Competitive agency with moral hazard.∗

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Abstract

Principals seek to enter a productive relationship with agents by posting general incentive contracts. A contract is exclusive and must solve both the ex post moral hazard and the ex ante competitive search problem (participation). Menus of contracts do not help hence (single) contract posting is optimal. Principal competition restores some bargaining power with the agents, to whom principals must offer a rent to attract them. Transferring utility to the agents is best achieved by improving the insurance properties of the incentive contract. This implies lower effort in equilibrium and therefore lower welfare. A planner can completely internalize the negative externality generated by principal competition and restore the first best. We also compare rationing rules; uniform public randomization is optimal.

Keywords: moral hazard, asymmetric information, contracts, directed search, search frictions. JEL Classification: D82.

1 Introduction

When looking for a new job a worker may search over a set of opportunities and decide which prospective employer(s) to contact on the basis of the advertised employment conditions. In turn, employers competing over workers set employment conditions so as to attract prospective candidates. That is, the search process influences contract design, and contract design influences search. This heuristic description is well understood by search theorists when information is symmetric,

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and to a lesser extent, under adverse selection. In this paper we consider search and moral hazard together.

To do so we cast the principal-agent relationship in a market environment with matching frictions. We use the directed search framework, in which a large number of principals seek to match with one of many agents (see for example Peters, 1984, 1991, 2000; Shimer, 1996; Moen, 1997; Julien, King and Kennes, 2000 and Burdett, Shi and Wright, 2001). Principals simultaneously post, and commit to, contractual terms. Agents observe all contracts, which thus direct their search activity. “Searching” means finding the best probability distribution over the set of contract offers. Upon matching a principal and a single agent enter into a productive relationship under moral hazard.

This framework is useful on several accounts. First it allows us to study principal competition in an agency model in a tractable manner. Because search is directed by the principals’ offers, any one contract affects the others, however only through the search process. Here competition directly affects the participation problem only, not the incentive problem (it does indirectly only). Thus we can circumvent the difficulties of common agency while still meaningfully exploring the interaction of competition and moral hazard. This stands in contrast to the works of Attar et al (2006, 2007a, 2007b), Martimort (2004), Aubert (2005) or Célier (2012), for example. In each of these, an agent may be party to more than one contract at once; this decision interferes with both the participation and the moral hazard constraint.¹

Second, this work endogenizes the relevant outside option: here “participation” really means participating in the search process (i.e. assigning some positive probability to a given contract offer). That decision is made on the basis of the array of contract offers and the market conditions, not any exogenously given outside option.

Last, search followed by one-to-one (i.e. exclusive) contracting depicts many economic applications such as employment, procurement or financial contracting such a banking relationships and the dealing of over-the-counter (OTC) instruments. In all these cases there is no centralized, organized market, so search frictions matter. In contrast, for example, Tirole (2006), Holmström

¹This is slightly beyond the scope of this discussion but nonetheless interesting to note that common agency can lead to the first-best solution under moral hazard (Attar et al 2007, Martimort, 2004), or to the collapse of incentives (Aubert, 2005). In this paper moral hazard is preserved thanks to the assumption of one-to-one contracting.
and Tirole (2011) and Rochet (2008) study situations where lenders are competitive and a single borrower is subject to moral hazard. Because participation is always secured the only distortions stem from the standard moral hazard problem. Here search frictions allow for market power on the principals’ side. This is important to identify the distortions that (imperfect) competition for agents introduces in the contract.

Our game allows for a very large strategy space: in general contracts may depend on the number of agents present at the time of contracting. Yet we present a remarkably simple characterization. We also show that search frictions have important welfare consequences. To attract agents, principals must make sufficiently generous contract offers. Exclusive contracting generates congestion, like a capacity constraint; in response agents assign a probability distribution over the array of offers. This mixed strategy equilibrium (in search) necessarily implies some coordination failure, so that a principal contracts with an agent with a probability always less than one. This introduces a trade-off between improving offers to attract agents and the principal’s payoff conditional on a match. The first-order conditions we derive exactly characterize this trade-off.

This new trade-off induces welfare losses. Facing risk-averse agents, principals most efficiently transfer utility by improving the insurance properties of the contract they offer. While this is welfare enhancing for a fixed level of effort, it also weakens the incentives for said effort. On balance effort decreases sufficiently to offset the insurance-based welfare gain. Thus principal competition is welfare decreasing. The reason is that transfers do more than allocating rents; in a moral hazard problem they are essential to spur surplus-enhancing effort. All these results are robust to a change in the agent rationing rule.\footnote{This rule determines what to do if more than one agent matches with a principal, and therefore influence the agents’ search decision.}

This paper also contributes to search theory. We affirm the result of Selcuk (2012) whereby contract posting (the equivalent of fixed-price posting) is optimal. That is, menus of contracts contingent on the number of agents showing up are suboptimal. The reason however differs from Selcuk’s, who shows that risk-averse buyers dislike a lottery over a menu of transfers. Instead we establish that principals do not like a lottery over a menu of actions because the total cost of an action is convex. Hence a single contract always dominates; it is this simplification that affords us a simple characterization. Furthermore, this result extends to the choice of rationing rule. That is,
using a uniform public randomization device is the unique optimal rationing rule.

Few papers combine moral hazard with a notion of market. Zaharieva (2010) studies a dynamic model of a labor market with search and moral hazard where output is restricted to be binary, contracts are linear and agents are protected by limited liability. In contrast we stay with a static model but allow for general contracts that may even depend on the match profile. We also steer away from limited liability to allow for other applications than labor markets (see also Jewitt, Kadan and Swinkels 2008 paper for a general analysis with bounded payments). Tsuyuhara (2001) studies a labor market problem with on the job search. Effort translates into a probability of success, which is a binary random variable. Emphasis is laid on the macroeconomic implications of unobserved behavior, such as labor mobility, wage dispersion and business cycle behavior. In contrast our paper contributes to understanding the impact of market competition on the optimal contract. Moen and Rosen’s work (2011) investigate incentives in a competitive search model. In addition to unobservable effort, they assume a match-specific stochastic component to output; that is, a type. The relationship between type and effort follows Laffont and Tirole (1986), and so turns the problem into one of adverse selection. They show that informational rents are useful in this frictional market: they help attracting agents. Roger (2014) studies the problem of two principals competing to attract a single agent to contract under moral hazard in a Hotelling-like model, where in addition that agent’s location is private information. Depending on the transportation cost the principals are either local monopolists or compete for the agent. Effort is non-monotonic in that transportation cost: fierce competition (high substitutability) leads principals to aggressively bid utility to attract the agent. Here too the need to attract an agent weakens the incentives. However in the present model participation becomes probabilistic because of the coordination failure induced by search, and the participation probability depends on market tightness.

In the next section we specify the model and re-state the benchmark model. Section 3 is the heart of the analysis; it starts with the search problem and characterizes the symmetric equilibrium. We derive welfare implications in Section 4 and then present concluding remarks. All proofs and some supplementary material can be found in the Appendix.
2 Model

2.1 Description

There is a large set $N$ of $N$ homogenous agents and a large set $M$ of $M$ homogenous principals seeking to form bilateral relationships subject to moral hazard. Let $\Theta = \frac{N}{M} < \infty$, but $N, M \to \infty$; this ratio denotes market tightness. An agent’s utility is $u(t) - c(a)$, where $u(\cdot)$ is increasing and concave, $t$ is the transfer received and $a \in A \subset \mathbb{R}$ the action chosen at cost $c(a)$ increasing and convex. An action $a$ governs the distribution $F(x|a)$ of outcomes $x \in \mathcal{X} \equiv [x, \bar{x}] \subset \mathbb{R}$, with everywhere positive density $f(x|a)$. The likelihood ratio $f_a/f$ is increasing concave in $x$; it follows that $F(x|a) < F(x|a')$ for $a' > a$ (first-order stochastic dominance). Whenever $t$ is a function $t(x)$, that function is taken to be equicontinuous as in Holmström (1977, 1979).

The game unfolds as follows. The principals seek to employ one agent (only) by simultaneously posting anonymous contracts $C_j = (t_j(x), a_j)$, $j \in M$ with $t_j(x) = (t^1_j(x), \ldots, t^N_j(x))$, $a_j = (a^1_j(x), \ldots, a^N_j(x))$: each offers a transfer and prescribes an action that may depend on the number $n$ of agents showing up, but not on who those agents are. Let $C$ denote the vector of contracts with element $C_j$. The restriction to anonymous contract is without loss because agents are homogenous. An agent can work for at most one principal, and a principal only needs one agent. Observing contract offers, agents decide which principal(s) to visit. Denote by $\sigma_i$ the probability distribution that agent $i$ assigns over the vector $C$ of $M$ contract offers, i.e. $\sigma_i = (\sigma^1_i, \sigma^2_i, \ldots, \sigma^M_i)$. Each $\sigma^j_i$ is the probability that $i$ visits $j$ given $C$. Upon visiting a principal, agents may accept or reject the contract offered. If rejecting, they receive $u_0$. If no agent goes to a principal, that principal simply receives 0.

We consider two posting mechanisms. The first one induces uniform rationing: if more than one agent visit the same principal, the latter selects one only using a public randomization device (such as a coin flip). The second mechanism, which we study as an extension, allows the principal to elicit bids ex post in order to select the sole agent. Because agents cannot commit to an action (it is not observable), bids are transfer offers the agents are willing to receive. For completeness, the timing is:

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These are, essentially, smooth functions. Examples include Lipschitz continuous functions and $C^1$ functions. There is no loss in limiting attention to continuous function, and only at most an arbitrary small loss in relying on equicontinuity; more can be availed from the authors upon request.
1. principals post contracts;

2. agents decide which principal(s) to visit; rationing occurs;

3. if accepting a contract, an agent selects an action;

4. payoffs are realized.

A strategy for a principal is to post a menu of contracts \( C^j \) that may depend on \( n \). For an agent \( i \) it is to decide on the (vector of) probability of a visit \( \sigma_i = (\sigma^1_i, \sigma^2_i, ..., \sigma^N_i) \), and on which action \( a_n, n \in \mathcal{N} \) to take upon accepting a contract.

Without loss we look for symmetric, subgame-perfect equilibria of this game, which amounts imposing that agents use mixed strategies off the equilibrium path in the continuation game. This is not restrictive: Bland and Loertscher (2012) provide a refinement that selects mixed strategies in the buyers’ continuation game. They show that all equilibria other than directed search equilibria violate a monotonicity property of buyers’ strategies because they require at least one buyer to visit a seller with higher probability after this seller increases his price. Their result provides a rationale for focusing on directed search equilibrium with buyers mixing off the equilibrium path. Furthermore, Galenianos and Kircher (2012) show that in canonical directed search models, the symmetric equilibrium is also unique. In other words there are no asymmetric equilibria if one focuses on agents playing mixed strategies off the equilibrium path. Hence our results are quite robust. An equilibrium is:-

- a vector of transfers and actions \((t, a)\) (with elements \(t_n(x), a_n, n \in \mathcal{N}\)) offered by principals as best response to each other and the agents’ selection strategies \( \sigma \) and

- a vector of selection strategies \( \sigma \) as best response to observed contracts \((t, a)\).

Throughout we suppose that the first-order approach is valid, see Jewitt (1988) for details.

### 2.2 Preliminaries: the standard problem

The canonical model features a single agent and a single principal. Let the linear functionals

\[
\pi(t, a) \equiv \int_X [z - t(z)] dF(z|a)
\]

\[
U(t, a) \equiv \int_X u(t(z))dF(z|a) - c(a)
\]
represent each party’s payoffs. Because \( t(\cdot) \) is equicontinuous, \( u(\cdot) \) is continuous and \( X \) is bounded, these are well defined and bounded. The principal’s problem is to maximize \( \pi(t, a) \) by choice of the contract variables \( t(x), a, \) subject to \( U(t, a) \geq u_0 \) and \( U_a = 0 \) and with \( u_0 \) known. The solution is characterized by the conditions (see Holmström, 1979; Jewitt, 1988)

\[
\frac{1}{u'} = \lambda + \mu \frac{f_a}{f} \tag{2.1}
\]

and

\[
\pi_a + \mu U_{aa} = 0 \tag{2.2}
\]

where \( \lambda, \mu > 0 \) are Lagrange multipliers, together with the two aforementioned constraints. The terms \(-f(x|a) \equiv \delta \pi(t, a)\) and \( u'(t(x))f(x|a) \equiv \delta U(t, a)\) are standard Fréchet derivatives with respect to the function \( t \). We note that \( \lambda \equiv \lambda(u_0) > 0 \) means that the participation cost is determined in terms of the agent’s outside option only.

3 Search and moral hazard

For any contract (that is, any pairing of principal and agent), let \( a^*_n \) denote the agent’s optimal action given some transfer \( t^n_j \). Conditional on contracting with principal \( j \) an agent receives the post-match expected utility (sometimes also called rent), \( U_j(a^*_n, t^n) \). Let \( U \) be the vector with element \( U_j, j = 1, 2, ..., M \). What exactly \( U \) amounts to depends on the nature of the information, the rationing rule and how many other agents visit the same principal; let that number be \( n \leq N \). So each \( U_j \) is itself a vector with element \( U_j(a^*_n, t^n), n = 1, 2, ..., N \). To be clear, in what follows we focus on uniform rationing.\(^4\)

3.1 Search

At the search stage, each agent selects a distribution \( \sigma_i \) over the set \( U \) to maximize her expected utility (given contract offers \( C \)). That expected utility depends both on her choice of \( \sigma_i \equiv \sigma_i(U^1, U^2, ..., U^M) \) as a function of the vectors of rents \( U^j \) (with each element a function of \( n \)) and on how many other agents (\( n \)) may be present at the same principal. This vector \( U \) is a large

\(^4\)Uniform rationing is not a restriction in the class of price posting mechanisms.
object; our first order of business is to simplify matters. Given the binomial distribution

\[ B_n(N, \sigma^j) = \binom{N}{n} \left( \prod_{i} \sigma^j_i \right) \left( \prod_{i} (1 - \sigma^j_i) \right) \]

an agent’s expected utility from conducting some search and matching with principal \( j \) reads:

\[ W^j_P(\sigma(\mathbf{U})) \equiv \sum_{n=0}^{N-1} B_n \left( N - 1, \sigma^j \right) \left[ \frac{1}{n+1} U^j(t_n, a_n) + \left( 1 - \frac{n}{n+1} \right) u_0 \right] \quad (3.1) \]

The first term in the square brackets is the expected payoff to agent \( i \) when contracting with principal \( j \). Contracting may occur if \( i \) is alone (i.e. \( n = 0 \)), or for any other realization of \( n \leq N \). When the agent either fails to match or, if matching, fails to contract with the principal, she receives her exogenous outside option \( u_0 \); this is the second term.

Expression (3.1) is cumbersome to work with. Fortunately, because agents are ex ante homogeneous, we may restrict attention to symmetric search strategies for the agents.

**Assumption 1** Homogeneity property: Given \( \mathbf{U} \), \( \sigma^j_i(\mathbf{U}) = \sigma^j(\mathbf{U}) \forall i \in \mathcal{N} \).

This assumption is standard in directed search models. It allows us to simplify the binomial distribution to

\[ B_n(N, \sigma^j) = \binom{N}{n} \sigma^j \left( 1 - \sigma^j \right)^{N-n} \]

Then

\[ W^j_P(\sigma(\mathbf{U})) \equiv \sum_{n=0}^{N-1} B_n \left( N - 1, \sigma^j \right) \left[ \frac{1}{n+1} U^j(t_n, a_n) + \left( 1 - \frac{n}{n+1} \right) u_0 \right] \]

Further simplification of \( W^j_P(\sigma(\mathbf{U})) \) is possible by extending the result of Selcuk (2012) to this setting. The optimal transfer function \( t^*_n(x) \) is in fact invariant in \( n \). This extension is not immediate because the social surplus, \( \mathbb{E}[x|a] - c(a) \) here, is endogenous to \( t \) (via \( a \)), whereas it is fixed in Selcuk’s model. Our proof relies on a different approach altogether, which suggests a different motivation.

**Proposition 1** There is a unique symmetric equilibrium, in which all principals use a unique tariff; that is, \( \forall j \in \mathcal{M}, t^*_n(x) = t(x) \) and \( a^*_n = a^* \).
Proposition 1 is substantive and simplifying. It is substantive in that it claims both the optimal action and the optimal transfer function are independent of the actual number \( n \) of agents visiting anyone principal. Selcuk (2012) argues that using \( n \)-contingent transfers amounts to exposing agents to a lottery over payoffs, which is costly with risk-averse agents. To establish the result we show that the \emph{principals} prefer a single contract because it minimizes the cost of implementing their preferred action. That cost is convex and increasing in the action (equivalently, the principals’ payoffs are concave decreasing). Principals are better off avoiding a lottery over actions, again because of curvature. This result does not invalidate Selcuk’s, which may well apply here (but is more difficult to establish). Rather it complements it and is sufficient to limit attention to \( n \)-invariant transfers.

\textbf{Remark 1} Coles and Eeckhout (2003) show that meeting-contingent prices induce indeterminacy of equilibrium. As in Selcuk (2012) this indeterminacy disappears here. The reason, as in Selcuk (2012), is that the distribution of rents matters because of risk aversion.

Proposition 1 is also simplifying. With a unique transfer function and uniform rationing, the only relevant events are whether the agent matches with any principal. So we can make use of the fact that the limit of the binomial distribution converges to a Poisson distribution parametrized by market tightness as \( N, M \to \infty \). Let \( \theta(U) \) denote the expected queue length at principal \( j \) given the (now simplified) vector of offers \( C \). Because \( \theta = \mathbb{E}[n]/M \), in general \( \theta(U) \neq \Theta \) – except at equilibrium. An agent’s expected utility now reads

\[
W^j(U) = 1 - e^{-\frac{\theta(U)}{\theta(U)}} U^j + \left( 1 - \frac{1 - e^{-\theta(U)}}{\theta(U)} \right) u_0.
\]

(3.2)

In Section B.1 of the Appendix we show that the limit of \( W^j \) defined by (3.1) is indeed (3.2) (under Assumption 1). So indeed there is no loss in analyzing a large market.

\subsection{3.2 Characterization under moral hazard}

Assumption 1 allows us to treat the problem as symmetric and thus to drop the superscript \( j \). The principal maximizes

\textbf{Problem 1}

\[
\max_{t, a, \theta} \left[ 1 - e^{-\theta} \right] \int_X [z - t(z)] dF(z|a)
\]
subject to

\[ W(U) \geq \tilde{W} \] (3.3)

\[ \int_{\mathcal{X}} u(t(z)) dF(z|a) - c(a) \geq u_0 \] (3.4)

\[ \int_{\mathcal{X}} u(t(z)) dF_a(z|a) - c'(a) = 0 \] (3.5)

where (3.5) is the first-order condition of an agent’s problem and (3.3) is the Market Utility Property (MUP). Any competing principal must offer at least what the market offers (McAfee, 1993; Peters, 2000). The MUP replaces

\[ \sigma \in \arg \max \langle \sigma W^T \rangle, \]

(where the superscript \( T \) means transpose) and so embodies the requirement that for a profile of contracts inducing the payoff profile \( U \), the profile of distributions \( \sigma \) be a Nash equilibrium of the search subgame. This allows us to treat \( W^{-j} \) as exogenous in the principal’s program. The original game can now be handled as an optimization problem.

**Proposition 2** The solution to Problem 1 is characterized by the necessary and sufficient first-order conditions

\[ \int_{\mathcal{X}} [z - t(z)]dF_a(z|a) + \mu \left[ \int_{\mathcal{X}} [z - t(z)]dF_{aa}(z|a) - c''(a) \right] = 0 \] (3.6)

and

\[ \frac{1}{u'} = \lambda + \mu \frac{f_a}{f} + \frac{\theta e^{-\theta}}{1 - e^{-\theta} - \theta e^{-\theta}} \frac{\pi}{U - u_0} \] (3.7)

with \( U > u_0 \), \( \lambda = 0 \) and \( \mu > 0 \) and \( \theta(U) = \Theta \).

Expression (3.6) is standard in a moral hazard problem. It results from subgame perfection: for any transfer \( t(x) \) a principal offers, the agent will choose the action that is optimal for her – after contracting and knowing the transfer function \( t(x) \). So search does not (directly) enter this equation.

The first remark about Condition (3.7) is that it defines a fixed-point problem in the space of transfer functions. This problem is simpler than it first appears as one notices that \( \pi \) and \( U \) are not just linear functionals, but expected values. That is, for a given function \( t \), \( \pi \in \mathbb{R} \) and \( U \in \mathbb{R} \); it then follows that \( \theta(U) \in \mathbb{R} \). Thus (3.7) rewrites

\[ \frac{1}{u'} = \alpha + \mu \frac{f_a}{f}, \quad \alpha \in \mathbb{R}, \]
which mimics (2.1). So the two first-order conditions (3.6), (3.7) and the constraints (3.5) and (3.3) completely identify the solution (for details see Roger, 2014). Here the market-given participation constraint (3.3) replaces the standard participation constraint (3.4).

Second, Condition (3.7) shows that the slope of the transfer is related to the likelihood ratio $f_a/f$, as we know from Holmström (1977, 1979). However the additional term includes $(\theta e^{-\theta})/(1 - e^{-\theta} - \theta e^{-\theta})$, which defines precisely the probability of moving from one agent $(\theta e^{-\theta})$ to at least two agents $(1 - e^{-\theta} - \theta e^{-\theta})$ being present to contract with. This is the hazard rate of the distribution of the expected number of agents visiting a principal (recall $E[n|\sigma] = \theta$), given the equilibrium strategy profile in the search subgame.

The participation constraint (3.4) is not active in Problem 1 because the principal has to contend with a different problem than in the standard model. Here, attracting at least one agent (away from other principals) occurs with probability $1 - e^{-\theta}$ only. Hence principals face a trade-off between incentives and participation probability. As a result, the multiplier $\lambda \equiv \lambda(u_0)$ is naught here, and instead the market-given constraint (3.3) binds.

### 4 Welfare considerations

Proposition 2 shows that principal competition affords the agent some form of effective bargaining power. This has welfare consequences in this model because the agents’ action is productive: it matters for the total surplus to be shared. In this section we establish two results showing that adding search frictions to moral hazard worsens the outcome.

Define equilibrium social welfare as

$$W(a) \equiv \max \{M, N\} \cdot \left(1 - e^{-\theta}\right) \left[\pi(t, a) + U(t, a)\right] + \left[M \cdot e^{-\theta} + (N - M)I_{N>M}\right] u_0,$$

where $I_{N>M} = 1$ and zero if $N \leq M$. There are at most $M$ or $N$ productive matches, whichever is greater. If $N > M$ some agents fail to match; they receive their outside option $u_0$. First,

**Lemma 1** Social welfare increases with the agents’ actions.

Because a higher action shifts the distribution $F(x|a)$ of the output in a first order sense, it is obvious that the social surplus of a dyad is increasing in the agent’s action – all things otherwise equal. It is also true in equilibrium: although a higher action is more expensive, it remains preferred by the principal. Therefore
Proposition 3 Under search with friction, the optimal action $a^U$ solving (3.6) is lower than the standard optimal action $a^S$ solving (2.2).

The intuition for this result is remarkably simple. Competing principals must transfer utility to agents to attract them. Facing risk-averse agents this is most efficiently achieved by reducing the variability of the transfer: offering better insurance. As a result agents face weaker incentives to exert effort.

Second, the planner would choose a different allocation, even if constrained by moral hazard, for a given number of principals and agents. More precisely, if the planner could design the optimal contract, he would simply select the second-best contract characterized by (2.1) and (2.2).

Proposition 4 Fix $M$ and $N$. The decentralized matching game under moral hazard does not implement the utilitarian planner’s. The decentralized matching solution induces less effort than the planner’s solution.

This result echoes Proposition 3. The utilitarian planner seeks to maximize the surplus to be shared, which amounts to implementing the second-best action and leaves no rent to the agents (this is the logic of Lemma 1). Here the planner is not constrained by the MUP (3.3), because he does not care which principal fails to not contract, and so needs not attract agents by giving away rents. Put another way, the probability that any one principal contracts with any one agent is always given by $\theta(U) = \Theta$ in equilibrium, regardless of the rent level $U$. So the principal selects $U$ so as to maximize welfare: $U = u_0$. That is, the negative externality on principals that stems from their competing for the agents disappears. In fact it disappears completely in that the planner’s preferred solution is the second-best solution.

Remark 2 Of course re-introducing the MUP (3.3) yields the decentralized allocation of Proposition 2. The reason is that principals (in the decentralized problem) and planner (in the centralized problem) set the terms of trade, and maximizing welfare is equivalent to maximizing the principals surplus, regardless of the search frictions. The point of Proposition 4 is that decentralized trading under frictions induces a profile of equilibrium actions $a^*$ that generates a lower welfare. Note however that allocation is Pareto optimal.

\footnotetext{5}{It is easy to see that varying welfare weights attached to the principals and agents may change this proposition.}
5 Extension: alternative rationing mechanism

We consider an alternative rationing rule whereby the principal is able to solicit bids from agents if more than one is present at the contracting stage. This mirrors entering an auction with a posted reserve price; bidding is active only if there are more than one agent. This is a form of renegotiation, however it can happen only with the probability that more than one agent be present. We call this *ex post bidding*.

The ensuing analysis complements Proposition 1 and is a worthwhile exercise on two more accounts. First it is often used in multistage procurement problems (e.g. bidders qualification followed by actual bidding). Second, the result of Proposition 1 does not immediately extend to this form of renegotiation: altering the rationing rule also alters the value of the MUP $\hat{W}$.

An agent’s expected utility includes a payoff if contracting when she is alone, a payoff if matching but not being alone (bidding) and the outside option if not matching at all. Here too we are able to simplify things.

**Lemma 2** Let $a^P$ denote the principal’s preferred action given $u_0$. In the *ex post* auction with principal $j$, agents bid transfers $t_i^n$ such that $U_j^j(t_n,a_n^*) = u_0 \forall a_n^*, n > 1$ and the winning bid is such that $\forall n > 1$, $a_n^* = a^P$.

Agents are symmetric and engage in Bertrand competition if more than one agent is present; they can only receive their outside option $u_0$. Under moral hazard, for any transfer $t$, there is a unique optimal action for the agent. So any credible offer is one that exactly mimics the standard second-best contract – with a unique transfer and a unique action. By Lemma 2, there can be only a single contract on offer in the case only one agent shows up and a single contract (the second-best contract) for any other number $n > 1$ of agents. Then the expected utility writes:

$$W_j^j(\sigma_i(U)) = \prod_{i=1}^{N-1} (1 - \sigma_i^j) U_j^j(t_1, a_1^*)$$

$$+ \frac{1 - \prod_{i=1}^{N} (1 - \sigma_i^j)}{\phi_j^j(N, U)} \prod_{i=1}^{N-1} (1 - \sigma_i^j) u_0 + \left(1 - \frac{1 - \prod_{i=1}^{N} (1 - \sigma_i^j)}{\phi_j^j(N, U)}\right) u_0$$

$$= \prod_{i=1}^{N-1} (1 - \sigma_i^j) U_j^j(a^*) + \left[1 - \prod_{i=1}^{N-1} (1 - \sigma_i^j) U_j^j(a^*)\right] u_0$$

The first term captures the probability that agent $i$ is alone at principal $j$. The second one reflects that at least another agent is present and $i$ contracts with the principal but is pushed to her outside
option by Lemma 2. The last term is the probability of not being alone and losing the ex post bidding game. Applying Assumption 1 and taking the limit again as $N, M \to \infty$ yields

$$W_A = e^{-\theta}U_A + \left(1 - e^{-\theta}\right)u_0$$

(5.2)

All things otherwise equal agents face a worse lottery (between rent and outside option) under this rationing rule that under uniform rationing. Problem 1 thus becomes

**Problem 2**

$$\max_{t_A, a_A, \theta} \theta e^{-\theta} \int_X [z - t_A(z)]dF(z|a) + \left[1 - \theta e^{-\theta} - e^{-\theta}\right] \int_X [z - t_{SB}(z;n)]dF(z|a_{SB})$$

s.t. (3.4), (3.5) and (5.2).

This program already accounts for the fact that the second-best solution is known, and given by the standard optimality conditions (2.1) and (2.2), when the agents are made to bid. Let $t_{SB}, a_{SB}$ solve the standard problem of Section 2.2, and $U_A, \pi_A, \pi_{SB}$ be defined as in that Section. We present the next Lemma for completeness of exposition.

**Lemma 3** Under ex post bidding the optimal transfer $t_A$ and optimal action $a_A$ are characterized by the necessary and sufficient first-order conditions

$$\int_X [z - t(z)]dF_a(z|a) + \mu \left[\int_X [z - t(z)]dF_{aa}(z|a) - c''(a)\right] = 0$$

(5.3)

and

$$\frac{1}{u'} = \lambda + \frac{f_a}{U_A - u_0} + \frac{\pi_{SB}}{U_A - u_0} + \frac{1 - \theta}{\theta} \frac{\pi_A}{U_A - u_0}$$

(5.4)

with $\lambda = 0, \mu > 0$ and $U_A > u_0$. Therefore, under ex post bidding, equilibrium outcomes are characterized by

- $\theta(U) = \Theta$;
- Conditions (5.3) and (5.4) if $n = 1$; and
- Conditions (2.1) and (2.2) if $n > 1$. 

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A rent is only paid when the agent is alone visiting the principal; otherwise the standard second-best solution prevails – in line with Lemma 2.

Note that $\theta$ needs not be less than 1; it is a queue length. The term $(1 - \theta)/\theta$ represents the change in the probability that the agent be alone, conditional on being alone. Thus when the queue is long $\theta > 1$, the last term is a “discount”. The term $\pi_{SB}/(U_A - u_0)$ represents a premium to compensate the fact that the principal solicits bids from the agents if more than one is present. Because $u'_0 > 0$ and $f_a/f = 0$ for some $x \in X$, $\pi_{SB} > \pi_A(1 - \theta)/\theta$ necessarily when $\theta > 1$.

We see that the rent $U$ depends on the rationing rule. So selecting the trading mechanism is an important aspect of the principals’ problem. Ex post bidding is a form of renegotiation between a single principal and multiple agents, whose outside option is $u_0$ at the renegotiation stage. It thus unwinds ex post the bargaining power that search frictions bestow agents with ex ante. Bearing this in mind, our last result is simple to understand.

**Proposition 5** For principals uniform rationing dominates ex post bidding.

This claim can be construed as an extension of Proposition 1, for its logic is the same: exposing agents to a lottery over rents is costly. Even if the second-best can be implemented when bidding is active, it cannot be the rationing rule that is used in equilibrium. The proof however differs a bit from that of Proposition 1 because the market utility $\bar{W}$ is endogenous to the rationing rule. So we need to check it is not profitable for a principal to unilaterally move away from a unique contract, and also that the converse is true. If all others use ex post bidding, principal $j$ can profitably deviate by offering a unique transfers under uniform rationing.

### 6 Discussion and conclusion

In this paper many principals compete to attract one of many agents, hence contract offers can in general depend on how many agents get to meet any given principal. That is, principals are allowed to offer menus of contracts contingent on the number of agents who are present at the time of contracting. We show that offering such menus is dominated by a single contract that the principals commit to. Here the reason is that a menu exposes the principals to a lottery over actions, the cost of which is convex. So principals are risk averse over actions and prefer a single contracts
that guarantees them a known action. In turn this result affords us a simple characterization of the optimal contract.

We allow for directed search and moral hazard where the social surplus depends on the action of the agent(s). Precisely because search is directed, search frictions affect the social surplus to be shared, not just the sharing rule. The reason is that principals compete to attract agents by increasing their rents. This weakens the agents’ incentive to exert effort. So markets and frictions matter a great deal in bilateral contracting decisions.

An agent’s rent depends on market tightness. The tighter the market, the harder the principals compete by increasing rents and the less the agents work. A (labor) market may be tight because entry is difficult (say, it requires specific skills) or regulated (e.g. licensing). An OTC market may be tight because it is highly specialized, or may become tight because of an underlying liquidity crisis. Our model shows then prices (spreads) should increase because of frictions, regardless of the risk of a contract. Rents also depend on the principals’ opportunity cost of not hiring an agent. If a relationship is very productive, principals are willing to bid rents up. In this paper search frictions restore some bargaining power on the side of the agents – at a cost. Search frictions, especially competitive search, allows us to derive this bargaining power endogenously.

We believe that combining agency with competitive search creates a natural environment for a canonical model of competitive agency. Search models of monetary policy may benefit of this innovation. It is already known that paying with debt is not the same as paying with cash, not because of record-keeping problems but because of ex post moral hazard (see DeMarzo, Kremer and Skrzypacz (2005) in the context of auctions). That is, there may be reasons that traders have to hold money balances beyond the standard credit rationing explanation. This is left to future work.
Appendix

This Appendix has two parts. The first one contains the proofs. In the second one the reader one can find additional material that supports some claims in the text.

A Proofs

Proof of Proposition 1: Suppose all other $N - 1$ principals use a single tariff $t(x)$. Form the
Lagrangian of

\[
\max_{\{t(x), a_n\}_{n=1}^N} \sum_{n=1}^N B_n(N, \sigma^j)\pi^j(t_n, a_n)
\]

s.t.

\[
\int_{X} u(t_n^j(z))dF(z|a_n) - c(a_n^j) \geq u_0, \quad \forall \ n \leq N \quad (A.1)
\]

\[
\int_{X} u(t_n^j(z))da_n F(z|a_n) - c'(a_n^j), \quad \forall \ n \leq N \quad (A.2)
\]

and

\[
W^j(\sigma^j(U)) \geq W(\sigma^*(U)) \quad (A.3)
\]

with $\sigma + (M - 1)\sigma^* = 1$. Attach multipliers $\lambda, \mu, \nu$ to each of these constraints. The impact of any $t_n$ on $\sigma$ can be ignored (envelope property). Let $\psi(n, N, \sigma^j) \equiv B_n(N, \sigma^j)$; the first order condition with respect to any $t_n(x)$ is

\[
\frac{d\psi}{d\sigma}\delta\sigma\pi - \psi f + \lambda u'f + \mu u'f + \nu \left[\frac{d\psi}{d\sigma}\delta\sigma U^j + \psi u'f + \delta u_0 \left[ N \left( (1 - \sigma)^{N-1} - \sigma^{N-1} \right) \right] \right] = 0
\]

However one can also derive with respect to $\sigma$ for any $t_n$:

\[
\frac{d\psi}{d\sigma}\pi + \nu \frac{d\psi}{d\sigma} U^j + u_0 \left[ N \left( (1 - \sigma)^{N-1} - \sigma^{N-1} \right) \right] = 0,
\]

so that any variation $\delta\sigma$ can be ignored. Hence the first-order condition with respect to the transfers $t_n(x)$ simplifies to

\[
\frac{1}{u'(t_n(x))} = \tilde{\lambda} + \tilde{\mu} \frac{f_{a_n}}{f} + \nu, \quad n = 1, 2, ..., N \quad (A.4)
\]

where $\tilde{\lambda} = \lambda \cdot \sigma^j N/B_n(N, \sigma^j) \geq 0$, $\tilde{\mu} = \mu \cdot \sigma^j N/B_n(N, \sigma^j) > 0$ and $\nu \geq 0$. From Jewitt, Kadan and Swinkels (2008, now JKS) we know that fixing action $a_n$ the solution $t_n$ to this equation is unique.
for each $n$. To see why here, fix $a$, then $\tilde{\mu}$ is fixed, so by (A.4) and monotonicity of $u$, $t_n$ must be unique. By the first-order condition $U_a = 0$ (under the conditions of the FOA), we also know that the (agent-) optimal action $a^*$ is unique for a given transfer function $t_n$.

To show the equilibrium contract is a fixed transfer and action independent of the matching states, let $v(x) \equiv u(t(x))$, $\eta = \nu + \tilde{\lambda}$ and $L(\cdot)$ denote the Lagrangian of the cost-minimization problem. The effective cost of implementing action $a_n$ defined as

$$C(a_n) = \max_{\eta, \tilde{\mu}} \min_{v} L(v)$$

$$= \max_{\eta, \tilde{\mu}} \eta \left[ W(\sigma(U)) + c(a_n) \right] + \tilde{\mu}c'(a_n) - \int \rho \left( \eta + \tilde{\mu} \frac{f_{an}}{f} \right) dF(z|a_n)$$

is convex in $a_n$ (see JKS). Here we make use again of the agent’s first-order condition $U_a = 0$ in the $W(\cdot)$ term. The first line is an application the Lagrange duality theorem. In the second line $\rho(y) \equiv \max_{v} (yv - u^{-1}(v))$ is a convex function for any $y$. Take any $a_1 \leq a_2 \leq \ldots \leq a_N$ (w.l.o.g.) induced by $t_1 \leq t_2 \leq \ldots \leq t_N$ and define

$$E[C(a_n)] = \sum_{n=1}^{N} \Pr(a_n)C(a_n)$$

This is a convex function for it is necessarily bounded (below and above). Furthermore, there also exists a convex function

$$C(E[a]) \equiv C \left( \sum_{n=1}^{N} \Pr(a_n)a_n \right)$$

For each $n$, let $a^*_n$ denote the optimal action; then $E[a^*] = \sum_{n=1}^{N} \Pr(a_n)a^*_n$ and

$$E[C(a^*_n)] \geq C(E[a^*])$$

which contradicts the premise that $t_1 \leq t_2 \leq \ldots \leq t_N$ are optimal given $E[a^*]$. ■

**Proof of Proposition 2:** Form the Lagrangean of Problem 1 where the multipliers are $\lambda, \mu$ and $\nu$ (on Condition (3.3)); optimizing with respect to $\theta$ allows us to ignore implicit effects of $(t, a)$ on $\theta(U)$, $U = (U^1, \ldots, U^M)$.

$$\begin{align*}
[1 - e^\theta] \left[ \pi_a + \lambda U_a + \mu U_{a0} \right] + \nu \frac{1 - e^\theta}{\theta} U_a &= 0 \\
[1 - e^\theta] \left[ -f + \lambda u' f + \mu u' f_a \right] + \nu \frac{1 - e^\theta}{\theta} u' f &= 0 \\
e^{-\theta} \left[ \pi + \lambda (U - u_0) + \mu U_a \right] + \nu \left[ \theta e^{-\theta} - (1 - e^{-\theta}) \right] \left( U - u_0 \right) &= 0
\end{align*}$$

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where \( \theta \equiv \theta(U) \), \( \delta \pi = -f, \delta U = u'f \), and \( \delta U_a = u'_af \). The complementary slackness conditions associated with (3.3) and (3.4) read \( \nu(W - \bar{W}) = 0 \) and \( \lambda(U - u_0) = 0 \). Using \( U_a = 0 \) immediately yields (3.6) from the first condition. Next, suppose \( \lambda > 0 \) then \( U = u_0 \) by complementary slackness; combining with \( U_a = 0 \) generates a contradiction by the third condition. So \( \lambda = 0 \) necessarily. To show \( \nu > 0 \), rewrite the second condition as
\[
\frac{\theta}{u'} = \mu \frac{f_a}{f} + \nu
\]
and integrate over \( \mathcal{X} : \nu = \mathbb{E}_\mathcal{X} \left[ \frac{\theta}{u'} \right] > 0 \). Then by the third condition we have \( \nu = \frac{\theta^2 e^{-\theta}}{1-e^{-\theta}-e^{-\pi}} \frac{\pi}{u-u_0} \). Substitute in the second condition and re-arrange. It is easy to show that \( \mu > 0 \) by adapting the proof of Jewitt (1988); see Roger (2014). Thus (3.7) is established. Roger (2014) shows that the term \( \frac{\theta^2 e^{-\theta}}{1-e^{-\theta}-e^{-\pi}} \frac{\pi}{u-u_0} \) can be treated as a Lagrange multiplier, so that (3.7) rewrites
\[
\frac{1}{u'} = \alpha + \mu \frac{f_a}{f} , \quad \alpha \in \mathbb{R}
\]
where \( f_a(x)/f(x) \) is solely determined by the action \( a \) and the multipliers \( (\alpha, \mu) \) are known to be unique.

Proof of Lemma 1: Social welfare \( W(a) \) rewrites
\[
W(a) \equiv \max\{M, N\} \cdot \left(1 - e^{-\theta}\right) \left[ \int_\mathcal{X} zdF(z|a) - T(a) + U(t, a) \right] + \left[M \cdot e^{-\theta} + (N - M)\mathbb{I}_{N>M}\right] u_0
\]
where \( T(a) \equiv \int t(x)dF(x|a) \) is known to be an increasing, concave function (Conlon, 2008). In equilibrium, \( \theta \equiv \theta(U) = \Theta \). Differentiate with respect to the action
\[
\frac{dW}{da} = \max\{M, N\} \cdot \frac{d\theta}{dU} U_a e^{-\theta} \left[ \int_\mathcal{X} zdF(z|a) - T(a) + U(t, a) \right] - u_0 + \max\{M, N\} \cdot \left(1 - e^{-\theta}\right) \left[ \int_\mathcal{X} zdF_a(z|a) - T_a + U_a \right]
\]
\[
= \max\{M, N\} \cdot \left(1 - e^{-\theta}\right) \left[ \int_\mathcal{X} zdF_a(z|a) - T_a \right],
\]
since \( U_a = 0 \). Therefore \( \frac{dW}{da} > (\leq)0 \Leftrightarrow \int_\mathcal{X} zdF_a(z|a) - T_a > (\leq)0 \). Because the multiplier \( \mu \) is known to be positive, the first-order conditions (3.6) and (5.3) immediately tell us that \( \int xF_a(x|a) - T_a > 0 \).

Proof of Proposition 3: The statement we want to prove is this: consider a rent level \( \bar{U} \in [u_0, U) \), where \( U \) is such that \( \|(1 - e^{-\theta})/\theta\|U + [1 - \|(1 - e^{-\theta})/\theta\]|u_0 = \bar{W} \); to the rent level \( \bar{U} \) must correspond a higher action.
First note that for each action \( a \), there exists a unique \( x' \) such that \( f_a(x')/f(x') = 0 \) the ratio in increasing and \( \int f_a(z|a)dz = 0 \). Let \( U(t^U, a^U) \) be such that Condition (3.3) – the market-induced participation constraint – binds. Fix the action \( a^U \) so that the distribution \( F(\cdot|a^U) \) is fixed. For any \( \overline{U} < U(t^U, a^U) \), construct an alternative transfer scheme \( t \) such that \( U(t, a^U) = \overline{U} \) (this needs not be an optimal scheme). There is some \( \tilde{x} \in \mathcal{X} \) such that
\[
\begin{align*}
t(x) &= \begin{cases} 
< t^U(x), & \text{for } x < \tilde{x}; \\
= t^U(x), & \text{for } x = \tilde{x}; \\
> t^U(x), & \text{for } x > \tilde{x}.
\end{cases}
\]
that is, \( t(\cdot) \) single-crosses \( t^U(\cdot) \) from below at the point \( \tilde{x} \). Because the action \( a^U \) remains fixed, \( \tilde{x} = x' \). So to the left of \( \tilde{x} \), \( f_a(x)/f(x) < 0 \) while to its right \( f_a(x)/f(x) > 0 \). Now,
\[
\begin{align*}
f_a(\tilde{x}) = c'(a^U) = \frac{1}{f} \int_{\tilde{x}}^\infty \frac{f}{f} dF(z | a^U) = 0.
\end{align*}
\]
since \( f_a(\tilde{x}) = c'(a^U) \) by the moral hazard constraint and the facts that \( u(t^U(x)) - u(t(x)) > 0 \) and \( f_a/f < 0 \) to the left of \( \tilde{x} \), and conversely to its right. So \( t(x) \) clears the moral hazard constraint (3.5). By concavity of \( U(\cdot, \cdot) \) and since \( c(a) \) is increasing and convex, \( U(t, a) < U(t^D, a^D) = \overline{U} \). ■

**Proof of Proposition 4:** Fix \( M, N \). In light of Proposition 1 one can use a unique tariff \( t(x) \).

The problem of a utilitarian planner is

**Problem 4**
\[
\max_{t,a} \left[ M (1 - e^{-\theta}) \pi(t, a) + N \left[ \frac{1 - e^{-\theta}}{\theta} U(t, a) + \left( \frac{1 - e^{-\theta}}{\theta} \right) \right] \right]
\]
The MUP (3.3) does not enter the planner’s problem because he can dictate the terms of trade to the principals. Attach multipliers $\beta, \gamma$ to each of these constraints. The first-order conditions read

$$M(1 - e^{-\theta})\pi_a + N\frac{1 - e^{-\theta}}{\theta}U_a + \beta U_a + \gamma U_{aa} = 0,$$

where the two $U_a$ terms are zero, and where $M = N/\theta$ by definition. This condition thus displays the standard envelop property. The second one reads

$$-M(1 - e^{-\theta})f + M\frac{1 - e^{-\theta}}{\theta}u'f + \beta u' f + \gamma u' f = 0,$$

which rewrites

$$\frac{1}{w'} - 1 = \hat{\beta} + \hat{\gamma} \frac{f_a}{f},$$

where $\hat{\beta} = \frac{\beta}{M(1 - e^{-\theta})}$ and $\hat{\gamma} = \frac{\gamma}{M(1 - e^{-\theta})}$ are just re-scalings. The point of interest is the complementary slackness condition

$$\beta [U(t, a) - u_0] = 0,$$

which establishes whether any level of rent is distributed to agents. For some value $\tilde{x}$, $u'(t(\tilde{x})) = 1$ and so at that point $\beta + \gamma \frac{f_a}{f} = 0$ too.

Case 1: $\beta = 0$.

Then necessarily $\tilde{x}$ is associated to some action $a_1$ such that $f_a(\tilde{x}|a_1)/f(\tilde{x}|a_1) = 0$.

Case 2: $\beta > 0$.

In this case $\tilde{x}$ is associated to some action $a_2$ such that $f_a(\tilde{x}|a_2)/f(\tilde{x}|a_2) = -\beta < 0$, which means that the likelihood ration $f_a/f$ crosses 0 to the right of $\tilde{x}$. That is to say $F(x|a_2)$ first-order stochastically dominates $F(x|a_1)$. By application of Lemma 4, it is preferred by the planner.

Proof of Lemma 2: Under symmetric information $a^*_n$ is fixed by the terms of the contract; as the solution of the principal’s optimization problem it necessarily corresponds to $a^P$. Under moral hazard, by (3.5) and the conditions of the first-order approach, if there exists an optimal action $a^P$ preferred by the principal, then there exists a (unique) transfer $t$ that implements $a^P$ (see JKS). As shown by Conlon (2008)

$$T(a) \equiv \int_{\mathcal{X}} t(z) dF(z|a)$$

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is monotone increasing and concave in $a$. Therefore, for any action $a$ an expected transfer defined by $T(a)$ exists, is well defined and is unique. Bertrand competition in transfers ensures that $U^j(a^P) = u_0$ and that any bid $t^j$ such that $a^*_n < a^P$ is dominated. ■

**Proof of Lemma 3:** As in the Proof of Proposition 2. ■

**Proof of Proposition 5:** Under uniform rationing, a principal’s expected cost of implementing action $a_P$ is $(1 - e^{-\Theta})C(a_P)$. Under ex post bidding, the only two relevant events are $n = 1$, which the principal confronts with unconditional probability $1 - e^{-\Theta} - \Theta e^{-\Theta}$ and $n > 1$, with unconditional probability $\Theta e^{-\Theta}$. Thus, conditional on contracting, the cost of $a_P$ is $C(a_P)$, while the cost of the lottery over $(a_A, a_{SB})$ is

$$
(1 - e^{-\Theta}) \left[ \frac{\Theta e^{-\Theta}}{1 - e^{-\Theta}} C(a_A) + \left(1 - \frac{\Theta e^{-\Theta}}{1 - e^{-\Theta}} \right) C(a_{SB}) \right]
$$

Apply Proposition 1, it is immediate that

$$
\mathbb{E}[C(a^*)] = \frac{\Theta e^{-\Theta}}{1 - e^{-\Theta}} C(a_A) + \left(1 - \frac{\Theta e^{-\Theta}}{1 - e^{-\Theta}} \right) C(a_{SB}) \geq C \left( \frac{\Theta e^{-\Theta}}{1 - e^{-\Theta}} a_A + \left(1 - \frac{\Theta e^{-\Theta}}{1 - e^{-\Theta}} \right) a_{SB} \right) = C (\mathbb{E}[a^*])
$$

Therefore an action $a' \geq \frac{\Theta e^{-\Theta}}{1 - e^{-\Theta}} a_A + \left(1 - \frac{\Theta e^{-\Theta}}{1 - e^{-\Theta}} \right) a_{SB}$ can be implemented at a cost no higher than $\mathbb{E}[C(a^*)]$. ■

**B Additional material**

**B.1 Small market: validation**

There is a finite number $N$ of agents and a finite number $M$ of principals. The rest of the environment is as described in the main text. To solve for an equilibrium in finite market, we postulate a candidate (symmetric) equilibrium contract $C^* = \{t^*_n(x), a^*_n\}_{n=1}^N$, posted by all Principals $k \in M \setminus j$, and consider the benefit of Principal $j$ deviating to contract $C^j = \{t^j_n(x), a^j_n\}_{n=1}^N$.

To simplify notation let $\sigma^j = \sigma$. Furthermore, let $B_n(N, \sigma) = \binom{N}{n} \sigma^n (1 - \sigma)^{N-n}$ the probability of $n$ agents selecting the deviating Principal. Similarly, let $B_n(N - 1, \sigma) = \binom{N-1}{n} \sigma^n (1 - \sigma)^{N-n-1}$ be the probability of $n$ other agents selecting the deviating Principal from the perspective of an agent
considering selecting that Principal. Both these functions are clearly continuous and differentiable in \( \sigma \). Noting that \( B_n(N-1,\sigma) = \frac{B_n(N,\sigma)}{\sigma N} \), the deviating Principal solves:

\[
\max_{\{t_n,a_n\}_{n=1}^N,\sigma \in (0,1)} \sum_{n=1}^N B_n(N,\sigma)\pi(t_n(x),a_n) \\
\text{s.t. } U(t_n,a_n) \geq u_0 \quad \text{(B.1)} \\
a_n \in \arg \max U(t_n,a_n) \quad \text{(B.2)} \\
W_P(\sigma) \geq W_P(\sigma^*) \quad \text{(B.3)} \\
\sigma + (M-1)\sigma^* = 1 \quad \text{(B.4)}
\]

where

\[
W_P(\sigma) \equiv \sum_{n=1}^N \frac{B_n(N,\sigma)}{\sigma N}(U(t_n,a_n) - u_0) + u_0,
\]

and

\[
W_P(\sigma^*) \equiv \sum_{n=1}^N \frac{B_n(N,\sigma^*)}{\sigma^* N}(U^*(t_n^*,a_n^*) - u_0) + u_0.
\]

\( \sigma^* \) represent agents selection strategy for any non deviating Principals. Note that it is responsive to \( \sigma \) so the Market Utility Property may not be applied. The Lagrangian for the problem is:

\[
\max_{\{t_n,a_n\}_{n=1}^N,\sigma \in (0,1)} \sum_{n=1}^N B_n(N,\sigma)\pi(t_n(x),a_n) + \mu U(a_n) + \lambda(U - u_0) + \nu[W_P(\sigma) - W_P(\sigma^*)].
\]

We optimize over \( \sigma \) as in Proposition 1 (envelope condition). The necessary conditions are:

\[
t_n : \pi t_n + \mu U a_n + \lambda U t_n + \nu \frac{U t_n}{\sigma N} = 0, \forall n \leq N
\]

\[
a_n : \pi a_n + \mu U a_n + \lambda U a_n + \nu \frac{U a_n}{\sigma N} = 0, \forall n \leq N
\]

\[
\sigma : \sum_{n=1}^N \frac{\partial B_n(N,\sigma)}{\partial \sigma} [\pi + \mu U a_n + \lambda(U - u_0)] + \nu \left[ \sum_{n=1}^N \frac{\partial B_n(N,\sigma)}{\partial \sigma} (U - u_0) - \sum_{n=1}^N \frac{\partial B_n(N,\sigma^*)}{\partial \sigma} \frac{\partial \sigma^*}{\partial \sigma} (U^* - u_0) \right] = 0
\]  

\( \frac{\partial \sigma^*}{\partial \sigma} = -\frac{1}{M-1} \). The complementary slackness conditions associated with (B.3) and (B.1) read \( \nu(W - W^*) = 0 \) and \( \lambda(U - u_0) = 0 \). Since we focus on symmetric equilibrium where \( \sigma = \sigma^* = \frac{1}{M} \),

\( U = U^* \), \( t_n = t_n^* \) and \( a_n = a_n^* \) for all \( n \), (B.5) becomes:

\[
\sum_{n=1}^N \frac{\partial B_n(N,\sigma)}{\partial \sigma} [\pi + \mu U a_n + \lambda(U - u_0)] + \nu \left[ \sum_{n=1}^N \frac{\partial B_n(N,\sigma)}{\partial \sigma} \frac{M}{M-1} (U - u_0) \right] = 0
\]

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Next, suppose $\lambda > 0$ then $U = u_0$ by complementary slackness; combining with $U_{a_n} = 0$ generates a contradiction by the third condition. So $\lambda = 0$ necessarily. To show $\nu > 0$, rewrite (B.5) again as

$$\nu = \frac{\sum_{n=1}^{N} \frac{\partial B_n(N, \sigma)}{\partial \sigma} \pi(t_n, a_n)}{- \sum_{n=1}^{N} \frac{\partial B_n(N, \sigma)}{\partial \sigma} \frac{M}{M-1} (U(t_n, a_n) - u_0)}.$$  

It is easy to show that $\frac{\partial B_n(N, \sigma)}{\partial \sigma} > 0$ and $\frac{\partial B_n(N, \sigma)}{\partial \sigma} < 0$ for all $n$. Therefore $\nu > 0$ and is unique and independent of $n$. Rewriting the first two necessary conditions again using $\sigma = \frac{1}{M}$ and $\Theta = \frac{N}{M}$,

$$\frac{1}{u_{t_n}} = \frac{\nu}{\Theta} + \mu \frac{f_n}{f}, \quad \forall n \leq N \quad \text{(B.6)}$$

$$\pi_{a_n} + \mu U_{a_n a_n} = 0, \quad \forall n \leq N.$$  

Here too we can make use of Proposition 1, which is not specific to a large market. Then things simplify further:

$$\nu = \frac{(1 - \frac{1}{M})^{N-1} \pi}{\frac{1}{M-1} (U - u_0)} > 0.$$  

Substituting in (B.6) yields

$$\frac{1}{u_t} = \mu \frac{f_a}{f} + \frac{\Theta (1 - \frac{1}{M})^{N-1} \pi}{\left(1 - (1 - \frac{1}{M})^{N} - \Theta (1 - \frac{1}{M})^{N-1} \frac{M}{M-1} (U - u_0)\right)},$$

and taking the limit as $N, M \to \infty$ (but maintaining $\Theta$ finite)

$$\frac{1}{w'} = \lambda + \mu \frac{f_a}{f} + \frac{\Theta e^{-\Theta}}{1 - e^{-\Theta} - \Theta e^{-\Theta}} \frac{\pi}{U - u_0}$$

just as in the large market.

**B.2 Symmetric information**

In the main text we show that search frictions are very costly because they lead principals to present the agents with contracts that distort their effort, and therefore lowers the total surplus to be shared. It is also true under symmetric information. To simplify the exposition we dispense with much of the analysis, most of which is carried out in the previous Sections.

**Claim 1** Under symmetric information, the optimal transfer and the optimal action are characterized by the conditions

$$\frac{1}{w'(t)} = \frac{\theta e^{-\theta}}{1 - e^{-\theta} - \theta e^{-\Theta}} \frac{\pi}{U - u_0} \quad \text{(B.7)}$$
and
\[ \frac{1}{u'(t)} = \frac{E_a[x|a]}{c'(a)} \]  \hspace{1cm} (B.8)

**Proof:** Note first that Condition (B.7) is also sufficient for the LHS increases in \( t \) while the RHS decreases. When the agent’s action is observable the constraints become
\[ U = u(t) - c(a) \geq u_0 \]  \hspace{1cm} (B.9)
where \( t \in \mathbb{R} \) is a scalar, and
\[ W^j \geq \hat{W} \]  \hspace{1cm} (B.10)
where the arguments entering (B.10) also reflect symmetric information. The problem becomes

**Problem 5**
\[ \max_{t,a,\theta} \left[ 1 - e^{-\theta} \right] \cdot \left[ \int_X zdF(z|a_n) - t_n \right] \]
\[ \text{s.t. (B.9) and (B.10)} \]

Let \( \lambda, \nu \) be the multipliers of (B.9) and (B.10), then
\[ (1 - e^{-\theta})[\pi_t + \lambda U_t] + \nu \left( \frac{1 - e^{-\theta}}{\theta} \right) U_t = 0 \]
\[ (1 - e^{-\theta})[\pi_a + \lambda U_a] + \nu \left( \frac{1 - e^{-\theta}}{\theta} \right) U_a = 0 \]
\[ e^{-\theta} \pi + \nu \left( \frac{e^{-\theta} \theta - (1 - e^{-\theta})}{\theta^2} \right) (U - u_0) = 0 \]
where \( \theta \equiv \theta(U), \pi_t = -1, \pi_a = E_a[x], U_t = u'(t) \) and \( U_a = -c'(a) \) – this is complete information and \( t \in \mathbb{R} \). From the last condition we must have \( \nu > 0 \), which immediately implies that \( \lambda = 0 \) by complementary slackness (i.e. \( \lambda(U - u_0) = 0 \)). The first two conditions jointly yield the Pareto-optimality condition; we also have \( \nu = \theta/u' = \theta \cdot E_a[x|a]/c'(a) = \frac{\theta^2 e^{-\theta}}{1 - \theta e^{-\theta} - e^{-\theta} \cdot \pi_F} \).  \( \Box \)

Condition (B.8) asserts that the allocation is Pareto-efficient. However, because of search frictions, the agents are able to keep a rent: \( U > u_0 \). Absent search frictions, that is, when (B.10) needs not be satisfied, the optimality conditions are (B.8) and \( \lambda = 1/u' \). So the allocation does entail a welfare loss that is attributable to principal competition with frictions.

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References


[22] Rochet, J.-C. (2008) “Why are there so many Banking Crises?” *Princeton University Press*


