GOVERNMENT DEBT IN AN INTERGENERATIONAL MODEL OF ECONOMIC GROWTH, ENDOGENOUS FERTILITY AND ELASTIC LABOR WITH AN APPLICATION TO JAPAN

by

Bei Li
Business School
University of Western Australia

and

Jie Zhang
Chongqing University, China and National University of Singapore

DISCUSSION PAPER 13.27
Government debt in an intergenerational model of economic growth, endogenous fertility, and elastic labor with an application to Japan

Bei Li

 Discipline of Economics, Business School, University of Western Australia

Jie Zhang

 Department of Economics, Chongqing University, China

Department of Economics, National University of Singapore, Singapore

July 12, 2013

Abstract

We derive the social optimum and optimal government debt in an intergenerational model of growth with fertility, elastic labor, human capital externalities and a non-convex feasible set. Debt through lump-sum taxation increases leisure and labor, reduces fertility, and can achieve the social optimum with education subsidization. Debt and education subsidization through labor income taxation can generate similar effects on fertility and labor. Quantitatively, the model can yield changes in output growth, fertility, and labor resembling observations in Japan with rising debt since 1970. From this calibration, the optimal debt-output ratio is 12.3% and excessive debt causes substantial welfare losses.

Keywords: Government debt; Education subsidy; Leisure; Fertility; Human capital externalities; Income taxes

JEL Classifications: H6; I2; J0; O1

Correspondence: Jie Zhang, School of Economics and Business Administration, Chongqing University, Chongqing, China 400030; Email: zhangjiecqu@cqu.edu.cn; Tel: +86 23 65102571; Jie Zhang, Department of Economics, National University of Singapore, Singapore 117570. Email: ecszj@nus.edu.sg; Tel: +65 6516 6024; Fax: +65 6775 2646.
1. Introduction.

Government debt has become increasingly worrisome in many industrial countries. In 2012, the ratio of government gross debt to GDP reached 238% in Japan, 158% in Greece, 127% in Italy, and 106% in the United States (IMF, 2013 Fiscal Monitor). Take Japan, once known for miraculous growth, as an example for what may go wrong with rising debt (see Figures 1 and 2): From 1970 to 2011, the ratio of government debt to GDP went up significantly from 11.9% to 230%, fertility declined dramatically from 2.1 to 1.4, the employment rate of population aged 15-64 rose moderately from 68% to 71% (more significantly for women), and the economy nearly stagnated for 20 years with an average annual growth rate of GDP at 1.09% from 1990 to 2011.

Government debt has long been at the center of macroeconomic policy analysis, particularly about its effects on equilibrium allocations. Yet, few studies have investigated how much government debt should be optimal or how much welfare losses may result from excessive debt. The existing results differ between life-cycle and dynastic (infinitely-lived-agent) models, between cases with or without a labor-leisure tradeoff, and between cases with or without endogenous fertility.

Using government debt to correct dynamic inefficiencies or externalities can be traced back to Diamond (1965) in a life-cycle model of overlapping generations. In the Diamond model, dynamic inefficiency is a necessary and sufficient condition for government debt to increase steady state welfare. This condition is no longer sufficient once labor is elastic as shown in Fanti and Spataro (2006). In a dynastic model, government debt financed by lump-sum taxes is neutral through counteracting intergenerational transfers in Barro (1974). With a leisure-labor tradeoff, however, government debt financed by labor income taxes is welfare-reducing through reducing labor and capital stock and increasing leisure in Burbidge (1983). In a two-period model with income taxes and elastic leisure, Bassetto and Kocherlakota (2004) argue that it is the present value, rather than the timing, of taxes that matters.

With endogenous fertility in a dynastic model, higher government debt reduces fertility and increases capital intensity by increasing the bequest cost of a child, as shown in Becker and Barro (1988), Lapan and Enders (1990), and Wildasin (1990). Government
debt can also promote per worker output growth, through reducing fertility and increasing labor and human capital investment per child, and improve efficiency through internalizing human capital externalities in a dynastic model in Zhang (2003, 2006). Without leisure, however, these models cannot capture the effect of taxing labor income or consumption spending for government debt on the leisure-labor tradeoff.\(^1\) Also, absent leisure, parental time would be allocated only between labor and childrearing, and thus a dramatic decline in fertility would mean a much more substantial increase in labor than what was observed in Japan in Figures 1 and 2.

In this paper, we explore the effects of government debt on an economy and optimal debt in an intergenerational model of endogenous growth, endogenous fertility, and elastic labor with human capital externalities, and explain the Japanese experiences since 1970 quantitatively. The tradeoff between the quantity of children and investment per child causes a non-convex feasible set as in Razin and Ben-Zion (1975). The human capital externality engenders high fertility and low education spending.\(^2\) With time intensive childrearing, high fertility goes hand in hand with low leisure or low labor.

We make several contributions in this rich model. First, we establish the social optimum from not only necessary conditions but also sufficient conditions facing a non-convex feasible set. Second, we demonstrate that the laissez faire not only has lower investment in education and higher fertility as known in the literature but also lower leisure and lower labor than does the social optimum.

Third, some new results emerge from our analysis of government debt financed by either a lump-sum or a labor income tax with or without education subsidies. Government debt serviced by lump-sum taxes increases both leisure and labor and reduces fertility, by increasing the bequest cost of a child, but cannot correct the under-spending on education. Combining government debt with education subsidies and

\(^1\) As shown in Turnovsky (2000), the effects of government policy may differ significantly with or without leisure in general, and so do the effects of government debt in particular as seen in the aforementioned literature.

\(^2\) Indeed, some empirical studies find evidence on human capital externalities in the determination of individuals’ earnings through channels such as ethnic groups, neighborhoods, work places, or state funding of schools; see, e.g., Borjas (1995), Rauch (1993), Davies (2002), and Moretti (2004a, 2004b).
lump-sum taxes can achieve the social optimum. Absent education subsidization, debt serviced by labor income taxation increases leisure and induces spending away from education toward consumption. In the meanwhile, debt serviced by labor income taxation may decrease (increase) labor if the taste for leisure is sufficiently strong (weak). This is because a rise in the labor income tax reduces the opportunity cost of time for leisure and childrearing and reduces the after-tax return to labor and human capital, counteracting the forces of government debt. When government debt is serviced by labor income taxes together with education subsidies, the socially optimal allocation of output can be achieved, but fertility, labor and leisure remain sub-optimal. However, in both scenarios government debt serviced by labor-income taxation can mitigate the efficiency loss of human capital externality under plausible conditions albeit it cannot achieve the social optimum.

Last but not least, our model produces quantitative changes in aggregate output growth, fertility, and labor resembling observations in Japan when the debt-output ratio is raised from 0% to 85%, and yields plausible welfare implications. From this empirically plausible parameterization, the optimal debt-output ratio with labor income taxes and education subsidies is 12.3%, which is close to the case of Japan in 1970. Excessive debt at 85% reduces (annual) aggregate output growth from 3.78% in the above optimal case to 1.52%, reduces fertility from 2.17 to 1.45, increases labor by 2 percentage points, and causes a large welfare loss equivalent to more than a 30% decline in consumption for all generations. When leisure were excluded, however, the model with the same values for the rest of parameters would predict much larger increases in labor (over 30 percentage points) than what was observed in Japan, and would predict even larger losses in welfare.

The remainder of the paper proceeds as follows. Section 2 introduces the model. Section 3 determines the social optimum. Section 4 characterizes the competitive equilibrium and analyzes optimal government policies. Section 5 presents quantitative implications with an application to Japan. The last section concludes the paper.
2. The model

This model has an infinite number of discrete periods \( t = 0,1,2\ldots \). Each period has two overlapping generations of identical children and identical adults. Children embody human capital without making any decision. Each adult agent allocates one unit of time endowment to labor, leisure, and childrearing. Rearing a child needs \( \frac{1}{v} \) fixed units of time, implying an upper bound, \( 1/v \), on fertility, \( n_t \). Each adult generation has a large mass of \( L_t \) that evolves according to \( L_{t+1} = n_t L_t \).

The utility of an adult, \( V(t) \), depends on his own consumption, \( c_t \), the number of children, \( n_t \), leisure, \( z_t \), and average utility per child, \( V(t+1) \):

\[
V(t) = U(c_t, n_t, z_t) + \alpha V(t+1), \quad 0 < \alpha < 1.
\]  

(1)

where \( \alpha \) is the taste for per child welfare and the subjective discounting factor. To obtain a reduced-form solution for welfare analysis, we assume:

\[
U(c_t, n_t, z_t) = \ln c_t + \rho \ln n_t + \phi \ln z_t, \quad \rho, \phi > 0,
\]

where \( \rho \) stands for the taste for the number of children and \( \phi \) stands for the taste for leisure. This preference specification is an extension of that used in Razin and Ben-Zion (1975), Lapan and Enders (1990), and Zhang (2003, 2006) to incorporate leisure.

The production of final output, \( Y_t \), uses physical capital, \( K_t \), and effective labor, \( L_t l_t h_t \), according to a Cobb-Douglas function:

\[
Y_t = D K_t^\theta (L_t l_t h_t)^{1-\theta}, \quad D > 0, \quad 0 < \theta < 1,
\]

(2)

where \( l_t \) and \( h_t \) are per worker labor and per worker human capital, respectively. Physical and human capital depreciates fully in one period (plausible for a period of 30 years). To focus on the role of government debt, we do not consider the role of

\textsuperscript{3} The inclusion of average child utility in the parental preference conforms to the Millian Social Welfare Function (Mill, 1848). This is different from the Benthamite Social Welfare Function adopted by Becker and Barro (1988) where the utility of parents depends on the total utility of their children, and where an interior allocation exists only if the cost of rearing a child exceeds the discounted wage income of the child. The empirical relevance of requiring children as net financial burdens is unclear and particularly questionable in modern economies, however. Becker and Lewis (1973) include child quality, rather than child utility, in parental utility, where the Welfare Theorems do not apply. Also, Eckstein and Wolpin (1985) consider optimal population growth without including child utility or any other index of child quality in the parental preference, an approach that has no motive for parents to give bequests and make investment in child education.
government expenditure in production. Should government spending be allowed to contribute to production as in Glomm and Ravikumar (1994, 1997), then government debt would achieve a larger welfare gain.

The human capital of a child depends on investment, \(e_t\), parental human capital, \(h_t\), and the average human capital of the parent’s generation, \(\bar{h}_t\), according to

\[
h_{t+1} = Ae^\delta_t \left( h_t^\beta \bar{h}_t^{1-\beta} \right)^{1-\delta}, \quad A > 0, \quad 0 < \beta \leq 1, \quad 0 < \delta < 1. \tag{3}
\]

For \(\beta < 1\), there are positive spillovers from the average human capital of the parent’s generation, \(\bar{h}_t\), to every child’s learning as in Tamura (1991) and De la Croix and Doepke (2003). Empirical evidence on human capital externalities from various sources was mentioned earlier in Footnote 2.

Normalize the price of the final good to unity and let \(\bar{x}\) be the average of a variable \(x\). The wage rate per unit of effective labor, \(w_t\), and the rental price of capital, \(r_t\), are determined competitively:

\[
w_t = (1-\theta) D \mu_t^\theta, \tag{4}
\]
\[
r_t = \theta D \mu_t^{\theta-1}, \tag{5}
\]

where \(\mu_t = \bar{k}_t / (\bar{h}_t \bar{h}_t)\) is the ratio of physical capital to effective labor with \(\bar{k}_t = K_t / L_t\) standing for physical capital per worker. Accordingly, \(\bar{y}_t = Y_t / L_t = D \mu_t^{\theta} \bar{h}_t \bar{h}_t\) is per worker output.

A worker devotes \(\nu n_t\) units of time to rearing children, \(z_t\) to leisure, and the remaining \(l_t = 1-\nu n_t - z_t\) to working. At the beginning of adulthood, everyone receives a bequest \(a_t\) plus interest income from his parent. Wage earnings and bequest income are spent on consumption, the education of children, and bequests to children:

\[
c_t = a_t r_t + (1-\nu n_t - z_t) w_t h_t - \tau_t - e_t n_t - a_t n_t, \tag{6}
\]

This human capital externality in education differs from another type of human capital externality in production whereby average human capital generates spillovers to the productivity of each worker as in Lucas (1988). Since this externality also reduces private returns to human capital from the social rate, it should yield similar results.
where \( \tau_i \) is a tax (to be specified later). The feasible set from the budget constraint is not convex because of the tradeoff \( e, n_i, a_{t+1}, n_i \).

The government issues one-period bonds and collects taxes to service debt repayment:

\[
\bar{n}_t \bar{b}_{t+1} = \bar{b}_t \tau_t - \tau_t, \tag{7}
\]

where \( \bar{b}_t \in \mathbb{R} \) is the amount of outstanding debt per worker. Without uncertainties, government bonds and physical capital are perfect substitutes. The capital market clears when:

\[
K_t = L_t (\bar{a}_t - \bar{b}_t). \tag{8}
\]

In per worker terms, \( \bar{k}_t = \bar{a}_t - \bar{b}_t \). In equilibrium, we expect \( x = \bar{x} \) by symmetry.

3. The social optimum

By Millian optimality, the social planner maximizes the utility of the living adult generation in (1) subject to feasibility \( c_i = Dk_i^\theta [(1 - v n_i - z_i)h_i]^{-\delta} - e_i n_i - k_{t+1} n_i \) and education technology \( h_{i+1} = A e_i h_i^{1-\delta} \). Here, the social planner internalizes the externality by setting \( h_i = \bar{h}_t \). However, given the non-convexity in the feasible set due to the tradeoffs \( n_t e_i \) and \( n k_{t+1} \), we need to work out the sufficient conditions for optimality.

From the functional forms and the full depreciation of capital within one period, both fertility and the proportional allocations of time and output are stationary at all times, given any initial state. Thus, \( n_i = n \) and \( z_i = z \) for \( t \geq 0 \). Similarly, let \( \Gamma_c \equiv c_i / y_i \), \( \Gamma_k \equiv k_{t+1} n_i / y_i \), and \( \Gamma_e \equiv e_i n_i / y_i \) be the fraction of output consumed, the fraction of output invested in physical capital, and the fraction of output invested in human capital, respectively. The transformed feasibility is \( \Gamma_c = 1 - \Gamma_k - \Gamma_e \). Substituting these notations in \( h_{i+1} = A e_i h_i^{1-\delta} \), \( y_i = D \mu_i h_i \) and \( k_{t+1} n = \Gamma_k y_i \), we have

\[\text{Footnote: For the purpose of this paper, we bypass shocks of government spending in another line of studies (e.g., Auerbach and Gorodnichenko, 2012).}\]
\[ k_{t+1} = \Gamma_k \mu_i^{\theta \delta} k_t / n, \quad (9) \]
\[ h_{t+1} = A (\Gamma_e / n)^\delta D^\delta (1 - vn - z)^\delta \mu_i^{\delta \theta} h_t. \quad (10) \]

These equations determine the evolution of the ratio of physical capital to effective labor:

\[
\mu_{t+1} = \left[ \frac{\Gamma_k D^{1-\delta}}{n^{1-\delta} A \Gamma_e^\delta (1 - vn - z)^\delta} \right] \mu_i^{\theta (1-\delta)},
\]
\[
\mu_n = \left[ \frac{\Gamma_e D^{1-\delta}}{n^{1-\delta} A \Gamma_e^\delta (1 - vn - z)^\delta} \right]^{[1-\theta(1-\delta)]},
\]

where \( \mu_i \) is globally convergent toward its long-run level \( \mu_i^\infty \) because \( 0 < \theta (1-\delta) < 1 \).

Denote the growth rate of per worker income \( \gamma_i \) as \( 1 + \gamma_{y,i} \). In a steady-state balanced growth equilibrium, \( \gamma_k = \gamma_h = \gamma_y = \gamma \). Using these conditions and substituting \( \mu_n \) into (9) or (10) gives the long-run growth rate of per worker income:

\[
1 + \gamma = \left[ \left( A \left[ (\Gamma_e / n) (1 - vn - z) \right]^\delta \right) \mu_i^{\theta (1-\delta)} D^\delta \left( \Gamma_k / n \right)^\delta \right]^{1-\theta(1-\delta)}.
\]

The growth rate of per worker income depends positively on the fractions of output invested in physical and human capital per child and on the labor time but negatively on fertility. Accordingly, the growth rate of aggregate income is \( n (1 + \gamma) \).

Letting \( \Gamma = \theta (1-\delta) \) and solving the log-linear versions of these equations yields:

\[
\ln \mu_i = (1 - \Gamma') \ln \mu_i^\infty + \Gamma' \ln \mu_0.
\]
\[
\ln h_i = t \left[ \ln \left( AD^\delta \right) + \delta \ln \left( \Gamma_e / n \right) + \delta \ln \left( 1 - vn - z \right) \right] + \delta \theta \left[ t - \frac{1 - \Gamma'}{1 - \Gamma} \right] \ln \mu_n
\]
\[
+ \delta \theta \left[ \frac{1 - \Gamma'}{1 - \Gamma} \right] \ln \mu_0 + \ln h_0
\]

where \( \ln \mu_0 = \ln \left( k_0 / h_0 \right) - \ln \left( 1 - vn - z \right) \) and \( \mu_0 \) is a function of \( n \) and \( z \). Starting from any initial stocks of capital \( (k_0, h_0) \) and for any decision rules \( (n, z, \Gamma_e, \Gamma_k, \Gamma) \), we can track down the entire dynamic path of capital accumulation \( (\mu_i, \mu_n, h_i) \) for \( t \geq 0 \).

Now, rewrite (1) as
\[ V(0) = \sum_{t=0}^{\infty} \alpha^t \left( \ln c_t + \rho \ln n + \phi \ln z \right) \]

\[ = \sum_{t=0}^{\infty} \alpha^t \left[ \ln \Gamma_e + \ln D + \theta \ln \mu_t + \ln (1 - vn - z) + \ln h_t \right] + \left( \frac{\rho}{1 - \alpha} \right) \ln n + \left( \frac{\phi}{1 - \alpha} \right) \ln z. \]

Substituting the solution for \((\ln h_t, \ln \mu_t)\) into the above expression of \(V(0)\) gives the welfare expression in terms of the initial state and the proportional allocation rules:

\[ V(0) = B_0 + B(n, z, \Gamma_e, \Gamma_k) + \Psi \ln \left( \frac{k_0}{h_0} \right) + \left( \frac{1}{1 - \alpha} \right) \ln h_0, \]  
(11)

where \(B_0\) is a constant (independent of the initial state and of the decision rules) and

\[ B(n, z, \Gamma_e, \Gamma_k) = \left( \frac{1}{1 - \alpha} \right) \left[ \rho \ln n + \phi \ln z + \ln (1 - \Gamma_e - \Gamma_k) + \ln (1 - vn - z) \right] \]

\[ + \left[ \frac{\alpha \delta}{1 - \alpha} \right] \left[ \ln (\Gamma_e/n) + \ln (1 - vn - z) \right] + \Phi \left[ \ln \Gamma_k - \ln n - \delta \ln (\Gamma_e/n) - \delta \ln (1 - vn - z) \right] - \Psi \ln (1 - vn - z) \]

with

\[ \Phi = \left( \frac{1}{1 - \theta(1 - \delta)} \right) \left[ \frac{\alpha \theta (1 - \delta)(1 - \theta)}{(1 - \alpha)[1 - \alpha \theta(1 - \delta)]} + \frac{\alpha \delta \theta}{(1 - \alpha)^2} \right] = \frac{\alpha \theta [1 - \alpha (1 - \delta)]}{(1 - \alpha)^2 [1 - \alpha \theta (1 - \delta)]} > 0, \]

\[ \Psi = \frac{\theta [1 - \alpha (1 - \delta)]}{(1 - \alpha)[1 - \alpha \theta (1 - \delta)]} > 0. \]

Incorporating the feasibility and technologies, the individual’s welfare in (11) is fully characterized by the initial state \((k_0, h_0)\) and the proportional allocations of time and output.\(^6\) Maximizing utility in (11) can now be achieved by choosing \((n, z, \Gamma_e, \Gamma_k)\) to maximize \(B(n, z, \Gamma_e, \Gamma_k)\).

We give the social optimum below and relegate the proof to Appendix A.

---

\(^6\) When population is endogenous, the notion of Pareto efficiency becomes confusing since one may consider the unborn children in the future as well. For Pareto efficiency with endogenous population growth, see Golosov et al. (2007), Michel and Wigniolle (2007), and Conde-Ruiz et al. (2010). Our analysis focuses on the welfare of a representative parent living today who values the average welfare of children in addition to own consumption, leisure, and fertility.
Proposition 1. For $\rho > \rho' \equiv a\delta(1-\alpha)^{-1} + \Phi(1-\delta)(1-\alpha)$, the interior social optimum is

$$
\Gamma^*_e = \frac{a\delta(1-\theta)}{1-\alpha(1-\delta)},
$$

$$
\Gamma^*_h = a\theta,
$$

$$
\Gamma^*_c = 1 - \Gamma^*_e - \Gamma^*_h,
$$

$$
\eta^* = \frac{\rho\Gamma^*_c - \Gamma^*_h}{\nu[1-\theta + (\rho + \phi)\Gamma^*_c - \Gamma^*_h - \Gamma^*_e]},
$$

$$
z^* = \frac{\phi \Gamma^*_c}{1-\theta + (\rho + \phi)\Gamma^*_c - \Gamma^*_h - \Gamma^*_e},
$$

$$
l^* = \frac{1-\theta}{1-\theta + (\rho + \phi)\Gamma^*_c - \Gamma^*_h - \Gamma^*_e}.
$$

The solution and the sufficient condition (checked in the appendix) for the social optimum are new, to our knowledge, in this tractable endogenous growth model with fertility, leisure, and two types of capital. This result about the social optimum will be useful for policy analyses in the subsequent sections.

4. The competitive equilibrium and government policies

We now consider government policies in three cases in the competitive equilibrium. First, the government uses a lump-sum tax to service its debt and finance education subsidies to fully eliminate the efficiency losses of human capital externality. Second, the government uses a labor-income tax to service its debt without education subsidies. Third, government debt is serviced by a labor-income tax together with education subsidization. These cases are closely related to the literature on government debt. Considering these cases separately can help understand the forces at work as well.

4.1. Government debt with lump-sum taxation and education subsidization

In this case, the government budget constraint becomes:
\[ n_{t+1}h_{t+1} = h_{t}r_{t} - \tau_{t} + s_{t}e_{t}, \]  

where \( s_{t} \) is the rate of the education subsidy, and \( \tau_{t} \) is the lump-sum tax. Accordingly, the individual’s budget constraint becomes:

\[ c_{t} = a_{t}r_{t} + (1 - v)n_{t} - z_{t} \]  

\[ w_{t}h_{t} - \tau_{t} - (1 - s_{t})e_{t}n_{t} - a_{t+1}n_{t}. \]  

(13)

If leisure were absent, then this case would be the same as Zhang (2003) because the consumption tax at a constant rate therein would be lump-sum without leisure.

Given his own initial stocks \((a_{t}, h_{t})\), the prices, the government policy, and aggregate/average variables denoted in \( \Omega_{t} = (\bar{h}_{t}, r_{t}, w_{t}, \tau_{t}, s_{t}) \), the problem of a representative agent maximizing utility choosing \((a_{t+1}, h_{t+1}, n_{t}, z_{t})\) is formulated in the Bellman’s equation:

\[ V(a_{t}, h_{t}; \Omega_{t}) = \max_{a_{t+1}, h_{t+1}, n_{t}, z_{t}} \left[ \ln c_{t} + \rho \ln n_{t} + \phi \ln z_{t} + \alpha V(a_{t+1}, h_{t+1}; \Omega_{t+1}) \right], \]  

(14)

subject to

\[ c_{t} = a_{t}r_{t} + (1 - v)n_{t} - z_{t} \]  

\[ w_{t}h_{t} - \tau_{t} - (1 - s_{t})n_{t}(h_{t+1}/A)^{(1/\delta)} \left( h_{t}^{\rho} \bar{h}_{t}^{(1-\rho)} \right)^{-(1-\delta)/\delta} - n_{t}a_{t+1}, \]

where we have used (3) to substitute \( h_{t+1} \) for \( e_{t} \).

The first-order conditions are as follows:

\[ \frac{n_{t}}{c_{t}} = \frac{\alpha r_{t+1}}{c_{t+1}}, \]  

(15)

\[ \frac{n_{t}(1 - s_{t})}{c_{t}} = \frac{\alpha \delta (1 - v)n_{t+1} - z_{t+1}}{n_{t}(1 - \delta)} \]  

\[ w_{t}h_{t} - \tau_{t} - (1 - s_{t})n_{t+1}e_{t+1}, \]  

\[ \frac{\rho}{n_{t}}, \]  

(16)

\[ \frac{w_{t}h_{t} + (1 - s_{t})e_{t} + a_{t+1}}{c_{t}} = \frac{\rho}{n_{t}}, \]  

(17)

\[ \frac{w_{t}h_{t}}{c_{t}} = \frac{\phi}{z_{t}}. \]  

(18)

Government debt tends to have a positive direct effect on the bequest cost of a child via \( a_{t+1} \) in the left-hand side of (17) as known in the literature. Also known in the literature is that education subsidization has negative effects on the user cost of education and the
education cost of a child respectively in the left-hand sides of (16) and (17). We thus expect that higher government debt reduces fertility and that higher education subsidization increases education spending. These effects may also generate further responses from other variables through the tradeoff between labor and leisure and the tradeoff between the number of children and investments per child, which leads to new results below. Moreover, there are equilibrium-feedback effects through changes in prices.

The competitive equilibrium is characterized by (2)-(5), (8)-(10), (12)-(13), \( l_i = 1 - vn_i - z_i \) , (15)-(18), and \( \bar{x} = x \) for \( x = a,b,h,k,l,z,y,n \). Let \( \Gamma_b \equiv b_i n_i / y_i \) be the debt-output ratio and \( \Gamma_a \equiv a_i n_i / y_i \) be the fraction of output left as bequests. In the current model, fertility, proportional allocations of time and output, the debt-output ratio, and the subsidy rate are stationary. Starting with the initial period (time 0), the equilibrium solution for all \( t \geq 0 \) is given by

\[
\Gamma_e \equiv e_i n_i / y_i = \frac{\alpha \delta (1 - \theta)}{(1 - \delta) \left[ 1 - \alpha \beta (1 - \delta) \right]},
\]

\[
\Gamma_k = k_i n_i / y_i = \alpha \theta, \tag{20}
\]

\[
\Gamma_a = a_i n_i / y_i = \alpha \theta + \Gamma_b, \tag{21}
\]

\[
\Gamma_c = c_i / y_i = 1 - \Gamma_e - \Gamma_k, \tag{22}
\]

\[
n = \frac{\rho \Gamma_e - \Gamma_b - \Gamma_k - (1 - s) \Gamma_e}{\nu \left[ 1 - \theta + (\rho + \phi) \Gamma_e - \Gamma_b - \Gamma_k - (1 - s) \Gamma_e \right]}, \tag{23}
\]

\[
z = \frac{\phi \Gamma_e}{1 - \theta + (\rho + \phi) \Gamma_e - \Gamma_b - \Gamma_k - (1 - s) \Gamma_e}, \tag{24}
\]

\[
l = 1 - vn_i - z = \frac{1 - \theta}{1 - \theta + (\rho + \phi) \Gamma_e - \Gamma_b - \Gamma_k - (1 - s) \Gamma_e}. \tag{25}
\]

The social optimum corresponds to a special case of the competitive equilibrium solution in (19)-(25) without externality and without government intervention, i.e. \( \beta = 1, \ s = 0 \).
and \( b_t = 0 (\Gamma_b = 0) \) for all \( t \). That is, the Welfare Theorems apply here absent frictions.

Comparing the laissez faire with the social optimum gives:

**Lemma 1.** Suppose \( \rho > \varrho \equiv \alpha \delta (1 - \alpha)^{-1} + \Phi (1 - \delta) (1 - \alpha) \). The laissez faire allocation with \( \beta = 1 \) is socially optimal. In laissez-faire equilibrium with \( 0 < \beta < 1 \), fertility and the fraction of output spent on consumption are above their socially optimal levels, while leisure, labor, and the fraction of output spent on education are below their socially optimal levels, leading to lower levels of output per worker at all times.

**Proof.** The result in the case with \( \beta = 1 \) follows our previous discussion. Differentiating the solution above with respect to \( \beta \) yields: \( \partial \Gamma_c / \partial \beta > 0 \), \( \partial \Gamma_k / \partial \beta = 0 \), \( \partial \Gamma_c / \partial \beta < 0 \), \( \partial n / \partial \beta < 0 \), \( \partial z / \partial \beta > 0 \), and \( \partial l / \partial \beta > 0 \). From the relationship between the laissez-faire equilibrium for \( 0 < \beta < 1 \) and the social optimum \( (\beta = 1) \), we thus have \( \Gamma_c > \Gamma_c^* \), \( \Gamma_e < \Gamma_e^* \), and \( \Gamma_k < \Gamma_k^* \). As a result, \( (\Gamma_k + \Gamma_e) / \Gamma_c < (\Gamma_k^* + \Gamma_e^*) / \Gamma_c^* \). As in (23), a positive fertility with zero debt requires the taste for number of children to satisfy \( \rho \geq \gamma (\Gamma_k^* + \Gamma_e^*) / \Gamma_c^* \). Therefore, the sufficient condition for an interior social optimum \( \rho > \gamma = \alpha \delta (1 - \alpha)^{-1} + \Phi (1 - \delta) (1 - \alpha) \) in Proposition 1 is sufficient to guarantee an interior laissez-faire solution. *Q.E.D.*

The effects of human capital externalities in Lemma 1 are consistent with the common concerns in developing countries about high birth rates and low education spending, as noted in the aforementioned literature. A new result in Lemma 1 is the negative effects of the human capital externality on leisure and labor. Thus, if higher government debt increases leisure and labor, it can mitigate the efficiency loss of the externality.

We now look at the effects of government debt and education subsidization. From
(19) to (25), fertility, $n$, is decreasing in $\Gamma_b$, whereas leisure, $z$, and labor, $l$, are increasing in $\Gamma_b$. Meanwhile, higher education subsidies have a positive effect on the fraction of output spent on children’s education, a negative effect on the fraction of output on consumption, a negative effect on fertility, and a positive effect on labor. We summarize the results below.

**Proposition 2.** With a lump-sum tax, a rise in the debt-output ratio increases both leisure and labor but reduces fertility, without any effect on the proportional output allocation. A rise in the education subsidy rate increases both labor and the fraction of output for education but reduces both fertility and the fraction of output for consumption.

The results in Proposition 2 extend those in Zhang (2003, 2006) to capture a positive effect of higher government debt on leisure and therefore a more moderate positive effect on labor. This result allows us to explain the observed moderate increase in labor in Japan in Section 5. The positive effect of higher government debt on labor differs from the negative effect on labor in Burbidge (1983). Intuitively, higher government debt reduces fertility by increasing the bequest cost of having a child (hence increasing capital intensity). Through the tradeoff between the quantity and the quality of children, higher government debt increases per child education spending without changing the fraction of output spent on education. The result that higher government debt does not change the proportional allocation of output reflects the Ricardian neutrality. Consequently, output per worker and consumption per worker depend positively on government debt.

As a new result from the current model, higher government debt increases both leisure and labor at the same time, which is possible because the decline in fertility driven by government debt frees time from childrearing. Also new is the positive effect of higher education subsidies on labor through reducing fertility. These effects of government debt and education subsidies counteract those of the human capital externality.

We now explore whether government debt along with education subsidies under a
lump-sum tax can close the gaps between the competitive equilibrium solution with externalities and the social optimum. This has been achieved in Zhang (2003) without the leisure-labor tradeoff, where government debt and education subsidies are sufficient to fully close two gaps in fertility and education spending per child between the competitive equilibrium solution and the social optimum. With one more gap in the labor-leisure tradeoff in the present model, it is unclear whether the two instruments can still close the gaps altogether.

**Proposition 3.** Government debt and education subsidization serviced by lump-sum taxes achieve the social optimum for $0 < \beta < 1$ (with externalities) when it consists of

$$ s^* = \frac{\alpha(1-\beta)(1-\delta)}{1-\alpha\beta(1-\delta)} > 0, $$

$$ \Gamma_b^* = \frac{\alpha^2\delta(1-\delta)(1-\beta)(1-\theta)}{[1-\alpha\beta(1-\delta)][1-\alpha(1-\delta)]} > 0. $$

**Proof.** The optimal policy mix $(s^*, \Gamma_b^*)$ is chosen such that the competitive equilibrium solution becomes the same as the social planner’s solution given in Proposition 1. The optimal subsidy rate $s^*$ is set such that $\Gamma_e$ in (19) is equal to $\Gamma_e^*$ in Proposition 1. There is no effect of the externality and no effect of the policy $(s, \Gamma_b)$ on $\Gamma_k$. Thus, $\Gamma_e = \Gamma_e^*$ is obtained by the education subsidy rate $s^*$. The optimal debt level $\Gamma_b^*$ is set such that a common term $\Gamma_b + (1-s)\Gamma_e$ in (23), (24) and (25) is equal to $\Gamma_e^*$, at which fertility, leisure, and labor reach their socially optimal levels: $n = n^*$, $z = z^*$ and $l = l^*$. Clearly, in the absence of the externalities, the socially optimal levels of government debt and education subsidization should be equal to zero. Q.E.D.

Neither government debt alone nor education subsidization alone can achieve the social optimum in this model, albeit each of them tips the leisure-labor tradeoff and the
tradeoff between the quantity of children and investment per child in the right directions. Higher government debt has a negative effect on fertility according to (23) and a positive effect on leisure and labor according to (24) and (25). By reducing fertility, higher government debt can increase investments in human and physical capital per child, \( \Gamma_e/n \) and \( \Gamma_k/n \), in (19) and (20). From Proposition 2 and Lemma 1, however, government debt serviced by a lump-sum tax has no effect on the fraction of output spent on children’s education (\( \Gamma_e \)) and thus cannot fully resolve the problem of under-investment from the externality.

By contrast, higher education subsidization has a standard positive effect on the fraction of output spent on children’s education according to (19), a negative effect on fertility according to (23) as well as a positive effect on labor according to (25). When the fraction of output spent on children’s education is raised to its socially optimal level by education subsidization alone, one can verify that fertility would still be above its socially optimal level and that, at the same time, leisure and labor would still be below their socially optimal levels. Therefore, government debt and education subsidization financed by the lump-sum tax help to reduce fertility to its socially optimal level and correct the under-investment in education. When fertility reaches its socially optimal level, the remaining time available for leisure and labor is at its socially optimal level. The division between leisure and labor falls into their socially optimal levels as well because the division between them is not affected by the human capital externality according to (24) and (25).

The results here hinge on the use of a lump-sum tax. However, taxes across nations are usually functions of income or spending rather than lump-sum. We consider a labor income tax next for a comparison with another line of the literature.

### 4.2. Government debt with labor income taxation and without education subsidization

With a labor-income tax rate \( \pi_l \) and without education subsidies as in Burbidge (1983), the budget constraint is:
\[ c_i = a_i r_i + (1 - \nu_i - z_i) w_i (1 - \pi_i) h_i - \tau_i - e_i n_i - a_i \tau_i, \]  

(26)

where \( \tau_i > 0 \) if \( t = 0 \) and \( \tau_i = 0 \) if \( t > 0 \) is a transfer in the initial period from a permanent increase in the debt to output ratio. The government’s budget constraint becomes:

\[ n_i r_{t+1} = n_i r_t - \tau_i - \pi_i (1 - \nu_i - z_i) w_i h_i. \]  

(27)

The initial transfer from a rise in government debt allows for a one-period adjustment in order to obtain time-invariant decision rules in equilibrium. We rewrite the government budget constraint when \( t > 0 \) as:

\[ \Gamma_b = \frac{\Gamma}{\alpha} - \pi_i (1 - \theta). \]  

(28)

The capital market clearing condition is the same as (8).

The solutions for output and time allocations and fertility are given below:

\[ \Gamma_k = \frac{k_i}{y_i} = \alpha \theta, \]  

(29)

\[ \Gamma_a = \frac{a_i}{y_i} = \alpha \theta + \Gamma_b, \]  

(30)

\[ \Gamma_c = \frac{c_i}{y_i} = 1 - \Gamma_c - \Gamma_k, \]  

(31)

\[ \Gamma_e = \frac{e_i n_i}{y_i} = \frac{\delta[\alpha(1 - \theta) - (1 - \alpha) \Gamma_b]}{1 - \alpha \beta (1 - \delta)}, \]  

(32)

\[ n = \frac{\alpha \rho \Gamma_c - \Gamma_b - \Gamma_k - \Gamma_c}{\nu[\alpha(1 - \theta) - \Gamma_b + \alpha (\rho + \phi) \Gamma_c - \alpha \Gamma_k - \alpha \Gamma_e]}, \]  

(33)

\[ z = \frac{\alpha \phi \Gamma_e}{\alpha(1 - \theta) - \Gamma_b + \alpha (\rho + \phi) \Gamma_c - \alpha \Gamma_k - \alpha \Gamma_e}, \]  

(34)

\[ l = \frac{\alpha (1 - \theta) - (1 - \alpha) \Gamma_b}{\alpha(1 - \theta) - \Gamma_b + \alpha (\rho + \phi) \Gamma_c - \alpha \Gamma_k - \alpha \Gamma_e}. \]  

(35)

The labor income tax introduces a negative effect of higher government debt on education spending in (32) in comparison with (19) in which a lump-sum tax is used. Through the tradeoff between the quantity and quality of children, the decline in
education spending relative to income, $\Gamma_e$, caused by higher government debt creates an indirect positive effect of higher government debt on fertility in (33) and an indirect negative effect on labor in (35), in comparison with (23) and (25), respectively. Also, the labor income tax introduces a direct negative effect of higher government debt on labor in (35) as opposed to (25). To determine the net effects, differentiating the solutions above with respect to the debt-output ratio leads to:

**Proposition 4.** Without education subsidization, a rise in government debt serviced by a labor income tax increases leisure and the fraction of income for consumption, reduces the fraction of income for education, and has no effect on the fraction of income invested in physical capital. It also increases (reduces) labor if $\phi < (>) \alpha / (1 - \alpha) - \rho$. Moreover, it reduces fertility if

$$\rho < \alpha [1 - \alpha \beta (1 - \delta) + \phi(1 - \delta)(1 - \alpha \beta)] / \left[ (1 - \alpha)[1 - \alpha \beta (1 - \delta)] \right].$$

The rise in the labor income tax rate reduces the after-tax return on human capital and labor, tending to reduce education spending and labor. It also reduces the opportunity cost of spending time on leisure and childrearing, tending to increase leisure and fertility. These effects of the labor income tax on labor and fertility counteract those of government debt in Proposition 2, while the effect of the labor income tax on leisure reinforces that in Proposition 2. Thus, a rise in government debt serviced by the labor income tax increases leisure but its effects on fertility and labor depend on the strength of the taste for leisure and for the number of children. If the taste for the number of children is not too strong relative to the taste for the welfare of children, then increasing government debt serviced by labor income taxation reduces fertility and may increase labor. It is important to note that the condition for a negative effect of higher government debt on fertility is necessary but insufficient for a positive effect of higher government debt on labor, which can be verified by comparing the two conditions in Proposition 4. If the taste for leisure is relatively strong (weak), the rise in government debt with labor income taxation reduces (increases) labor.

These rich effects of government debt serviced by a labor income tax on leisure and
labor are important in their own right and can only arise when we consider endogenous fertility and the tradeoff between labor and leisure altogether. For instance, on the one hand, without endogenous fertility, higher government debt serviced by a labor income tax would increase leisure and reduce labor at the same time for certain, as in Burbidge (1983). On the other hand, without elastic leisure as in Zhang (2003, 2006), a decline in fertility due to greater government debt would go hand in hand with a rise in labor, because all the time units freed from rearing fewer children would add to labor. Thus, the former case with fixed fertility and with elastic leisure could not capture the rise in labor during the period of rising government debt observed in Japan. Whereas the latter case with endogenous fertility and without leisure would overstate the possible positive effect of higher government debt on labor in Japan.

We now characterize the optimal debt policy serviced by labor-income taxation without education subsidization. The solution for the welfare level as a function of government policy can be obtained by substituting the competitive equilibrium allocation in (29)-(35) into (11). The proof of the result is relegated to Appendix B.

**Proposition 5.** Without education subsidization, optimal government debt serviced by a labor income tax is unique and positive for $0 < \beta < 1$ (with externalities) if $\rho < \rho < \alpha / (1 - \alpha)$ and if $\phi < [1 - \alpha \beta (1 - \delta) - \alpha \delta] / [\alpha \delta (1 - \alpha)]$. It equals zero for $\beta = 1$ (without externalities).

The existence of optimal government debt financed by the labor income tax in Proposition 5 depends on the strength of the taste for the number of children (relative to the welfare of children) and on the taste for leisure in the presence of the externalities. This is because the labor income tax may increase fertility and reduce labor and education spending, which reinforces the effects of human capital externalities in Lemma 1 and counteracts the effects of government debt in Proposition 2. According to Proposition 4, government debt serviced by labor income taxation reduces fertility when the taste for the number of children relative to the welfare of children is bounded above
by a specific level. It is easy to verify that the upper bound on the taste for the number of children for optimal government debt in Proposition 5 to mitigate the effects of human capital externalities satisfies the condition for a negative fertility effect of higher government debt in Proposition 4. Under this condition, higher government debt serviced by labor income taxes may increase education spending per child relative to output per worker through the typical tradeoff between the quantity and the quality of children.

Also, when higher government debt reduces fertility, the subsequent decline in time for childrearing will increase leisure and may increase labor. According to Proposition 4, higher government debt serviced by labor income taxes increases labor when the taste for leisure is bounded above by a specific level. In this spirit, the upper bound on the taste for leisure for optimal government debt serviced by labor income taxes in Proposition 5 allows for a positive effect of higher government debt on labor. This restriction on the taste for leisure for optimal government debt is absent in Zhang (2003, 2006) that ignores leisure.

Under the restrictions in Proposition 5, the negative effect on fertility and the possible positive effects on education spending per child and labor of government debt serviced by labor income taxes counteract the effects of human capital externality. Thus, under these restrictions in Proposition 5, government debt serviced by labor income taxation can engender a net welfare gain in the presence of the externality.

The welfare implication in this case differs from that in Burbidge (1983) although government debt increases leisure in both cases. A key factor for the difference in the welfare implication is the existence of the human capital externality and the consideration of the tradeoff between the quantity and the quality of children. Without the externality, the competitive equilibrium allocation is socially optimal in this dynastic model and government debt would thus be welfare reducing because it always distorts fertility and leisure. If fertility were exogenous, the human capital externality would imply that leisure should be above its socially optimal level. In this latter case, a positive effect of higher government debt serviced by labor income taxes on leisure would be welfare reducing.
4.3. Debt policy with labor income taxation and education subsidization

This case is most relevant in practice because proportional income taxes and subsidies are used in many countries (see, e.g., Glomm and Ravikumar, 1998). Since education subsidization helps to mitigate under-spending on education in the presence of human capital externalities, it can help government debt financed by labor income taxes to do better in welfare terms. Now, the agent’s budget constraint becomes:

\[ c_t = a_t r_t + (1 - v n_t - z_t) w_t (1 - \pi_t) h_t - \tau_t - (1 - s) e_t n_t - a_{t+1} n_t, \tag{36} \]

The lump-sum tax or transfer will be forced to zero after the initial period in which it can be used to cancel out any residual change in the government budget constraint. The government runs a balanced budget according to:

\[ n_t h_{t+1} - \pi_t (1 - v n_t - z_t) w_t h_t + s e_t n_t. \tag{37} \]

The market clearing condition is the same as (8).

The solutions for proportional allocations of time and output and fertility are:

\[
\Gamma_e \equiv \frac{e_t n_t}{y_t} = \frac{\delta [\alpha (1 - \theta) - (1 - \alpha) \Gamma_b]}{\alpha \delta s + (1 - s) [1 - \alpha \beta (1 - \delta)]}, \tag{38}
\]

\[ \Gamma_k \equiv \frac{k_t n_t}{y_t} = \alpha \theta, \tag{39} \]

\[ \Gamma_a \equiv \frac{a_{t+1} n_t}{y_t} = \alpha \theta + \Gamma_b, \tag{40} \]

\[ \Gamma_c \equiv \frac{c_t}{y_t} = 1 - \Gamma_e - \Gamma_k, \tag{41} \]

\[ n = \frac{\alpha [\rho \Gamma_c - \Gamma_b - \Gamma_k - (1 - s) \Gamma_c]}{v [\alpha (1 - \theta) - \Gamma_b + \alpha (\rho + \phi) \Gamma_c - \alpha \Gamma_k - \alpha \Gamma_e]}, \tag{42} \]

\[ z = \frac{\alpha \phi \Gamma_c}{\alpha (1 - \theta) - \Gamma_b + \alpha (\rho + \phi) \Gamma_c - \alpha \Gamma_k - \alpha \Gamma_e}, \tag{43} \]

\[ l = \frac{\alpha (1 - \theta) - \alpha s \Gamma_c - (1 - \alpha) \Gamma_b}{\alpha (1 - \theta) - \Gamma_b + \alpha (\rho + \phi) \Gamma_c - \alpha \Gamma_k - \alpha \Gamma_e}. \tag{44} \]

As expected, the education subsidy helps to offset the negative effect of government debt on education spending under the labor income tax in (38). The labor income tax
weakens the positive effects of education subsidies on education spending and labor in (38) and (44), respectively, in comparison with (19) and (25) in which the lump-sum tax is used.

The government can choose a combination of government debt and education subsidies financed by labor income taxes to maximize social welfare in (11) subject to the government budget constraint, knowing the equilibrium solution in (38)-(44).

**Proposition 6.** The optimal government debt and education subsidization financed by labor-income taxation is such that:

\[
s^* = 1 - \frac{M_1 + \sqrt{M_4 - M_2[1 - \alpha \beta (1 - \delta) - \alpha \delta] - M_3 \alpha (1 - \theta)(1 - \delta)(1 - \alpha \beta)}}{2 \alpha \phi (1 - \theta)(1 - \delta)(1 - \alpha \beta) [1 - \alpha \beta (1 - \delta) - \alpha \delta]},
\]

where

\[
M_1 = (1 - \alpha)[1 - \alpha \beta (1 - \delta)][\rho - \alpha - \alpha (\rho + \phi)] + \alpha \phi \delta,
\]

\[
\{\alpha \delta (1 - \theta) + \alpha \theta [1 - \alpha (1 - \delta)] - \rho (1 - \alpha)[1 - \alpha \theta (1 - \delta)]},
\]

\[
M_2 = (1 - \alpha)[1 - \alpha \theta (1 - \delta)][\rho (1 - \alpha) - \alpha],
\]

\[
M_3 = [1 - \alpha (1 - \delta)][\alpha (1 + \rho + \phi) - (\rho + \phi)],
\]

\[
M_4 = \{M_2 \phi [1 - \alpha \beta (1 - \delta) - \alpha \delta] + M_3 \alpha (1 - \theta)(1 - \delta)(1 - \alpha \beta) - M_1 \}^2
\]

\[-4 \alpha \phi [1 - \alpha \beta (1 - \delta) - \alpha \delta](1 - \theta)(1 - \delta)(1 - \alpha \beta) M_2 M_3.
\]

\[
\Gamma_b^* = \frac{\alpha (1 - \theta)}{[1 - \alpha (1 - \delta)]} \left\{1 - \frac{(1 - s^*)[1 - \alpha \beta (1 - \delta) - \alpha \delta]}{1 - \alpha} \right\},
\]

\[
\pi_i^* = 1 - \frac{(1 - s^*)[1 - \alpha \beta (1 - \delta)]}{1 - \alpha (1 - \delta)}.
\]

We relegate a sketch of the proof to Appendix C (the full proof is lengthy and available upon request). As mentioned earlier, it is intuitive that the optimal debt policy financed by labor income taxes in Proposition 6 with education subsidization is better than that in Proposition 5 without education subsidization. However, the optimal policy in Proposition 6 cannot achieve the socially optimal level of fertility, because the labor
income tax tends to weaken the negative (positive) effect of government debt on fertility (labor) as well as the positive effects of education subsidies on education spending and labor. Thus, we regard the case in Proposition 6 as the second best policy in this paper, ranked below the socially optimal government debt and education subsidization financed by lump-sum taxes.

5. Numerical examples: An application to Japan

The numerical example is calibrated to the Japanese economy. The selection of Japan for calibration is based on several reasons. First, as mentioned earlier, the ratio of government debt to GDP rose significantly from the lowest among the OCED countries in the early 1970s to the highest today. Second, the bulk of Japanese government debt is internally held by Japanese investors, which allows our closed-economy model to do reasonably well. Third, like other OECD countries, Japan’s basic education (elementary and lower secondary) is compulsory and heavily subsidized by the government, which suggests a non-negligible degree of externality in formal education. Fourth, Japan’s dramatic demographic change since the 1970s necessitates an endogenous treatment of fertility. Last but not least, the altruistic dynasty model has been found to be consistent with the Japanese household data as in Hayashi et al. (1988) and Hayashi (1989).

It is a challenge to see whether the model can quantitatively capture changes in fertility, labor, and aggregate output growth in Figures 1 and 2 when the ratio of government debt to output rises substantially. Doing so provides an empirically plausible parameterization to determine the optimal debt-output ratio, the education-subsidy rate, and the consequences of excessive debt on the economy. Also, it is important to compare cases with or without leisure and gauge the differences in results quantitatively.

The values of the parameters are chosen in line with those in the literature when available, such as $\alpha = 0.6$, $\theta = 0.33$, and 30 years per period. The values for the rest of parameters are chosen to generate plausible values for fertility, labor, the proportional

---

7 In 2009, the public share of total expenditure on primary, secondary, and post-secondary non-tertiary education is 90.4% in Japan (OECD, Education at a glance 2012). Also, Japan’s intergenerational earnings mobility is documented higher than that in the United States, the United Kingdom, and France, implying a stronger spillover effect in determining children’s learning; see, e.g., Ichino et al. (2011) and Lefranc et al. (2008).
allocations of income, and the growth rate in Japan. In particular, the values of these parameters are chosen to generate changes in variables resembling observations in Figures 1 and 2 in Japan: $v = 0.22, \delta = 0.27, \rho = 1.455, \phi = 0.033, \beta = 0.4$, and $A = D = 3$. Here, the share parameter for physical inputs in education ($\delta = 0.27$) is smaller than the counterpart in production because education is less physical capital intensive than is production.\footnote{Bowen (1987) estimated that physical inputs account for 22% of the total explicit cost of acquiring higher education (excluding forgone earnings). When implicit costs (in-kind subsidies such as land) are considered, $\delta = 0.27$ appears plausible.}

A key parameter for human capital externality is $\beta$. Lefranc et al. (2008) gauges education mobility in terms of human capital transmission across generations by using five waves of Japan’s Social Stratification and Mobility Survey data from 1965 to 2005. The estimated coefficient on father’s education from regressing son’s education on father’s education and cohort dummies is 0.29 when the number of years of formal schooling is used as a proxy for educational attainment (human capital stock). This intergenerational education elasticity corresponds to $\beta(1-\delta)$ in the log linear version of education technology in (3). Applying their estimates, the degree of the externality relative to parental human capital across families, $1-\beta$, can be pinned down to 0.6 with $\beta = 0.4$ for Japan’s economy.

Table 1 gives simulation results for the optimal debt-output ratio, the optimal education subsidy rate, the proportional allocations, aggregate output growth, and the consumption-equivalent variations in welfare in two panels. Each panel has four cases: the laissez faire; the first-best case in Proposition 3; the second-best case in Proposition 6; and the case with an 85% debt-output ratio, an education subsidy at the same rate as in the second-best case, and a labor income tax. The reason for choosing an 85% debt-output ratio on the higher end is that fertility may have a minimum value at 1 or higher because a family may have strong demand for at least one child or one child of particular gender. The present model, like most others, does not capture such a possible minimum fertility value.
The equivalent consumption variations in the last row of Table 1 are in terms of the change in consumption $\Delta$ in every period such that the laissez faire case reaches the same welfare level as that in a case with government intervention type $i$:

$$V(0)^{\text{no-government}} + \frac{1}{1-\alpha} \ln(1+\Delta) = V(0)^{\text{government policy } i}.$$  

This is to add $\sum_{t=0}^{\infty} \alpha^t \ln(1+\Delta) = [\ln(1+\Delta)]/(1-\alpha)$ to the welfare in the laissez-faire case.

In the left panel of Table 1, the debt-output ratio is increased from 0 (laissez faire) to the first-best level, the second-best level, and 85%. Among these cases, the second-best case with a debt-output ratio at 12.3%, fertility at 2.17, and the annual growth rate of aggregate output at 3.78% is most close to the 1970’s observation in Japan in Figures 1 and 2. From this second-best debt ratio of 12.3% to the high debt level at 85%, the fertility rate declines from 2.17 to 1.45, labor increases from 50.1% to 52.6%, and the annual growth rate of aggregate output declines from 3.78% to 1.52%. Such changes are very similar to those documented in Figures 1 and 2 in Japan as well. The welfare gain of the second-best policy with the 12.3% debt-output ratio, a 50% education subsidy, and a 26.6% labor income tax rate is equivalent to topping up consumption in the laissez faire by 2.65% for each generation.

Moreover, the optimal debt-output ratio is equal to 6.2% in the first-best case under lump-sum taxation, together with education subsidization at 31.9%. The welfare gain is equivalent to topping up consumption in the laissez faire by 4.31%. In comparison, the second-best case yields much of the possible welfare gain from the laissez faire case (a 2.65% gain in terms of consumption compared to the highest possible 4.31% gain). Such optimal policies obtained from the Japanese experience suggest that most G20 countries have accumulated too much government debt (except Australia, China, Russia and Saudi Arabia with their respective debt to GDP ratios at 27%, 23%, 11%, and 4% in 2012 as documented in IMF 2013 Fiscal Monitor).

Taking the parameterization from the Japanese economy, what is the possible welfare loss at an excessively high ratio of government debt to output? When we set the ratio of
debt to output at 85% with a labor income tax in Table 1, the welfare loss is equivalent to a substantial 39.39% reduction in consumption each period with a 50% education subsidy rate. This example clearly illustrates significant welfare losses from excessive government debt.

Also, the rise in government debt has non-monotonic effects on aggregate output growth due to the various responses from fertility, labor, education spending, and investment in different cases with different taxes and with or without subsidies. The aggregate output growth rate is 3.49% in the laissez faire, rises to 3.58% in the first-best case, rises further to 3.78% in the second-best case, and eventually falls to 1.52% at the excessive debt ratio of 85%.

Finally, we compare cases with or without leisure. In the right panel of Table 1, we omit leisure by setting \( \phi = 0 \) as in Zhang (2003, 2006). Several points from the comparison deserve attention. First, absent leisure, the first-best case in the right panel has the same debt-output ratio and the same education subsidy rate as those in the left panel with leisure. Second, the second-best case has a much higher debt-output ratio (59.3%) and a much higher education subsidy rate (89.7%) without leisure than those with leisure. Third, when the debt-output ratio rises to 85%, fertility declines much further to 0.59, labor increases much further to 87.1%, and the welfare loss increases much further to a 45.1% of consumption reduction than those in the case with leisure. However, the changes in fertility and labor in the right panel without leisure are much larger than those in Figures 1 and 2. The comparison suggests that the case with leisure in this model should be more realistic than the case without leisure in the literature.

6. Conclusion
In this paper we have explored how government debt serviced by either a lump-sum tax or a labor income tax affects labor, leisure, fertility, investment in physical and human capital, and welfare, with or without education subsidies in an intergenerational model. We have also explored how to use government debt and education subsidization to mitigate the efficiency losses of human capital externalities and determined optimal government debt financed by lump-sum or labor income taxes. Moreover, we have
applied the model to the Japanese economy for quantitative implications. Our results include some new contributions.

The first contribution is the establishment of the social optimum from not only the necessary but also the sufficient conditions, facing the non-convexity of the feasible set and facing a labor-leisure tradeoff. In comparison, the human capital externality in the laissez faire has negative effects on both leisure and labor, in addition to its positive effect on fertility and negative effects on capital intensity and education investment as found in existing studies.

Our second contribution is the finding of complex effects of government debt on fertility, leisure, and labor. Higher government debt has a positive effect on leisure regardless of whether education spending is subsidized or whether government debt is serviced by lump-sum or labor income taxes. But the effect of higher government debt on labor depends on whether the tax is lump-sum or proportional to income. With lump-sum taxation, higher government debt increases labor. With labor income taxation, higher government debt may decrease (increase) labor if the taste for leisure is sufficiently strong (weak). The positive effects of government debt on leisure and possibly on labor counteract those of human capital externalities for possible welfare gains. The social optimum can be decentralized through government debt and education subsidization financed by lump-sum taxation. With labor income taxation, however, government debt cannot achieve the social optimum but still mitigates the welfare loss from the human capital externality and can do better when it is used together with education subsidization.

The third contribution is the quantitative implications of the model calibrated to the Japanese experiences. The second-best policy, consisting of government debt at 12.3% of output and an education subsidy rate at 50% financed by labor income taxes, generates similar fertility and a similar growth rate of output to those in the 1970’s Japanese economy. When we increase the debt-output ratio from the second-best level to 85%, the model produces a large decline in fertility, a moderate increase in labor, and a large decline in the rate of output growth, which resemble the observations in Japan. From this plausible calibration, we obtain a welfare gain of the first-best (second-best) policy from
the laissez faire equivalent to a 4.31% (2.65%) increase in consumption of all generations. However, a high debt-output ratio at 85% causes a large welfare loss equivalent to a 39% reduction in consumption for each generation, together with a 50% subsidy rate as in the second-best case.

Finally, our quantitative results based on the Japanese experiences may have useful policy implications, suggesting an optimal government debt-GDP ratio in the range of 10-20%, along with education subsidization and income taxation. Indeed, many developed countries today worry about excessive government debt, low GDP growth, and below-replacement fertility. In these countries, our results suggest substantial welfare losses from high government debt, which are in line with the increasing social discontent about government debt policies in recent years.
Appendix A.

Proof of Proposition 1. The socially optimal allocation rules maximizing 
\( B(n,z,\Gamma_e,\Gamma_k) \) are derived from the following first-order conditions with respect to 
\( (n,z,\Gamma_e,\Gamma_k) \) respectively.

\[
B_n = A_1 \frac{1}{n} - A_2 \frac{v}{1-vn^*-z} = 0, \quad (A1)
\]

\[
B_z = \frac{\phi}{(1-\alpha)} z^* - A_2 \frac{1}{1-vn^*-z^*} = 0, \quad (A2)
\]

\[
B_{\Gamma_v} = A_1 \frac{1}{\Gamma_v^*} - \frac{1}{(1-\alpha)(1-\Gamma_v^* - \Gamma_k^*)} = 0, \quad (A3)
\]

\[
B_{\Gamma_k} = -\frac{1}{(1-\alpha)(1-\Gamma_v^* - \Gamma_k^*)} \Phi \frac{1}{\Gamma_k^*} = 0, \quad (A4)
\]

where \( A_i = \rho(1-\alpha)^{-1} - \alpha\delta(1-\alpha)^{-2} - \Phi(1-\delta) > 0 \)

if \( \rho > \rho = \alpha \delta (1-\alpha)^{-1} + \Phi(1-\delta)(1-\alpha), \)

\[
A_2 = (1-\alpha)^{-1} + \alpha \delta (1-\alpha)^{-2} - \Phi \delta - \Psi = \frac{(1-\theta)[1-\alpha(1-\delta)]}{(1-\alpha)^2[1-\alpha \theta(1-\delta)]} > 0,
\]

\[
A_1 = \alpha \delta (1-\alpha)^{-2} - \Phi \delta = \frac{\alpha \delta (1-\theta)}{(1-\alpha)^2[1-\alpha \theta(1-\delta)]} > 0,
\]

with \( \Phi \) and \( \Psi \) given in (11). It can be verified that

\[
\rho = \alpha \delta (1-\alpha)^{-1} + \Phi(1-\delta)(1-\alpha) = (\Gamma_v^* + \Gamma_k^*) / \Gamma_v^*.
\]

The condition \( \rho > \rho \) guarantees that fertility and the other choice variables are positive. Equations (A3) and (A4) determine \( A_i \Gamma_k^* = \Phi \Gamma_v^* \). With this relationship, \( (\Gamma_v^*, \Gamma_k^*) \) can be solved as in Proposition 1 from (A3) and (A4).

Equations (A1) and (A2) imply

\[
n^* = \frac{A_1}{v [A_1 + A_2 + \phi (1-\alpha)^{-1}]}, \quad \frac{A_1}{n^*} = \frac{\phi v}{(1-\alpha)z^*}.
\]

Substituting the values of \( A_i \) and \( A_2 \) gives the social optimal fertility and leisure.
The second-order condition should ensure a negative definite Hessian matrix in the situation where the first-order conditions hold:

\[
\begin{bmatrix}
  B_{nn} & B_{nz} & B_{nx} & B_{nx} \\
  B_{zn} & B_{zz} & B_{zx} & B_{zx} \\
  B_{xn} & B_{xz} & B_{xx} & B_{xx} \\
  B_{xn} & B_{xz} & B_{xx} & B_{xx}
\end{bmatrix}.
\]

The elements of the Hessian matrix \((B_n = B_z = B_x = B_{xx} = 0)\) are given below:

\[
B_{nn} = -A_1 \frac{1}{n^2} - A_2 \frac{v}{(1-\nu n - z^*)^2} = -A_1 \frac{v}{n^*(1-\nu n - z^*)} - A_2 \frac{v}{n^*(1-\nu n - z^*)} < 0 \text{ if } \rho > \rho_
,
\]

\[
B_{zz} = -\frac{\phi}{(1-\alpha)z^*} - \frac{A_2}{(1-\nu n - z^*)^2} = -\frac{1}{z^*(1-\nu n - z^*)} - \frac{\phi}{(1-\alpha)z^*(1-\nu n - z^*)} < 0,
\]

\[
B_{nx,x} = -\frac{1}{(1-\alpha)\Gamma_c^{*2}} - \frac{\Phi}{\Gamma_c^*} = -\frac{1}{\Gamma_c^*\Gamma_k^*} < 0,
\]

\[
B_{nz} = -\frac{v}{(1-\nu n - z^*)^2} A_2 = -\frac{1}{(1-\nu n - z^*)n^*} = \frac{-\phi v}{(1-\alpha)z^*(1-\nu n - z^*)} < 0
\]

\[
B_{nx,x} = B_{nz} = B_{nx,x} = B_{nx,x} = 0.
\]

The 2\(\times\)2 principal minor is derived and signed as follows:

\[
B_{nn}B_{zz} - B_{nz}^2 = \frac{v}{z^*n^*(1-\alpha)(1-\nu n - z^*)^2} [(A_1 + A_2)][(1-\alpha)A_2 + \phi] - \phi A_2
\]

\[
= \frac{v}{z^*n^*(1-\alpha)(1-\nu n - z^*)^2} [(1-\alpha)(A_1 + A_2)A_2 + \phi A_2] > 0 \text{ if } \rho > \rho_
.
\]

The 3\(\times\)3 principal minor is obviously negative under \(\rho > \rho_\) because \(B_{nx,x} = B_{nz} = 0\) and because \(B_{nx,x} < 0\). The determinant of the full Hessian matrix is signed as follows:

\[
B_{nn}B_{zz}[B_{nx,x}B_{nx,x} - B_{nx,x}^2] - B_{nz}^2[B_{nx,x}B_{nx,x} - B_{nx,x}^2]
\]
\[
= [B_{m}B_{zz} - B_{m}^{2}][B_{\Gamma_{1}\Gamma_{2}} B_{\Gamma_{1}\Gamma_{2}} - B_{m}^{2}]
\]
\[
= [B_{m}B_{zz} - B_{m}^{2}]\left\{ \frac{1}{\Gamma_{e}^{2} \Gamma_{e}^{2} \Gamma_{e}^{2}} \left( \frac{1}{1-\alpha} + A_{\delta} \right) \left( \frac{1}{1-\alpha} + \Phi \right) - \frac{\Phi^{2}}{\Gamma_{e}^{2} \Gamma_{e}^{2}} \right\}
\]
\[
= [B_{m}B_{zz} - B_{m}^{2}]\left\{ \frac{1}{\Gamma_{e}^{2} \Gamma_{e}^{2} \Gamma_{e}^{2}} \left( \frac{1}{1-\alpha} + \Phi \right) + \frac{\Phi}{\Gamma_{e}^{2} \Gamma_{e}^{2}} \left( \frac{1}{1-\alpha} \right) \right\}
\]
which is positive under \( \rho > \rho' \) that implies \( [B_{m}B_{zz} - B_{m}^{2}] > 0 \). Q.E.D.

Appendix B.

Proof of Proposition 5. In this case, let us define \( B(\Gamma_{b}) = B(n(\Gamma_{b}), z(\Gamma_{b}), \Gamma_{e}(\Gamma_{b}), \Gamma_{e}) \)

based on the expressions in (11) and the solution in (29)-(35). The first-order condition \( B'(\Gamma_{b}) = 0 \) is:

\[
B'(\Gamma_{b}) = \frac{1}{1-\alpha} \left[ \frac{\rho n'}{n} + \frac{\phi z'}{z} + \frac{\Gamma_{e}'}{\Gamma_{e}} - \frac{(vn' + z')}{1-vn - z} \right] + \frac{\alpha \delta}{(1-\alpha)^{2}} \left[ \frac{\Gamma_{e}'}{\Gamma_{e}} - \frac{n'}{n} - \frac{(vn' + z')}{1-vn - z} \right] - \Phi \left[ \frac{n'}{n} + \frac{\delta n'}{\Gamma_{e}} - \frac{\delta (vn' + z')}{1-vn - z} \right] + \Psi (vn' + z') = 0.
\]

For notational ease, let \( D_{n} \) be the common denominator of \( vn, z, \) and \( l \) in (33)-(35),

and let \( D^{o}, D^{z}, \) and \( D^{l} \) be the numerators in (33)- (35), respectively. Also, let us define

\( \Theta \equiv (1-\alpha \theta)[1-\alpha \beta(1-\delta)] - \alpha \delta(1-\theta) > 0 \). Then, we have

\[
B'(\Gamma_{b}) = 0 = \frac{F(\Gamma_{b})}{(1-\alpha)^{2} D_{n} D^{o} D^{z} D^{l} [\alpha(1-\theta) - (1-\alpha)\Gamma_{b}] [\Theta + \delta(1-\alpha)\Gamma_{b}]}
\]

where

\[
F(\Gamma_{b}) \equiv \delta (1-\alpha) \{(1-\alpha)[\alpha(1-\theta) - (1-\alpha)\Gamma_{b}] - \alpha \Theta - \alpha \delta (1-\alpha)\Gamma_{b} + \Phi (1-\alpha)^{2} \Theta
+ \Phi \delta (1-\alpha)^{3} \Gamma_{b} \} D_{n} D^{o} D^{z} D^{l} + \alpha (1-\alpha \theta)[\rho (1-\alpha) - \alpha \delta - \Phi (1-\alpha)^{2}
(1-\delta)](D^{o})^{2} D^{l} [\Theta + \delta(1-\alpha)\Gamma_{b}] \{(1-\alpha \beta(1-\delta))[(1-\alpha)\rho - \alpha - \alpha \phi
[1-\alpha \delta (1-\delta)] + \phi^{2} \alpha(1-\alpha)(1-\alpha \theta)\Gamma_{b} - \alpha \beta (1-\delta) - \alpha \delta \phi (1-\alpha)^{2} - \Theta (1-\alpha)^{2}\}
[\Theta + \delta(1-\alpha)\Gamma_{b}] - \alpha (1-\alpha \theta)[1-\alpha + \alpha \delta - \delta \phi (1-\alpha)^{2} - \Psi (1-\alpha)^{2}\]
[1-\alpha \beta (1-\delta)](1-\alpha)\Gamma_{b} + a_{1} \Gamma_{b} + a_{0} = 0.
\]

This reduces to \( F(\Gamma_{b}) = a_{5} \Gamma_{b}^{5} + a_{4} \Gamma_{b}^{4} + a_{3} \Gamma_{b}^{3} + a_{2} \Gamma_{b}^{2} + a_{1} \Gamma_{b} + a_{0} = 0 \).
A unique optimal $\Gamma_b$ exists if (i) $B'(\Gamma_b) > 0$ at $\Gamma_b = 0$ and (ii) $B^*(\Gamma_b) < 0$ at $\Gamma_b^*$ such that $B'(\Gamma_b^*) = 0$. The condition for (i) $B'(\Gamma_b) > 0$ at $\Gamma_b = 0$ is $a_0 > 0$:

$$a_0 = \frac{1}{\alpha(1-\theta)[1-\alpha\beta(1-\delta)]\Theta} = \alpha^2 \phi(1-\alpha)\{\alpha\delta(1-\alpha)(1-\theta) - \alpha\Theta + \delta\Phi(1-\alpha)^2\}$$

$$\{\alpha(1+\rho+\phi)\Theta - \alpha\Theta[1-\alpha\beta(1-\delta)]\} \{\rho\Theta - \alpha\Theta[1-\alpha\beta(1-\delta)] - \alpha\delta(1-\theta)\} + \alpha^2 \phi(1-\theta)(1-\alpha\beta(1-\delta)]\Theta[\rho(1-\alpha) - \alpha\delta - \Phi(1-\alpha)(1-\alpha)^2]\}$$

$$\{1-$$

$$
\alpha\beta(1-\delta)(1-\alpha\rho - \alpha\delta) - \alpha\beta(1-\delta)]\rho\Theta - \alpha\Theta[1-\alpha\beta(1-\delta)] - \alpha\delta(1-\theta)\} - \alpha^2 \phi(1-\alpha\theta)(1-\alpha\beta(1-\delta)]\} \{1-$$

$$\alpha + \alpha\delta - \delta\Phi(1-\alpha)^2 - \Psi(1-\alpha)^2\}[(1-\alpha)\rho - \alpha + \phi(1-\alpha)]\} \rho\Theta - \alpha\Theta[1-\alpha\beta(1-\delta)] - \alpha\delta(1-\theta)\}.$$ 

Using the definition of $(\Phi, \Psi)$ in (11), we rewrite it as

$$a_0 = \left\{\frac{\alpha(1-\theta)[1-\alpha\beta(1-\delta)]\Theta}{1-\alpha\theta(1-\delta)}\right\} \alpha^4 \phi(1-\theta)(1-\alpha\theta)(1-\delta)(1-\beta)f,$$

where

$$f \equiv \{\rho\Theta - \alpha\Theta[1-\alpha\beta(1-\delta)] - \alpha\delta(1-\theta)\} \{\alpha\delta(1-\alpha)[1-\alpha\beta(1-\delta)] + \phi(1-\alpha)\{1-\alpha\beta(1-\delta)](1-\alpha\theta)(1-\alpha\delta) + \alpha\delta\} + \alpha^2 \delta^2 \phi(1-\alpha)(1-\theta)\} +$$

$$\{1-$$

$$\alpha - \rho(1-\alpha)\}\Theta\phi[\alpha[1-\alpha\theta(1-\delta)] + (1-\alpha\theta)(1-\alpha\delta)]\} \{1-\alpha\beta(1-\delta)]\} +$$

$$\alpha\phi[1-\alpha\beta(1-\delta)] + \alpha\delta\Theta \rho(1-\theta)\} \{\alpha - \rho(1-\alpha)\} - \alpha\delta\Phi(1-\alpha) +$$

$$\{1+$$

$$\rho\Theta - \alpha\Theta[1-\alpha\beta(1-\delta)] - \alpha\delta(1-\theta)\} + \alpha\phi(1-\alpha)]\{1-$$

$$\alpha\theta(1-\delta)]\} \delta + \Theta[1-\alpha\beta(1-\delta)]\} + \alpha^2 \delta\phi(1-\theta)(1-\alpha\beta(1-\delta)] +$$

$$\alpha\phi(1-\theta)[1-\alpha\beta(1-\delta)]\} \{\alpha(\delta\delta + (1-\delta)(1-\alpha\theta)](1-\delta)(1-\alpha\beta) +$$

$$\{1-$$

$$\alpha\theta(1-\delta)]\} - \alpha\phi(1-\alpha)(1-\alpha\theta)\} \{1-$$

$$\alpha\beta(1-\delta)\} \} - \alpha\phi\Theta[1-\alpha\beta(1-\delta)]\{1-$$

$$\alpha\theta(1-\delta)]\}.$$

Observe that if $\beta = 1$ then $a_0 = 0$ and hence $B'(\Gamma_b) = 0$ at $\Gamma_b = 0$. For $0 < \beta < 1$,

$B'(\Gamma_b) > 0$ at $\Gamma_b = 0$ under the condition that $\alpha$ is sufficiently large relative to $\rho$ and $\phi$,

as shown in Figure 3 which is based on the parameterization $\beta = 0.4, \alpha = 0.6, \delta = 0.27,$

$\rho = 0.93, \theta = 0.33, \phi = 0.33$, and $\nu = 0.1$.

We now look at the second-order condition:

$$B^*(\Gamma_b) = \Pi_0[(n'/n)^2 - n'/n] + \Pi_1[(vn' + z')^2/(1-vn - z)] + (vn'' + z'')/(1-vn - z)] +$$

$$\Pi_2[(z'/z)^2 - z'/z] + \Pi_3(\Gamma'_c/\Gamma_c)^2 + \Pi_4(\Gamma'_c/\Gamma_c)^2,$$

where the coefficients of the variables are
\[ \Pi_b = -\rho(1-\alpha)[1-\alpha\theta(1-\delta)] + \alpha\{\delta + \theta(1-\alpha)(1-\delta)\} \]

\[ (\Pi_0 < 0 \text{ if } \rho > \underline{\rho} \text{ as in Proposition 1}) \]

\[ \Pi_1 = -\frac{(1-\theta)(1-\alpha)(1-\delta)}{(1-\alpha)^2[1-\alpha\theta(1-\delta)]} < 0 , \]

\[ \Pi_2 = -\frac{\phi}{1-\alpha} < 0 , \quad \Pi_3 = -\frac{1}{1-\alpha} < 0 , \quad \Pi_4 = -\frac{\alpha\delta(1-\theta)}{(1-\alpha)^2[1-\alpha\theta(1-\delta)]} < 0 . \]

From (33) and (34), the second derivatives of fertility and leisure with respect to \( \Gamma_b \) are:

\[ n'' = -\frac{2\alpha(1-\alpha\theta)[1-\alpha\beta(1-\delta)]}{\nu D^3_n} \{[1-\alpha\beta(1-\delta)][(1-\alpha)(\rho - \alpha - \alpha\phi) + \alpha\phi\delta]} \]

\[ [-1 + \alpha\beta(1-\delta) + \alpha\delta(1-\alpha)(1 + \rho + \phi)] , \]

\[ z'' = -\frac{2\alpha\phi(1-\alpha\theta)[1-\alpha\beta(1-\delta)]}{D^3_n} [1-\alpha\beta(1-\delta) - \alpha\delta] \]

\[ [-1 + \alpha\beta(1-\delta) + \alpha\delta(1-\alpha)(1 + \rho + \phi)]. \]

Here, \( \Omega_i \equiv \rho(1-\alpha)(1-\delta) - \alpha\delta(1-\theta) \) should be positive for a valid solution for fertility with a zero government debt level. When government debt is positive with the externality, this condition on \( \rho \) is not enough for positive fertility. For this reason, we further assume \( \Pi_0 < 0 \), i.e., \( \rho > \underline{\rho} \), where \( \underline{\rho} \equiv \alpha\delta(1-\alpha)^{-1} + \Phi(1-\delta)(1-\alpha) \) as defined in Proposition 1.

Let \( \Omega_2 \equiv [1-\alpha\beta(1-\delta)][(1-\alpha)(\rho - \alpha) - \alpha\phi[1-\alpha\beta(1-\delta) - \delta]] < 0 \) be the condition for \( \frac{dn}{d\Gamma_b} < 0 \). Let \( \Omega_3 \equiv -[1-\alpha\beta(1-\delta) - \alpha\delta(1-\alpha)(1 + \rho + \phi)] \) which is negative under \( \phi < [1-\alpha\beta(1-\delta) - \alpha\delta]/[\alpha\delta(1-\alpha)] \) and \( \rho < \alpha/(1-\alpha) \). In particular, \( \rho < \alpha/(1-\alpha) \) is a sufficient condition for \( \frac{dn}{d\Gamma_b} < 0 \).

With these notations, we use \( B'(\Gamma_b) = 0 \) to rewrite \( B^*(\Gamma_b) \) as

\[ B^*(\Gamma_b) = \frac{G(\Gamma_b)}{(D_n)^2(D^2_b)^2(D^3_b)^2[\Theta + \delta(1-\alpha)\Gamma_b]\alpha(1-\theta)-(1-\alpha)\Gamma_b}]^2} \]
where

\[
G(\Gamma_b) = \Pi_b \alpha^2 (1 - \alpha \theta)^2 [1 - \alpha \beta (1 - \delta)]^2 \Omega_2^2 (D^2)^2 (D')^2 [\Theta + \delta (1 - \alpha) \Gamma_b] \alpha (1 - \theta) -
(1 - \alpha) \Gamma_b^2 + \Pi_b \alpha^2 (1 - \alpha \theta)^2 [1 - \alpha \beta (1 - \delta)]^2 [(1 - \alpha) (\rho + \phi) - \alpha \Gamma_b^2 (D')^2 (D^2)^2
[\Theta + \delta (1 - \alpha) \Gamma_b] \alpha (1 - \theta) - (1 - \alpha) \Gamma_b^2 + \Pi_b \alpha^2 \phi^2 (1 - \alpha \theta)^2 [1 - \alpha \beta (1 - \delta)]^2
[1 - \alpha \beta (1 - \delta) - \alpha \beta] (D')^2 (D^2)^2 [\Theta + \delta (1 - \alpha) \Gamma_b] \alpha (1 - \theta) - (1 - \alpha) \Gamma_b^2 -
\delta^2 (1 - \alpha) (D')^2 (D^2)^2 [\Theta + \delta (1 - \alpha) \Gamma_b] \alpha (1 - \theta) - (1 - \alpha) \Gamma_b^2 + \Pi_b (1 - \alpha)^2 (D')^2
(D')^2 (D^2)^2 [\Theta + \delta (1 - \alpha) \Gamma_b] \alpha (1 - \theta) - (1 - \alpha) \Gamma_b^2 - 2 \Pi_b \Omega_2 (1 - \alpha) (D')^2 (D^2)^2 [\Theta + \delta (1 - \alpha) \Gamma_b] \alpha (1 - \theta) - (1 - \alpha) \Gamma_b^2.
\]

Clearly, all the right-hand terms above are negative under the restrictions \( \Pi_j < 0 \) for all \( i \) and \( \Omega_j < 0 \) for \( j = 2, 3 \). As argued above, \( B'(\Gamma_b) \) and \( B''(\Gamma_b) \) are continuous and well defined for all interior solutions of the variable \((n, z, l, \Gamma_c, \Gamma_e, \Gamma_k)\). Thus, the result \( B''(\Gamma_b^*) < 0 \) at \( \Gamma_b^* \) such that \( B'(\Gamma_b^*) = 0 \) and the fact that \( B'(0) > 0 \) under the stated conditions together imply the existence and uniqueness of the positive, optimal ratio of government debt to output in \( 0 \leq \Gamma_b \leq \Gamma_b^* \). If there were other positive, optimal debt-output ratios such that \( B'(\Gamma_b^*) = 0 \), at least one of them would violate \( B''(\Gamma_b^*) < 0 \) by argument of continuity over \( 0 \leq \Gamma_b \leq \Gamma_b^* \). See Figure 3 for a numerical illustration.

Q.E.D.

Appendix C.

Proof of Proposition 6. In this case, define \( B(\Gamma_b, s) = B(n(\Gamma_b, s), z(\Gamma_b, s), \Gamma_e(\Gamma_b, s), \Gamma_k) \) based on the expressions in (11) and the solutions in (38)-(43). Differentiating (38), (41), (42) and (43) with respect to \( \Gamma_b \) and \( s \) and substitute them into the first order conditions

\[
\frac{\partial B(\Gamma_b, s)}{\partial \Gamma_b} = 0 \text{ and } \frac{\partial B(\Gamma_b, s)}{\partial s} = 0
\]

as follows to maximize the welfare:
Solving (A5) and (A6) simultaneously gives $s^*$ and $\Gamma_b^*$ as in Proposition 6 and it is easy to verify $\Gamma_e\left(\Gamma_b^*, s^*\right) = \alpha\delta(1-\theta)/[1-\alpha(1-\delta)] = \Gamma_e^*$ as in Proposition 1, which indicates the socially optimal output allocation can be achieved. However, this optimal policy cannot achieve the socially optimal levels for fertility, leisure, and labor. \textit{Q.E.D.}
References:


### Table 1. Numerical results (calibrated to Japan’s economy)

Parameters: $\alpha = 0.6, \delta = 0.27, \rho = 1.455, \theta = 0.33, \nu = 0.22, \beta = 0.4, A = D = 3$

<table>
<thead>
<tr>
<th>$\phi = 0.033$</th>
<th>Laissez faire</th>
<th>First best</th>
<th>Second best</th>
<th>Debt&amp; subsidy</th>
<th>$\phi = 0$</th>
<th>Laissez faire</th>
<th>First best</th>
<th>Second best</th>
<th>Debt&amp; subsidy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_b$ (%)</td>
<td>--</td>
<td>6.2</td>
<td>12.3</td>
<td>85.0</td>
<td>--</td>
<td>6.2</td>
<td>59.3</td>
<td>85.0</td>
<td></td>
</tr>
<tr>
<td>$s$ (%)</td>
<td>--</td>
<td>31.9</td>
<td>50.0</td>
<td>50.0</td>
<td>--</td>
<td>31.9</td>
<td>89.7</td>
<td>89.7</td>
<td></td>
</tr>
<tr>
<td>$\pi_l$ (%)</td>
<td>--</td>
<td>--</td>
<td>26.6</td>
<td>87.1</td>
<td>--</td>
<td>--</td>
<td>84.9</td>
<td>94.3</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_e$ (%)</td>
<td>13.2</td>
<td>19.3</td>
<td>19.3</td>
<td>3.4</td>
<td>13.2</td>
<td>19.3</td>
<td>19.3</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_e$ (%)</td>
<td>67.0</td>
<td>60.9</td>
<td>60.9</td>
<td>76.8</td>
<td>67.0</td>
<td>60.9</td>
<td>60.9</td>
<td>72.9</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_h$ (%)</td>
<td>19.8</td>
<td>19.8</td>
<td>19.8</td>
<td>19.8</td>
<td>19.8</td>
<td>19.8</td>
<td>19.8</td>
<td>19.8</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>2.19</td>
<td>1.90</td>
<td>2.17</td>
<td>1.45</td>
<td>2.23</td>
<td>1.93</td>
<td>1.93</td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>0.0165</td>
<td>0.0170</td>
<td>0.0205</td>
<td>0.1543</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>$l$</td>
<td>0.5008</td>
<td>0.5655</td>
<td>0.5015</td>
<td>0.5257</td>
<td>0.5092</td>
<td>0.5752</td>
<td>0.5752</td>
<td>0.8710</td>
<td></td>
</tr>
<tr>
<td>$G^a$ (%)</td>
<td>3.49</td>
<td>3.58</td>
<td>3.78</td>
<td>1.52</td>
<td>3.54</td>
<td>3.63</td>
<td>3.63</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>$\Delta^b$ (%)</td>
<td>--</td>
<td>4.31</td>
<td>2.65</td>
<td>-39.39</td>
<td>--</td>
<td>4.31</td>
<td>4.31</td>
<td>-45.07</td>
<td></td>
</tr>
</tbody>
</table>

a: $G$ denotes discounted annual GDP growth on the balanced growth path.

b: $\Delta$ denotes equivalent variation in consumption (%) such that the laissez-faire welfare level is raised to that in each of the other cases.
Sources: Government debt is based on International Monetary Fund World Historical Public Debt Database (September 2012 version); total fertility rates are from World Bank; most recent fertility rates are based on estimations that differ across sources (e.g. 1.21 from CIA World Factbook).

Sources: OECD Statistics
Figure 3. Welfare with government debt and a labor income tax

Note: The parameterization is $\beta = 0.4, \alpha = 0.6, \delta = 0.27, \rho = 0.93, \theta = 0.33, \phi = 0.33, \nu = 0.1$. 

![Graph showing welfare with government debt and labor income tax.](image-url)
## ECONOMICS DISCUSSION PAPERS

### 2011

<table>
<thead>
<tr>
<th>DP NUMBER</th>
<th>AUTHORS</th>
<th>TITLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.01</td>
<td>Robertson, P.E.</td>
<td>DEEP IMPACT: CHINA AND THE WORLD ECONOMY</td>
</tr>
<tr>
<td>11.02</td>
<td>Kang, C. and Lee, S.H.</td>
<td>BEING KNOWLEDGEABLE OR SOCIABLE? DIFFERENCES IN RELATIVE IMPORTANCE OF COGNITIVE AND NON-COGNITIVE SKILLS</td>
</tr>
<tr>
<td>11.03</td>
<td>Turkington, D.</td>
<td>DIFFERENT CONCEPTS OF MATRIX CALCULUS</td>
</tr>
<tr>
<td>11.04</td>
<td>Golley, J. and Tyers, R.</td>
<td>CONTRASTING GIANTS: DEMOGRAPHIC CHANGE AND ECONOMIC PERFORMANCE IN CHINA AND INDIA</td>
</tr>
<tr>
<td>11.05</td>
<td>Collins, J., Baer, B. and Weber, E.J.</td>
<td>ECONOMIC GROWTH AND EVOLUTION: PARENTAL PREFERENCE FOR QUALITY AND QUANTITY OF OFFSPRING</td>
</tr>
<tr>
<td>11.06</td>
<td>Turkington, D.</td>
<td>ON THE DIFFERENTIATION OF THE LOG LIKELIHOOD FUNCTION USING MATRIX CALCULUS</td>
</tr>
<tr>
<td>11.07</td>
<td>Groenewold, N. and Paterson, J.E.H.</td>
<td>STOCK PRICES AND EXCHANGE RATES IN AUSTRALIA: ARE COMMODITY PRICES THE MISSING LINK?</td>
</tr>
<tr>
<td>11.08</td>
<td>Chen, A. and Groenewold, N.</td>
<td>REDUCING REGIONAL DISPARITIES IN CHINA: IS INVESTMENT ALLOCATION POLICY EFFECTIVE?</td>
</tr>
<tr>
<td>11.09</td>
<td>Williams, A., Birch, E. and Hancock, P.</td>
<td>THE IMPACT OF ON-LINE LECTURE RECORDINGS ON STUDENT PERFORMANCE</td>
</tr>
<tr>
<td>11.10</td>
<td>Pawley, J. and Weber, E.J.</td>
<td>INVESTMENT AND TECHNICAL PROGRESS IN THE G7 COUNTRIES AND AUSTRALIA</td>
</tr>
<tr>
<td>11.11</td>
<td>Tyers, R.</td>
<td>AN ELEMENTAL MACROECONOMIC MODEL FOR APPLIED ANALYSIS AT UNDERGRADUATE LEVEL</td>
</tr>
<tr>
<td>11.12</td>
<td>Clements, K.W. and Gao, G.</td>
<td>QUALITY, QUANTITY, SPENDING AND PRICES</td>
</tr>
<tr>
<td>11.13</td>
<td>Tyers, R. and Zhang, Y.</td>
<td>JAPAN’S ECONOMIC RECOVERY: INSIGHTS FROM MULTI-REGION DYNAMICS</td>
</tr>
<tr>
<td>Page</td>
<td>Authors</td>
<td>Title</td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>11.14</td>
<td>McLure, M.</td>
<td>A. C. PIGOU’S REJECTION OF PARETO’S LAW</td>
</tr>
<tr>
<td>11.15</td>
<td>Kristoffersen, I.</td>
<td>THE SUBJECTIVE WELLBEING SCALE: HOW REASONABLE IS THE CARDINALITY ASSUMPTION?</td>
</tr>
<tr>
<td>11.16</td>
<td>Clements, K.W., Izan, H.Y. and Lan, Y.</td>
<td>VOLATILITY AND STOCK PRICE INDEXES</td>
</tr>
<tr>
<td>11.17</td>
<td>Parkinson, M.</td>
<td>SHANN MEMORIAL LECTURE 2011: SUSTAINABLE WELLBEING – AN ECONOMIC FUTURE FOR AUSTRALIA</td>
</tr>
<tr>
<td>11.18</td>
<td>Chen, A. and Groenewold, N.</td>
<td>THE NATIONAL AND REGIONAL EFFECTS OF FISCAL DECENTRALISATION IN CHINA</td>
</tr>
<tr>
<td>11.20</td>
<td>Wu, Y.</td>
<td>GAS MARKET INTEGRATION: GLOBAL TRENDS AND IMPLICATIONS FOR THE EAS REGION</td>
</tr>
<tr>
<td>11.21</td>
<td>Fu, D., Wu, Y. and Tang, Y.</td>
<td>DOES INNOVATION MATTER FOR CHINESE HIGH-TECH EXPORTS? A FIRM-LEVEL ANALYSIS</td>
</tr>
<tr>
<td>11.22</td>
<td>Fu, D. and Wu, Y.</td>
<td>EXPORT WAGE PREMIUM IN CHINA’S MANUFACTURING SECTOR: A FIRM LEVEL ANALYSIS</td>
</tr>
<tr>
<td>11.23</td>
<td>Li, B. and Zhang, J.</td>
<td>SUBSIDIES IN AN ECONOMY WITH ENDOGENOUS CYCLES OVER NEOCLASSICAL INVESTMENT AND NEO-SCHUMPETERIAN INNOVATION REGIMES</td>
</tr>
<tr>
<td>11.24</td>
<td>Krey, B., Widmer, P.K. and Zweifel, P.</td>
<td>EFFICIENT PROVISION OF ELECTRICITY FOR THE UNITED STATES AND SWITZERLAND</td>
</tr>
<tr>
<td>11.25</td>
<td>Wu, Y.</td>
<td>ENERGY INTENSITY AND ITS DETERMINANTS IN CHINA’S REGIONAL ECONOMIES</td>
</tr>
<tr>
<td>DP NUMBER</td>
<td>AUTHORS</td>
<td>TITLE</td>
</tr>
<tr>
<td>-----------</td>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>12.01</td>
<td>Clements, K.W., Gao, G., and Simpson, T.</td>
<td>DISPARITIES IN INCOMES AND PRICES INTERNATIONALLY</td>
</tr>
<tr>
<td>12.02</td>
<td>Tyers, R.</td>
<td>THE RISE AND ROBUSTNESS OF ECONOMIC FREEDOM IN CHINA</td>
</tr>
<tr>
<td>12.03</td>
<td>Golley, J. and Tyers, R.</td>
<td>DEMOGRAPHIC DIVIDENDS, DEPENDENCIES AND ECONOMIC GROWTH IN CHINA AND INDIA</td>
</tr>
<tr>
<td>12.04</td>
<td>Tyers, R.</td>
<td>LOOKING INWARD FOR GROWTH</td>
</tr>
<tr>
<td>12.05</td>
<td>Knight, K. and McLure, M.</td>
<td>THE ELUSIVE ARTHUR PIGOU</td>
</tr>
<tr>
<td>12.06</td>
<td>McLure, M.</td>
<td>ONE HUNDRED YEARS FROM TODAY: A. C. PIGOU’S WEALTH AND WELFARE</td>
</tr>
<tr>
<td>12.07</td>
<td>Khuu, A. and Weber, E.J.</td>
<td>HOW AUSTRALIAN FARMERS DEAL WITH RISK</td>
</tr>
<tr>
<td>12.08</td>
<td>Chen, M. and Clements, K.W.</td>
<td>PATTERNS IN WORLD METALS PRICES</td>
</tr>
<tr>
<td>12.09</td>
<td>Clements, K.W.</td>
<td>UWA ECONOMICS HONOURS</td>
</tr>
<tr>
<td>12.10</td>
<td>Golley, J. and Tyers, R.</td>
<td>CHINA’S GENDER IMBALANCE AND ITS ECONOMIC PERFORMANCE</td>
</tr>
<tr>
<td>12.11</td>
<td>Weber, E.J.</td>
<td>AUSTRALIAN FISCAL POLICY IN THE AFTERMATH OF THE GLOBAL FINANCIAL CRISIS</td>
</tr>
<tr>
<td>12.12</td>
<td>Hartley, P.R. and Medlock III, K.B.</td>
<td>CHANGES IN THE OPERATIONAL EFFICIENCY OF NATIONAL OIL COMPANIES</td>
</tr>
<tr>
<td>12.13</td>
<td>Li, L.</td>
<td>HOW MUCH ARE RESOURCE PROJECTS WORTH? A CAPITAL MARKET PERSPECTIVE</td>
</tr>
<tr>
<td>12.14</td>
<td>Chen, A. and Groenewold, N.</td>
<td>THE REGIONAL ECONOMIC EFFECTS OF A REDUCTION IN CARBON EMISSIONS AND AN EVALUATION OF OFFSETTING POLICIES IN CHINA</td>
</tr>
<tr>
<td>12.15</td>
<td>Collins, J., Baer, B. and Weber, E.J.</td>
<td>SEXUAL SELECTION, CONSPICUOUS CONSUMPTION AND ECONOMIC GROWTH</td>
</tr>
<tr>
<td>12.16</td>
<td>Wu, Y.</td>
<td>TRENDS AND PROSPECTS IN CHINA’S R&amp;D SECTOR</td>
</tr>
<tr>
<td>12.17</td>
<td>Cheong, T.S. and Wu, Y.</td>
<td>INTRA-PROVINCIAL INEQUALITY IN CHINA: AN ANALYSIS OF COUNTY-LEVEL DATA</td>
</tr>
<tr>
<td>12.18</td>
<td>Cheong, T.S.</td>
<td>THE PATTERNS OF REGIONAL INEQUALITY IN CHINA</td>
</tr>
<tr>
<td>12.19</td>
<td>Wu, Y.</td>
<td>ELECTRICITY MARKET INTEGRATION: GLOBAL TRENDS AND IMPLICATIONS FOR THE EAS REGION</td>
</tr>
<tr>
<td>12.20</td>
<td>Knight, K.</td>
<td>EXEGESIS OF DIGITAL TEXT FROM THE HISTORY OF ECONOMIC THOUGHT: A COMPARATIVE EXPLORATORY TEST</td>
</tr>
<tr>
<td>12.21</td>
<td>Chatterjee, I.</td>
<td>COSTLY REPORTING, EX-POST MONITORING, AND COMMERCIAL PIRACY: A GAME THEORETIC ANALYSIS</td>
</tr>
<tr>
<td>12.22</td>
<td>Pen, S.E.</td>
<td>QUALITY-CONSTANT ILLICIT DRUG PRICES</td>
</tr>
<tr>
<td>12.23</td>
<td>Cheong, T.S. and Wu, Y.</td>
<td>REGIONAL DISPARITY, TRANSITIONAL DYNAMICS AND CONVERGENCE IN CHINA</td>
</tr>
<tr>
<td>DP NUMBER</td>
<td>AUTHORS</td>
<td>TITLE</td>
</tr>
<tr>
<td>-----------</td>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>13.01</td>
<td>Chen, M., Clements, K.W. and Gao, G.</td>
<td>THREE FACTS ABOUT WORLD METAL PRICES</td>
</tr>
<tr>
<td>13.02</td>
<td>Collins, J. and Richards, O.</td>
<td>EVOLUTION, FERTILITY AND THE AGEING POPULATION</td>
</tr>
<tr>
<td>13.04</td>
<td>Robitaille, M.C. and Chatterjee, I.</td>
<td>MOTHERS-IN-LAW AND SON PREFERENCE IN INDIA</td>
</tr>
<tr>
<td>13.05</td>
<td>Clements, K.W. and Izan, I.H.Y.</td>
<td>REPORT ON THE 25TH PHD CONFERENCE IN ECONOMICS AND BUSINESS</td>
</tr>
<tr>
<td>13.06</td>
<td>Walker, A. and Tyers, R.</td>
<td>QUANTIFYING AUSTRALIA’S “THREE SPEED” BOOM</td>
</tr>
<tr>
<td>13.07</td>
<td>Yu, F. and Wu, Y.</td>
<td>PATENT EXAMINATION AND DISGUISED PROTECTION</td>
</tr>
<tr>
<td>13.08</td>
<td>Yu, F. and Wu, Y.</td>
<td>PATENT CITATIONS AND KNOWLEDGE SPILLOVERS: AN ANALYSIS OF CHINESE PATENTS REGISTER IN THE US</td>
</tr>
<tr>
<td>13.09</td>
<td>Chatterjee, I. and Saha, B.</td>
<td>BARGAINING DELEGATION IN MONOPOLY</td>
</tr>
<tr>
<td>13.10</td>
<td>Cheong, T.S. and Wu, Y.</td>
<td>GLOBALIZATION AND REGIONAL INEQUALITY IN CHINA</td>
</tr>
<tr>
<td>13.11</td>
<td>Cheong, T.S. and Wu, Y.</td>
<td>INEQUALITY AND CRIME RATES IN CHINA</td>
</tr>
<tr>
<td>13.12</td>
<td>Robertson, P.E. and Ye, L.</td>
<td>ON THE EXISTENCE OF A MIDDLE INCOME TRAP</td>
</tr>
<tr>
<td>13.13</td>
<td>Robertson, P.E.</td>
<td>THE GLOBAL IMPACT OF CHINA’S GROWTH</td>
</tr>
<tr>
<td>13.14</td>
<td>Hanaki, N., Jacquemet, N., Luchini, S., and Zylbersztejn, A.</td>
<td>BOUNDED RATIONALITY AND STRATEGIC UNCERTAINTY IN A SIMPLE DOMINANCE SOLVABLE GAME</td>
</tr>
<tr>
<td>13.15</td>
<td>Okatch, Z., Siddique, A. and Rammohan, A.</td>
<td>DETERMINANTS OF INCOME INEQUALITY IN BOTSWANA</td>
</tr>
<tr>
<td>13.16</td>
<td>Clements, K.W. and Gao, G.</td>
<td>A MULTI-MARKET APPROACH TO MEASURING THE CYCLE</td>
</tr>
</tbody>
</table>

ECONOMICS DISCUSSION PAPERS
2013
| 13.17 | Chatterjee, I. and Ray, R. | THE ROLE OF INSTITUTIONS IN THE INCIDENCE OF CRIME AND CORRUPTION |
| 13.18 | Fu, D. and Wu, Y. | EXPORT SURVIVAL PATTERN AND DETERMINANTS OF CHINESE MANUFACTURING FIRMS |
| 13.19 | Shi, X., Wu, Y. and Zhao, D. | KNOWLEDGE INTENSIVE BUSINESS SERVICES AND THEIR IMPACT ON INNOVATION IN CHINA |
| 13.20 | Tyers, R., Zhang, Y. and Cheong, T.S. | CHINA’S SAVING AND GLOBAL ECONOMIC PERFORMANCE |
| 13.22 | Hartley, P.R. | THE FUTURE OF LONG-TERM LNG CONTRACTS |
| 13.23 | Tyers, R. | A SIMPLE MODEL TO STUDY GLOBAL MACROECONOMIC INTERDEPENDENCE |
| 13.24 | McLure, M. | REFLECTIONS ON THE QUANTITY THEORY: PIGOU IN 1917 AND PARETO IN 1920-21 |
| 13.27 | Li, B. and Zhang, J. | GOVERNMENT DEBT IN AN INTERGENERATIONAL MODEL OF ECONOMIC GROWTH, ENDOGENOUS FERTILITY, AND ELASTIC LABOR WITH AN APPLICATION TO JAPAN |
| 13.28 | Robitaille, M. and Chatterjee, I. | SEX-SELECTIVE ABORTIONS AND INFANT MORTALITY IN INDIA: THE ROLE OF PARENTS’ STATED SON PREFERENCE |
| 13.29 | Ezzati, P. | ANALYSIS OF VOLATILITY SPILLOVER EFFECTS: TWO-STAGE PROCEDURE BASED ON A MODIFIED GARCH-M |
| 13.30 | Robertson, P. E. | DOES A FREE MARKET ECONOMY MAKE AUSTRALIA MORE OR LESS SECURE IN A GLOBALISED WORLD? |
| 13.31 | Das, S., Ghate, C. and Robertson, P. E. | REMOTENESS AND UNBALANCED GROWTH: UNDERSTANDING DIVERGENCE ACROSS INDIAN DISTRICTS |
| 13.32 | Robertson, P.E. and Sin, A. | MEASURING HARD POWER: CHINA’S ECONOMIC GROWTH AND MILITARY CAPACITY |