi} - q}{\tilde{\pi}} \geq m
\]  
(3)

which means that the investor can increase the size of the margin loan; but how much more can be borrowed? The investor can borrow up to \( \tilde{q} \), which is determined as

\[
\tilde{q} = \frac{\tilde{\pi} (1 - m)}{m} = q \cdot \frac{\tilde{w}}{w} = q \cdot \frac{m + r - 1}{m}
\]  
(4)

where the new wealth is \( \tilde{w} = \tilde{\pi} - q \).

If \( r < 1 \), then

\[
\tilde{\pi} < \pi \quad \Rightarrow \quad \frac{\tilde{\pi} - q}{\pi} < m
\]  
(5)

which means that the investor will receive a margin call. The margin call can be fulfilled in two ways: by cash deposit or by partial liquidation of the portfolio. If we consider a cash deposit \( c \), then the minimum amount of \( c \) needed to satisfy the margin requirement follows from

\[
\frac{\tilde{\pi} - (q - c)}{\tilde{\pi}} = m \quad \Rightarrow \quad c = q - (1 - m)\tilde{\pi}.
\]  
(6)

In this case, the new margin loan becomes:

\[
\tilde{q} = q - c = (1 - m)\tilde{\pi} = (1 - m)r \cdot \pi = q \cdot r.
\]  
(7)

If we then consider a partial portfolio liquidation to fulfil the margin call, a portion \( \alpha \) of the portfolio is liquidated and its proceeds are used to reduce the size of the loan. The minimum \( \alpha \) portion can be determined from

\[
\frac{(1 - \alpha)\tilde{\pi} - (q - \alpha\tilde{\pi})}{(1 - \alpha)\tilde{\pi}} = m \quad \Rightarrow \quad \alpha = \frac{1 - m}{m} \cdot \frac{1 - r}{r}.
\]  
(8)
In this case, the new margin loan becomes:
\[
\tilde{q} = q - \alpha \bar{\pi} = q - \frac{1 - m}{m} \cdot \frac{1 - r}{r} \cdot \bar{\pi} = q \cdot \frac{m + r - 1}{m}.
\] (9)

Note that the term \(m + r - 1/m\) that appears in (4) and (9) will be (since \(m < 1\)) greater than \(r\) when \(r > 1\) and less than \(r\) when \(r \leq 1\). Equations (4), (7) and (9) demonstrate that the size of the margin loan is affected by the changes in the asset price. Furthermore, it is apparent that margin requirements have a direct influence on the size of margin loans. This theoretical finding is empirically corroborated by Hsieh and Miller (1990), who found a negative relationship between margin requirements and the amount of margin credit outstanding, i.e., margin debt. The following examples illustrate these assertions.

**Example 1.** Suppose that a margin requirement is 50% and that the asset price increases by 20%, i.e., \(m = 0.5\) and \(r = 1.2\). Based on (4), the new maximal margin loan is \(\tilde{q} = 1.4 \cdot q\), i.e., it increases by 40%. Now suppose that the asset price decreases by 20%, i.e., \(r = 0.8\). If the investor decides to use cash to fulfil the margin call, then, based on (7), the new margin loan is \(\tilde{q} = 0.8 \cdot q\), i.e., it decreases by 20%. If, however, the investor decides to use partial liquidation to fulfil the margin call, then, based on (9), the new margin loan is \(\tilde{q} = 0.6 \cdot q\), i.e., it decreases by 40%.

Example 1 illustrates the effect of asset price movements on margin loans and shows that the apparent inflationary effect of the stock market must be taken into account in order to isolate the effect of the change in the margin rules on the size of margin loans.

**Example 2.** If the margin requirement is 50%, i.e., \(m = 0.5\), then, based on (1), the maximal margin loan is \(q = w\). Now suppose that the asset price is fixed and the margin requirement increases by 20%, i.e., \(m = 0.6\). Based on (1), the new margin loan is \(\tilde{q} = 2/3 \cdot w = 2/3 \cdot q\), i.e., it decreases by 33%. Suppose, however, that the margin requirement decreases by 20%, i.e., \(m = 0.4\). Then, based on (1), the new margin loan is \(\tilde{q} = 3/2 \cdot w = 3/2 \cdot q\), i.e., it increases by 50%.

Example 2 illustrates the effect of change in margin requirements on margin loans. This example underscores the central question raised in this paper.

This paper posits the following hypothesis to be empirically investigated:

**Hypothesis.** The change in margin rules initiated from 2005 until 2008 has accelerated the growth in margin debt across the U.S. stock market.

This change should be particularly apparent during Phase II, as this phase included a wider range of securities. The change in margin rules, which mostly implied a decrease in margin requirements \((m)\), facilitated investors’ access to margin loans \((q)\) as shown in Example 2. Both the lower initial and maintenance margin requirements gradually encouraged new investors to open margin accounts and existing investors to diversify their investment positions. This change in investment behaviour should be observable in the monthly margin debt series for the U.S. stock market over time, which is the only publicly available data. Section 4 empirically investigates whether this change in margin rules can be detected and dated in the margin debt series. A common way to test for such a change is to use structural change tests, which are discussed in the next section.
3. Methods

As with most financial time series such as prices and indices, margin debt is non-stationary (see Figure 3). Therefore, the more popular structural change tests are not appropriate because they typically require the assumption of stationarity. Furthermore, there is no prescribed structural model for margin debt. Consequently, this paper employs a first-order autoregressive model (AR), which is commonly used in existing finance literature for non-stationary series. Specifically, the model used for margin debt is expressed as

\[ y_t = \rho_t y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d} \ (0, \sigma^2) \]  

(10)

for the sample \( t \in [1, T] \), where \( \rho_t \) is the AR(1) parameter and \( y_t \) is the margin debt series. When \( \rho_t = 1 \), \( y_t \) is a random walk. In some instances, the data series \( y_t \) may need to be demeaned and de-trended as discussed in Homm and Breitung (2012). All data manipulations are detailed in Section 4.

This paper’s hypothesis is tested with a modified Chow (1960) test for structural change that allows for non-stationary time series. Homm and Breitung (2012) proposed this modification of the Chow (1960) test and found that it provided a reliable estimator for the break date and that the test exhibited high power. In the spirit of Chow (1960), model (10) can be re-written under the assumption that \( \rho_t = 1 \) for \( t = 1, \ldots, [\tau^* T] \) and \( \rho_t - 1 = \delta \) for \( t = [\tau^* T] + 1, \ldots, T \), as follows:

\[ \Delta y_t = \delta \left( y_{t-1} 1_{\{t > [\tau^* T]\}} \right) + \varepsilon_t \]  

(11)

where \( 1_{\{\cdot\}} \) is an indicator function, \( \tau^* \in (0, 1) \), \([\tau^* T]\) is an unknown break point and \( \lfloor \cdot \rfloor \) denotes the largest integer that is smaller than the argument.

The null hypothesis of no structural change

\[ H_0 : \delta = 0 \quad \text{for} \ t = 1, \ldots, [\tau^* T], \ldots, T, \]  

(12)

is tested against the alternative that there is a structural change at an unknown break point:

\[ H_1 : \delta > 0 \quad \text{for} \ t > [\tau^* T]. \]  

(13)

These hypotheses clearly demonstrate this paper’s premise. The margin rules have been gradually amended during the pilot programme. The amendments co-exist alongside the old rules rather than supplanting them. It is natural to expect that investors progressively adapted their investment strategies as brokers began implementing the new portfolio margin requirements and offering them to their clients. Furthermore, the one-sided alternative hypothesis accounts for the growth in the margin debt due to the change in rules.

A simple way to test for the accelerated growth in margin debt is to compute a t-statistic for the difference between the mean growth of \( y_t \) before and after the break \([\tau^* T]\). The result of this test is reported in Section 4. One problem with this approach, however, is that it does not allow for an estimation of the break date \([\tau^* T]\). This issue is addressed in the tests below.
Under the null hypothesis (12), the t-statistic is calculated from

\[
DFC_\tau = \sum_{t=\lfloor \tau T \rfloor + 1}^{T} \Delta y_t y_{t-1} \overline{\delta_\tau} \sqrt{\sum_{t=\lfloor \tau T \rfloor + 1}^{T} y_t^2 - 1}
\]

with \( \overline{\delta_\tau} = \frac{1}{T-2} \sum_{t=2}^{T} \left( \Delta y_t - \hat{\delta}_{OLS} y_{t-1} \mathbb{1}_{\{t > \lfloor \tau T \rfloor \}} \right)^2 \), where \( \hat{\delta}_{OLS} \) is the OLS estimator of \( \delta \). The supremum Chow-type DF statistic to test for a structural change is

\[
\supDFC(\tau_0) = \sup_{\tau \in [0, 1-\tau_0]} DFC_\tau \overset{d}{\rightarrow} \sup_{\tau \in [0, 1-\tau_0]} \frac{\int_{\tau}^{1} W(r) dW(r)}{\sqrt{\int_{\tau}^{1} W(r)^2 dr}}
\]

where \( W \) is a standard Brownian motion and \( \overset{d}{\rightarrow} \) means convergence in distribution. The test is computed in the interval \( \tau \in [0, 1-\tau_0] \) and \( \tau_0 \) is usually fixed at 0.1 for estimation consistency. The null hypothesis \( \delta = 0 \) is rejected for large values of \( \supDFC(\tau_0) \). The test is also a one-sided “supWald” test statistic proposed by Andrews (1993).

Under the alternative hypothesis (13), the break date \( \lfloor \tau^* T \rfloor \) is unknown. \( \tau^* \) can be estimated by using the value

\[
\hat{\tau}_{DFC} = \arg \max_{\tau \in [0, 1-\tau_0]} DFC_\tau
\]

in the Chow-type statistic. \( \hat{\tau}_{DFC} \) maximises the likelihood function with respect to the break date.

Various other statistics have been suggested by Homm and Breitung (2012) to test for structural change when the time series is non-stationary. These statistics are modifications of the Bhargava (1986), Kim (2000) and Busetti and Taylor (2004) tests, which originally focused on testing for a change from stationary to non-stationary regimes or vice versa. Homm and Breitung (2012) adapted these tests such that \( y_t \) is a random walk under the null hypothesis with \( \rho_t = 1 \) for all \( t \) and under the alternative of a change from \( \rho_t = 1 \) for \( t = 1, \ldots, \lfloor \tau^* T \rfloor \) to \( \rho_t = \rho^* > 1 \) for \( t = \lfloor \tau^* T \rfloor + 1, \ldots, T \). These hypotheses also capture the structural change that would be induced from the amendments in margin rules resulting in the accelerated growth of the margin debt. Among the aforementioned statistics, the modification of the Busetti and Taylor (2004) test has a higher statistical power.

This paper utilises the modified Busetti and Taylor (2004) test to confirm the evidence found with the \( \supDFC \) test, which is defined as

\[
BT_\tau = \frac{1}{\sigma^2_0 (T - \lfloor \tau T \rfloor)^2} \sum_{t=\lfloor \tau T \rfloor + 1}^{T} (y_T - y_{t-1})^2.
\]

The \( BT_\tau \) test is based on the sum of squared forecast errors of forecasting the final value \( y_T \) from the periods \( y_{\lfloor \tau T \rfloor + 1}, \ldots, y_{T-1} \) under the null hypothesis. Note that the variance
estimator, $\hat{\sigma}^2_0$, uses the entire sample. The supremum test can be written as

$$\text{sup}_{\tau \in [0, 1-\tau_0]} B_{T\tau} \overset{d}{=} \sup_{\tau \in [0, 1-\tau_0]} \left( (1-\tau)^{-2} \int_\tau^1 W(1-r)^2 dr \right).$$

The supBT test rejects the null for large values of supBT($\tau_0$). For further details on the asymptotic distribution, refer to Homm and Breitung (2012).

The Busetti and Taylor (2004) statistic can be adequately modified to estimate $\tau^*$ as

$$\hat{\tau}_{\text{BT}} = \arg \max_{\tau \in [0, 1-\tau_0]} \Lambda(\tau), \quad \text{where } \Lambda(\tau) = \frac{\sum_{t=\lfloor\tau T\rfloor+1}^{T} (y_T - y_{t-1})^2 / (T - \lfloor \tau T \rfloor)^2}{\sum_{t=1}^{\lfloor\tau T\rfloor+1} (\Delta y_t)^2 / [\tau T]^2}$$

following Homm and Breitung (2012).\footnote{Note that $\Lambda(\tau)$ is not transcribed in the published paper but is available in a working paper version by the same authors.} As discussed in Homm and Breitung (2012), however, the $\hat{\tau}_{\text{DFC}}$ estimator in (16) has a cleaner theoretical justification and is more accurate in Monte Carlo simulations.

For practical reasons, Homm and Breitung (2012) recommended excluding observations after a potential peak in the time series that would arise in the post-break sampling interval $[\lfloor \tau^* T \rfloor + 1, T]$. The inclusion of observations after a potential peak can strongly decrease the power of both the supDFC and supBT tests. This peak can appear as a result of extraneous events affecting the time series. For example, the sub-prime mortgage crisis and the global financial crisis negatively affected all investments. Consequently, these events, which were unrelated to the change in margin rules, substantially decreased margin loans. This does not imply that there are no long run implications to the change in margin rules, but only that it is difficult to test for these effects over long periods of time. This is essentially a statistical limitation of these tests. Furthermore, cutting off the sampling interval at the peak requires either visual inspection of the time series or relying on knowledge of these external events. This shortcoming can be dealt with by adapting the critical values as further discussed below.

The empirical results provided by the supDFC are also checked against those produced by the Phillips et al. (2011) tests. One advantage of these tests is that they do not require excluding observations after a potential peak. Phillips et al. (2011) proposed a right-tail forward recursive Augmented Dicky-Fuller (ADF) test and a supremum ADF test for structural changes in a financial time series. Phillips et al. (2011) primarily applied these techniques to investigate the possibility of a bubble formation in asset markets. Other relevant research in existing literature includes Caballero et al. (2008), Phillips and Yu (2011), and Phillips et al. (2012) which generalised the methods proposed by Phillips et al. (2011) to detect and date multiple bubbles. This framework is similar to the supBT test and is also a suitable way to conduct robustness checks when investigating a structural change in margin debt.

Phillips et al. (2011) tested for structural change by setting $\rho_t = \rho$ in model (10) and testing the null $\rho = 1$ for all $t$ against the right-tailed alternative $\rho > 1$ for all $t$. This setup is commonly used in the context of unit root tests such as the ADF test, but it typically
assumes that \( y_t \) is a stationary series under the left-tailed alternative hypothesis, i.e., \( \rho < 1 \). Note that, unlike the supBT test, the alternative hypothesis \( \rho > 1 \) holds for the entire sample and does not allow for \( y_t \) to follow a random walk. This shortcoming is not encountered with the supDFC or the supBT tests.

The ADF statistic is simply the t-statistic of \( \rho_t \) in model (10) with a constant term; because no augmented lag terms are included, as recommended and discussed in Phillips et al. (2012) and Homm and Breitung (2012), the statistic used is the simple Dicky-Fuller (DF) statistic. The sequence of DF statistics is calculated through forward recursive regressions by repeatedly computing them with each estimation of (10) where the data subsample is incremented by one observation at a time. The regressions are based on the initial sample window \([1, [\tau_0 T]]\) sequentially augmented by additional observations up to \([\tau T]\) for \( \tau_0 \leq \tau \leq 1 \), where \( \tau_0 \) is fixed to the smallest value required for estimation efficiency, usually 0.1. This initial window has little impact on the empirical results presented in Section 4. Under the null, the corresponding recursive t-statistic is denoted as

\[
\text{DF}_\tau = \frac{\hat{\rho}_\tau - 1}{\hat{\sigma}_{\rho,\tau}}
\]

where \( \hat{\rho}_\tau \) is the OLS estimator of \( \rho \) and \( \hat{\sigma}_{\rho,\tau} \) is the estimator for the standard deviation of \( \rho \) for the subsample \([1, [\tau T]]\). The supremum DF-statistic and its limiting distribution are

\[
\sup \text{DF}(\tau_0) = \sup_{\tau \in [\tau_0, 1]} \text{DF}_\tau \xrightarrow{d} \sup_{\tau \in [\tau_0, 1]} \frac{\int_0^\tau W(r) dW(r)}{\int_0^\tau W(r)^2 dr}.
\]

The value of \( \sup \text{DF}(\tau_0) \) is compared to right-tailed critical values obtained from the asymptotic distribution in (21) or finite sample simulations in order to test for structural change.

The date of the structural change is located by comparing each \( \text{DF}_\tau \) statistic from the recursive \( \text{DF}_\tau \) sequence with \( \tau \in [\tau_0, 1] \) against the corresponding right-tailed critical values of the asymptotic distribution in (21) at \( \tau \). It follows that the estimate proposed by Phillips et al. (2011) is written as

\[
\hat{\tau}_{DF} = \inf_{\tau \in [\tau_0, 1]} \{ \tau : \text{DF}_\tau > \text{cv}_\tau^{\beta_\tau} \}.
\]

\( \hat{\tau}_{DF} \) is the estimated date of the structural change.

All critical values for all the tests used in this paper are simulated and their values are based on 10,000 replications. The model used to simulate the critical values in this paper is

\[
y_t = d T^{-\eta} + \rho_t y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, 1) \quad \forall t = 1, \ldots, T
\]

where \( \rho_t = 1, d = 1, \eta = 1 \) and \( y_0 \sim N(0, 1) \) for the supDF test as in Phillips et al. (2012), and \( \rho_t = 1, \eta = 0 \) and \( y_0 = 0 \) for the supBT and supDFC tests as in Homm and Breitung (2012). The finite sample critical values are generated using \( T = 73 \) observations for the sample truncated at the peak of the margin debt series and \( T = 140 \) observations for the full sample. For the asymptotic critical values \( T = 5000 \) observations. Both the finite sample and asymptotic critical values are obtained from taking the 90th, 95th and 99th percentiles of the simulated distribution; this is applied for all tests considered.
As noted earlier, a sequence of DF\(_{\tau}\) tests is calculated recursively for the entire sample. In order to identify the break date, a corresponding sequence of DF\(_{\tau}\) critical values is also simulated 10,000 times. The 90\(^{th}\), 95\(^{th}\) and 99\(^{th}\) quantiles of the sequence of DF\(_{\tau}\) critical values can then be chosen to determine the break date. In addition, both the supDFC and supBT rely on the assumption that a potential peak after the structural change is identified, which implies that the sample must be resized accordingly. Therefore, the simulated critical values should take this assumption into account.\(^3\) This is achieved by simulating (23) where \(T = 5000\), identifying the peak (maximum) of the simulated series and including the appropriate amount of observations prior to the identified peak. Again, this process is repeated 10,000 times.

4. Evidence

Data

The variable of interest is margin debt (MD\(_t\)) collected from the FINRA website for the entire U.S. stock market (aggregate) and for only the NYSE.\(^4\) Note that the margin debt series for the NYSE represents about 90\% of the aggregate U.S. stock market margin debt series on average. The data is available monthly and the sample spans from July 2001 until February 2013. The starting date of the sample is based on findings by Phillips, Wu, and Yu (2011) and Phillips, Shi, and Yu (2012) that the dot com asset bubble ended somewhere between March 2001 and June 2001. The sample used in this paper begins after this event. During this episode, investors taking margin loans exhibited bullish and then bearish behaviour consistent with the market tendencies. The margin debt series fluctuated with the market because the amount borrowed was only invested in securities in the stock market.

In order to test whether the margin debt changed as a result of the new margin rules, it is important to net out the inflationary effect of the stock market. As shown in equations (4), (7) and (9) and illustrated in Example 1, the size of margin loans is affected by the changes in stock prices. Therefore, MD\(_t\) is adjusted for the overall market trend captured by the year-on-year growth of the S&P 500 index as follows:

\[
\pi^{SP500}_t = \log \left( \frac{P^{SP500}_t}{P^{SP500}_{t-12}} \right).
\]

The data for the S&P 500 index is collected from Yahoo! Finance.\(^5\) The margin debt series is adjusted and then log-transformed as follows:

\[
md_t = \log \left( MD_t \times (1 - \pi^{SP500}_t) \right).
\]

There are other methods for adjusting the margin debt series. One way is to divide the margin debt by the stock market prices. For example, the margin debt can be deflated

\(^3\) The authors are grateful to Richard Gerlach for suggesting this.

\(^4\) http://www.finra.org

\(^5\) http://finance.yahoo.com
by the S&P 500 index: \[ md_t = \log\left(\frac{MD_t}{P_{SP500}^t}\right). \] Although this approach accounts for market sentiments and potential market exuberances, the market inflation adjustment more accurately matches the model and illustration in Section 2.

The supDFC and supBT tests require an identification of the peak of the series, as discussed earlier. The peak of \( md \) was in July 2007, which corresponds to the beginning of the subprime mortgage crisis. Note that both the 2007 subprime mortgage crisis and 2008 financial crisis were external events that affected the borrowing behaviour of investors regardless of the change in margin rules. In the results below, robustness checks are conducted to see whether the results for the supDFC and supBT tests change if the observations between July 2007 and the 2008 financial crisis are included after the peak of \( md \).

**Results**

As a first pass, the results for the test of the difference between the mean growth rates of \( md_t \) provide supporting evidence of a structural change at the start of Phase I and Phase II of the pilot programme. This is true for both the NYSE and the aggregate U.S. stock exchange. All the p-values are presented in Table 1 and show that there is a statistically significant change in the mean growth rate of \( md_t \) at the 5% level. The statistical significance is higher for July 2006.

<table>
<thead>
<tr>
<th>Margin Debt series</th>
<th>Supposed break dates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>July 2005 (Phase I)</td>
</tr>
<tr>
<td>Aggregate U.S. stock market</td>
<td>0.0177</td>
</tr>
<tr>
<td>NYSE</td>
<td>0.0113</td>
</tr>
</tbody>
</table>

Notes: There are two margin debt series: the NYSE and the aggregate U.S. stock market. Both are logged and adjusted for S&P500 market inflation. The results reported in this table are the p-values for the test of difference between mean growth rates of margin debt before and after the break \([\tau_T]\). \( H_0: \mu_{n1} = \mu_{n2} \) is tested against \( H_1: \mu_{n1} \leq \mu_{n2} \) with \( n_1 = [1, [\tau_T]], \ n_2 = [[\tau_T] + 1, T] \) and where \( \mu_{n1} \) is the pre-break mean growth rate and \( \mu_{n2} \) is the post-break mean growth rate. The sample considered spans from July 2001 until July 2007, the peak of the \( md_t \) series.

The test results presented in Tables 2 and 3 provide statistical evidence for structural change in margin debt, both in the NYSE and the aggregate U.S. stock exchange. The supDFC and supBT tests are all significant at the 5% level, indicating that there is a structural change in the margin debt series, as shown in Table 2. As expected, the more realistic critical values (denoted by *) that account for a peak in the simulated model are larger than the critical values that do not account for the presence of a peak. The results hold for both the aggregate U.S. stock market as well as the NYSE. The supDF tests are significant at the 5% level, which suggests that margin debt deviated from a random walk and increased

---

The results presented in this section are partially produced with codes from Shu-Ping Shi, available at [https://sites.google.com/site/shupingshi/](https://sites.google.com/site/shupingshi/), and from Jörg Breitung, available at [http://www.ect.uni-bonn.de/mitarbeiter/Joerg-Breitung/](http://www.ect.uni-bonn.de/mitarbeiter/Joerg-Breitung/).
rapidly at a given point in time. Notice that the finite sample critical values (based on 140 observations) and the asymptotic critical values are relatively close to each other.

<table>
<thead>
<tr>
<th></th>
<th>supDFC</th>
<th>supBT</th>
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<tbody>
<tr>
<td>Aggregate U.S. stock market</td>
<td>3.3903</td>
<td>4.6916</td>
</tr>
<tr>
<td>NYSE</td>
<td>3.4931</td>
<td>5.0797</td>
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</table>

Finite sample critical values*

<p>| | | |</p>
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</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>3.1738</td>
<td>5.4157</td>
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<tr>
<td>5%</td>
<td>2.6217</td>
<td>4.0994</td>
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<tr>
<td>10%</td>
<td>2.3241</td>
<td>3.4691</td>
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Finite sample critical values

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<table>
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<th></th>
<th></th>
<th></th>
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</thead>
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<tr>
<td>1%</td>
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<tr>
<td>5%</td>
<td>1.6414</td>
<td>2.6478</td>
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<td>10%</td>
<td>1.2781</td>
<td>2.0666</td>
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Asymptotic critical values

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<td>1%</td>
<td>2.3216</td>
<td>3.7973</td>
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<td>5%</td>
<td>1.6169</td>
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<td>10%</td>
<td>1.2346</td>
<td>1.9175</td>
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</tbody>
</table>

Notes: There are two margin debt series: the NYSE and the aggregate U.S. stock market. Both are logged and adjusted for S&P500 market inflation. The series are modelled as \( y_t = \rho \cdot y_{t-1} + \varepsilon_t \), with \( \varepsilon_t \sim \text{i.i.d} (0, \sigma^2) \). The table presents the test results for the Homm and Breitung (2012) modified Chow (1960) test, supDFC(\( \tau_0 \)), as in (14) and (15) and the modified Busetti and Taylor (2004) test, supBT(\( \tau_0 \)), as in (17) and (18). See Section 3 for details. The asymptotic critical values rely on \( T = 5000 \) observations, whereas the finite sample critical values rely on \( T = 73 \) observations, as the sample is truncated at the peak of the series. The critical values denoted by * are calculated from simulated series ending in a peak at the 73rd observation (or July 2007). All critical values are based on 10,000 replications.

The estimated date of the structural change in margin debt corresponds to Phase II of the pilot programme. The results are shown in Table 4 and depicted in Figures 4 and 5. Both the supDFC and supBT tests, \( \hat{\tau}_{DFC} \) and \( \hat{\tau}_{BT} \), estimate the date of the structural change at August 2006, which occurs in Phase II of the pilot programme. The supDF test, however, dates \( \hat{\tau}_{DF} \) the structural change to November 2006, which also occurs in Phase II. It is interesting to note that none of the estimated dates occur in Phase I. This is consistent with expectations, as very few market instruments (only broad-based indices and exchange-traded funds derivatives) were included in that early Phase and it was not anticipated to have a significant impact. Both the statistical test results and the estimated dates of structural change in the margin debt support this paper’s hypothesis.

A series of robustness checks is conducted for the supDFC and supBT tests and shown in Table 5. The observations between July 2007 and September 2008 are included after the peak of \( m_d_t \), as discussed in the Data Section. The supDFC and supBT test results
### Table 3: Testing for structural change in margin debt – supDF tests

<table>
<thead>
<tr>
<th>Margin debt series</th>
<th>supDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate U.S. stock market</td>
<td>1.8343</td>
</tr>
<tr>
<td>NYSE</td>
<td>2.0861</td>
</tr>
</tbody>
</table>

#### Finite sample critical values

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>2.0184</td>
</tr>
<tr>
<td>5%</td>
<td>1.4271</td>
</tr>
<tr>
<td>10%</td>
<td>1.1488</td>
</tr>
</tbody>
</table>

#### Asymptotic critical values

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>2.0505</td>
</tr>
<tr>
<td>5%</td>
<td>1.5488</td>
</tr>
<tr>
<td>10%</td>
<td>1.2513</td>
</tr>
</tbody>
</table>

Notes: There are two margin debt series: the NYSE and the aggregate U.S. stock market. Both are logged and adjusted for S&P500 market inflation. The series are modelled as $y_t = \rho y_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim \text{i.i.d}(0, \sigma^2)$. The table presents the test results for the supDF test proposed by Phillips, Wu, and Yu (2011) and as in (20) and (21). See Section 3 for details. The asymptotic critical values rely on $T = 5000$ observations, whereas the finite sample critical values rely on $T = 140$ observations. All critical values are based on 10,000 replications.

### Table 4: Dating the structural change

<table>
<thead>
<tr>
<th>Margin Debt series</th>
<th>$\hat{\tau}_{DF}$</th>
<th>$\hat{\tau}_{DFC}$</th>
<th>$\hat{\tau}_{BT}$</th>
</tr>
</thead>
</table>

Notes: There are two margin debt series: the NYSE and the aggregate U.S. stock market. Both are logged and adjusted for S&P500 market inflation. The date $\hat{\tau}_{DF}$ in (22) is found by comparing each $DF_\tau$ statistics as in (20) against the corresponding right-tailed critical values of the asymptotic distribution in (21) at each time period $\tau$. Both $\hat{\tau}_{DFC}$ and $\hat{\tau}_{BT}$ maximise the likelihood function with respect to the break date and are computed from equations (16) and (19), respectively. See section 3 for details.
are mildly significant at the 10% level when comparing them to their respective finite and asymptotic sample critical values. These results are not surprising, however. As discussed in Section 3, the supDFC and supBT tests have low power when including observations after a peak or in the presence of multiple structural changes. Therefore, the alternative set of critical values (denoted by \( \ast \)) is calculated from a simulated series whose peak is located at the 73\(^{rd} \) observation (or July 2007) and ends 14 observations after the peak (or September 2008). This is a simple way of accounting for the financial turmoil of 2007-2008. When comparing the two tests to the new set of critical values, the supDFC test rejects the null at the 5% level, while the supBT test results remain unchanged, as its \( \ast \) critical values are similar to its unadjusted finite sample counterpart. Despite the test results, the estimated date of the structural change remains the same as those exposed in Table 4, and thus are not reproduced here to avoid repetition.

As discussed in the Data Section, an alternative way to account for market trends is to deflate margin debt by the S&P 500 index. The results found with this deflated series corroborate those presented above. There is a structural change that appears to correspond to Phase II of the pilot programme. Therefore, the results are not presented in this paper and are available upon request.

Table 5: Testing for a structural change in margin debt – robustness check

<table>
<thead>
<tr>
<th>Test statistics</th>
<th>supDFC</th>
<th>supBT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate U.S. stock market</td>
<td>1.3969</td>
<td>2.5551</td>
</tr>
<tr>
<td>NYSE</td>
<td>1.1682</td>
<td>2.3467</td>
</tr>
</tbody>
</table>

Finite sample critical values*

| 1% | 4.0090 |
| 5% | 2.6511 |
| 10% | 2.0898 |

Finite sample critical values

| 1% | 3.9651 |
| 5% | 2.6301 |
| 10% | 2.0640 |

Notes: There are two margin debt series: the NYSE and the aggregate U.S. stock market. Both are logged and adjusted for S&P500 market inflation. The series are modelled as \( y_t = \rho_t y_{t-1} + \varepsilon_t \), with \( \varepsilon_t \sim \text{i.i.d} (0, \sigma^2) \). The table presents the test results for the Homm and Breitung (2012) modified Chow (1960) test, supDFC(\( \tau_0 \)), as in (14) and (15) and the modified Busetti and Taylor (2004) test, supBT(\( \tau_0 \)), as in (17) and (18). See Section 3 for details. The asymptotic critical values rely on \( T = 5000 \) observations, whereas the finite sample critical values rely on \( T = 73 \) observations, as the sample is truncated at the peak of the series. The critical values denoted by \( \ast \) are calculated from simulated series whose peak is located at the 73\(^{rd} \) observation (or July 2007), but ending 14 observations after the peak (or September 2008). All critical values are based on 10,000 replications.
Figure 4: Dating the structural change in the inflation adjusted aggregate margin debt. Figures (a) and (b) show the supDFC and supBT tests for the aggregate U.S. margin debt. Both figures present the inflation adjusted margin debt (⋯) and the sequence of tests (—). The vertical lines indicate the estimated location of the structural change.

Figure 5: Dating the structural change in the inflation adjusted aggregate margin debt. Figures (a) and (b) show DF tests for the aggregate U.S. margin debt and the NYSE margin debt respectively. Both figures present the inflation adjusted margin debt (⋯), the sequence of DF tests (—) and the 95% critical value sequence (- -). The vertical lines indicate the estimated location of the structural change.
5. Discussion

Section 4 presents statistical evidence that the beginning of a rapid growth in margin debt can be dated to the time of the change in margin rules between 2005 and 2008. The timing of this effect has been estimated to be in Phase II of the pilot programme, as anticipated. Indeed, Phase II of the pilot programme gave investors additional incentives to enter the financial market. With the subprime mortgage crisis in July 2007 and the subsequent global financial crisis, these investment portfolios lost their value and their owners received margin calls from their brokerage firms. In order to fulfil their obligations, investors were forced to sell equity. Ensuing from the global financial meltdown, many investors defaulted on their margin accounts, thus creating substantial losses.

When a margin call is issued, investors are typically given 3 business days to deposit additional equity to fulfil the margin call. If the margin call remains unfulfilled, brokerage firms have the right to liquidate all or part of the investment portfolio. Brokerage firms can choose which positions to liquidate and in what order. Without the complete knowledge of the risk associated with different parts of the portfolio, a partial liquidation may increase the potential loss and the required margin, thus leading to further liquidation. A simple example would be selling long call options from a debit call spread (see Figure 2). The maximum loss on that debit call spread is bound by the cost of the transaction (or $21,450), whereas the maximum loss on the naked short call is uncapped (Cohen, 2005). The general consensus within existing literature is that strategy-based margining methodology is very conservative; when it is applied by brokerage firms, it produces margin requirements that often significantly overestimate the risk associated with a portfolio. This is what motivated the amendments to the margin rules and the introduction of portfolio margining methodology. However, Coffman et al. (2010) and Matsypura and Timkovsky (2012, 2013) showed that strategy-based margining, when coupled with complex option spreads (SEC, 2003, p.4) produced much lower margin requirements and a more accurate estimation of risk.

The major advantage of strategy-based margining is that it always provides a portfolio liquidation strategy. As discussed in the Introduction, in order to determine margin requirements, strategy-based margining decomposes a portfolio into option spreads and then calculates margin requirements for each spread individually. This means that investors always know the risk structure of their portfolio and can liquidate the appropriate investment positions, if necessary. In contrast, the portfolio margining treats the entire portfolio as a “black box” (see Figure 1). If partial liquidation becomes necessary, it may be very difficult to determine which part of the portfolio should be liquidated. In addition, portfolio margining often produces significantly lower margin requirements (see Figure 2) and, as a result, makes it easier to enter investment positions. Thus, portfolio margining methodology effectively plays a role of a trap with an easy entrance and a difficult exit.

In the current climate of record low interest rates, borrowing is cheap. This climate, combined with bull markets, has stimulated investors to take on margin loans and buy more stock than they can otherwise afford. As margin loans again become increasingly popular with investors, margin debt has been rapidly climbing; as of September 2013, margin debt has reached levels similar to the peak attained in July 2007. This trend is likely to continue as long as these exceptional financial conditions prevail. This market speculation should
be watched closely in order to avoid the types of snowballing effects observed in the 1929 stock market crash. Margin rules should be adequately tailored to apply the appropriate margining methodology to investment portfolios.

It is apparent from the model discussed in Section 2 that, regardless of the margining methodology, increasing margin requirements can potentially curb investors’ risk taking behaviour and margin loans-based speculations in the stock market. Furthermore, the empirical evidence presented in this paper supports the premise that margin requirements affect margin loans. A number of researchers have considered how interactions between financial regulations, institutions and markets impacted the 2008 financial crisis and how these interactions could or should change (see Carey et al., 2012). Because margin requirements are monetary policy tools of the U.S. Federal Reserve Board, it is important to further investigate the efficacy of strengthening and loosening these requirements to prevent high risks of defaulting on margin loans, particularly in times of financial crisis.

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